THE

The Arithmetician's GUIDE:

Being a NEW, IMPROVED, and COMPENDIOUS

SYSTEM OF PRACTICAL ARITHMETIC.

Designed either for the Use of SCHOOLS, or the benefit of private Persons; and adapted to the Capacities of Beginners.

In THREE PARTS.

PART I.
Containing Definitions, Axioms, and all the Rules in Arithmetic of Whole Numbers.

PART II.
Containing Vulgar and Decimal Arithmetic, variously applied.

PART III.
Consists of Geometrical Definitions and Mensuration.

By JOHN EADON,
Writing Master and Teacher of the Mathematics, in Sheffield.

SHEFFIELD:
Printed for W. WARD; and Sold by S. CROWDER, and J. WILKIE, in *London*; and by the Author.

MDCCLXVI.

This Book belongs to
Matthew Jepson, Cooper
Ecclesfield, having bought
John Darlings, share and
John Jepson presented
him with his Share

October 25th 1822

M. J.

TO THE WORTHY
TRUSTEES,
OF THE
Free Writing School,
IN
SHEFFIELD.

GENTLEMEN,

THE Experience I have had of your Zeal in the faithful discharge of your Trust, the concern you seem to have for the Improvement of Youth placed at this SCHOOL *for* Education, *and your Dispositions to promote every Design that tends to the Advancement of useful Literature, make me hope, that this Treatise, which aims that way, may venture to shelter itself under your Patronage.*

That

DEDICATION.

That a mutual Agreement and Harmony may long ſubſiſt amongſt you, that you may long continue an Honour to your Town, Encouragers of Science, Benefactors to Society, Happy in Yourſelves, your Families and every other circumſtance of Life, is the ſincere Wiſh of him, who with the deepeſt ſenſe of Gratitude, ſubſcribes himſelf,

Your Faithful, Obedient,

And moſt humble Servant,

J. EADON.

PREFACE.

HE muſt be little verſed in the common affairs of life, who does not know the great uſefulneſs of Arithmetic: as no buſineſs can be carried on without the help of numbers; no trade or commerce exerciſed without regular accompts. And as the utility of Arithmetic is thus extenſive, ſo there are extant many Treatiſes on the ſubject, ſome of which are excellent in their method. But in my opinion there is ſtill room for a general, conciſe, and rational piece, better adapted to the uſe of Schools, as well as of thoſe who have not the aſſiſtance of a Maſter: for all the Treatiſes that have occurred to me, have either all, or none of their examples wrought at large; which, in the firſt place, renders them unfit for a Maſter in his School; and in the ſecond place, not fit for thoſe who would acquire a competent ſkill in Arithmetic, without the aſſiſtance of a teacher. To remove this defect, is the intention of the following Treatiſe; and in what manner my deſign is executed, is wholly ſubmited to the impartial Reader.

In the courſe of this undertaking, I have exerted my utmoſt endeavours, to deliver the definitions and rules in as brief a manner as poſſible; to make them, at the ſame time general, and yet free from ſuperfluities, which commonly attend definitions of this kind; and have given ſuch notes after them, as deſcribe ſome particulars not flowing from the general rules themſelves, but tending to explain them, or facilitate the operations. To

each rule alſo there is annexed, a great variety of the beſt choſen examples, with their anſwers, the greateſt part of which are new, and which the reader cannot fail of underſtanding, by means of the previous examples wrought at large. After the four firſt rules in ſimple and compound numbers, I have expreſſed the operations in algebraic ſigns, which in my opinion is far the beſt method, for it not only ſhortens the work, but the operation, when finiſhed, is itſelf a rule by inſpection.

And it may be noted, that, where fractions are in the anſwers to any of the examples in the rules preceding vulgar fractions, they are wrote down as they aroſe without any abbreviations; but all fractions in the anſwers afterwards are abbreviated as much as poſſible. I have treated the four fundamental rules, viz. Addition, Subtraction, Multiplication, and Diviſion, firſt in ſimple numbers, and afterwards in compound; for by teaching them in this order, difficulties do not ſo faſt ariſe. Yet I have delivered them in ſuch a manner, as to have little or no dependence on each other, that they may be taught in what order every Maſter chuſes.

I think it quite needleſs for the firſt ſimple rules to be copied out into the Scholars books; but if any Maſter chuſes to have it done, I would adviſe him to make his Scholars run over the examples firſt upon their ſlates; then begin the rules again, and write them, with a few examples to each, in their books; and thus he may with eaſe mix the ſimple and compound rules together. Tho' I ſaid before, there is nothing ſuperfluous in any of the problems or general rules; yet the judicious teacher may omit any notes, examples, or particular caſes he may think fit. And if every Scholar has a printed book, he may either

either write all the rules and examples in his accompt book, or may omit the Rules, Notes, and Tables, and write down only the operation of each example, especially where the rules are long and tedious. This book may also be of peculiar service to boys when taken from School, either in making a farther progress by their own application, or in the retainment of what they have already learned. I have, contrary to the practice of some Masters, inserted several wrought examples in each rule; having found by repeated experience, that a Scholar will sooner get both the rule and its meaning, by seeing a few examples wrought, than by any other method. But any Master, if he pleases, may soon make new questions of those that are solved, by only changing a figure in the data; and when his Scholars have gone through the operation, may make them invert the question in order to prove the truth of their work; that is, make a new question to prove the old one; and this will be no bad exercise for them.

This method of altering the data of a question is of great utility in a School; for, as Mr. Dilworth observes, "some boys, lazily inclined, when they see another at "work upon the same question, will be apt to make his "operation pass for their own." But these little forgeries are soon detected by the vigilance of the Tutor. And this is best and most expeditiously performed by changing a figure in the data; for then the Scholar must of course go thro' the whole operation in order to obtain the solution, as also thro' a second operation, in order to prove the truth of the first.

It is much the best way to enter upon Vulgar and Decimal Fractions, immediately after Reduction, if time will

will permit; but there are a great number of Scholars that have not time to learn Vulgar and Decimal Fractions with their various application, who yet of course, ought to learn the rule of Three, Practice, Tare and Tret, Interest, Rebate or Discount, and Equation of Payments, so essentially requisite in the common affairs of Life; and for that reason I have treated them in whole numbers: but the judicious Master may teach them in what order he pleases, regard being had to the capacity of his Pupil, and the time he has to spend therein. I have omited Duodecimal Arithmetic, because its place is better supplied, in all cases, by Decimal Fractions.

In this Treatise are contained many useful and interesting particulars, which might here be mentioned, but I rather chose to refer to the Work, and to let the Book speak for itself, only requesting that theReader will not too hastily censure and condemn it; but that after impartially perusing it, he will candidly excuse such defects as may occur to him, and have escaped my observation; and if he should find a *transposition* of a *letter*, or a false figure, to excuse it, as Errors of the Press will inevitably creep in, and some may have escaped my notice.

It is not in the least pretended that all things treated of here are new; for I have collected some (suitable to my plan) from the Writings of my predecessors, and here make this public acknowledgement for the assistance.

N. B. Those who may have occasion to learn Merchants Accompts, I would recommend to their perusal, ROOSE's BOOK-KEEPING, as being in my opinion, the best extant, both in Theory and Practice.

CONTENTS.

ERRATA.

The third Queſtion in Double Fellowſhip, page 189, *read as follows.*

Six merchants trade after this manner. A puts in 50l. for 6 months, and 60l. for 4 months. B puts in 90l. for 8 months. C puts in 160l. for 5 months, and 100l. for 4 months. D puts in 200l. for 7 months, and 100l for 5 months. E puts in 300l. for 10 months, and 100l. for 2 months. F puts in 400l. for 3 months, 200l. for 4 months, and 200l. for 5 months. They gained 686,4l. what is the ſhare of each merchant.

Page	Line	for	read
8	3 b	490720	490740
18	4	1166	1186
29	9	10 years	100
35	10	21 poles	31 poles.
39	9 b	x×x	x×8
43	3b	174l 6s. $2\frac{1}{2}$d	874l. 6s. $2\frac{1}{2}$d.
51	13 b	1882471	1883471
76	6	4l. 10s.	14l 10s.
77	10	continued	contained
79	1 b	3l. 9s. $2\frac{1}{4}$d. Anſw.	83l. 9s. $2\frac{1}{4}$d. Anſw.
88	11	Anſ. 218l. 19s. $3\frac{1}{2}$d.	Anſw. 222l. 15s. $0\frac{1}{2}$d.
89	13	14l. 7s. 6d.	14l 17s. 6d.
97	10	3 cwt. Cloff.	1 cwt. Cloff.
100	5	13l. 13s.	13l. 10s.
128	10 b	334	336
	11 b	383 the com. Denom.	384 the com. Denom.
183	5	Anſw. $27\frac{2}{7}$	Anſw. $28\frac{2}{7}$
238	7&9 b	384 42 & for 384,3	384
244	6	24975	499500
245	8	4s.	3s.
252	9	$\frac{6144 \div 3 = 2072}{}$	$\frac{6144 \div 3 = 2048}{}$
253	2 b	6144—3÷2	6144—3÷2—1,
255	9	12285×048	12285×2048

Note, b denotes from the bottom.

The Reader is deſired to Correct theſe.

John

INTRODUCTION,

MATHEMATICS originally ſignified any diſcipline or learning, (Matheſis :) But now it is that ſcience which contemplates and treats of all kinds of quantities that are capable of being numbered or meaſured. That part which relates to number only, is called *Arithmetic* ; and that which relates to meaſure in general, whether length, breadth, motion, force, &c. is called *Geometry*. When theſe two are converſant about multitude and magnitude abſtractedly conſidered, they are called pure or abſtract Mathematics, and are the foundation of all the other parts. When they are applied to particular ſubjects, they are called mixt Mathematics. Mathematics are alſo called ſpeculative, ſo far as they are concerned in finding out true propoſitions ; and practical, as they relate to uſe, and are applied to practice.

QUANTITY is whatever will admit of augmentation and diminution ; or is capable of any ſort of eſtimation or menſuration.

A PROPOSITION is ſomething propoſed to be proved or demonſtrated.

A THEOREM is a demonſtrable propoſition laid down as an acknowledged truth; and a ſet of ſuch theorems is called a Theory.

A PROBLEM is a queſtion requiring ſomething to be done. A limited Problem is that which has but one anſwer. An unlimitted Problem is that which has an

 infinite

infinite number of anſwers. A determinate Problem is that which has a certain number of anſwers.

SOLUTION of a Problem is the anſwer given to it. A numerical Solution is the anſwer in numbers. A geometrical Solution is an anſwer by the principles of Geometry. A mechanical Solution is one which is gained by trials.

A LEMMA ſignifies a propoſition, which ſerves previouſly to prepare the way for the more eaſy apprehenſion of the demonſtration of ſome Theorem.

A CORROLLARY, or *Conſectary*, is a conſequence drawn from a propoſition already demonſtrated.

A SCHOLIUM is a remark made on any propoſition, corrollary or diſcourſe.

PRINCIPLES are the firſt grounds, rules, or foundations, of any Science; as Definitions, Axioms, Poſtulates, and Hypotheſes.

A DEFINITION is our explication of any word or term in any Science; which explication ought to be clear, and to contain no word or term, but what is already underſtood.

An AXIOM, or *Maxim*, is ſuch a common, plain, ſelf-evident propoſition, as nobody can deny without contradicting common ſenſe and reaſon. And nothing ought to be allowed for an Axiom but what is thus clear and ſelf evident: as, *that nothing can act where it is not*; *that a thing cannot be, and not be, at the ſame time*; *that the whole is greater than a part thereof*; *that no body can naturally go into nothing*: Out of an infinite number of ſelf evident truths that occur

occur to the mind, men select such as are general, and of most use in demonstrating any science, and lay them up in store to have recourse to, as need requires. And tho' in their reasoning they do not always mention such and such Axioms; yet the mind perceives the force of them, without stopping to hear them particularly repeated or named.

A POSTULATE, or *Petition*, is something required to be granted, which is too evident to be denied.

An HYPOTHESIS is a supposition assumed to be true, by which a man is to argue, and build his reasoning upon.

DEMONSTRATION, is a chain of arguments depending on one another, and founded primarily on first and self evident Principles, or plain Propositions established and proved from them, and at last ending in the invincible proof of the thing to be demonstrated.

METHOD is the art of disposing a train of arguments, in right order, either to find out the truth, or falsehood of a Proposition; or to demonstrate it to others when we have found it out. This is either analytical or synthetical.

ANALYSIS, or the *Analytic Method*, is the art of finding out the truth of a Proposition, by supposing the thing to be done; and going back step by step, till we arrive at some known truth. This is called the *method of invention*, and is generally used in Algebra.

SYNTHESIS, or the *Synthetic Method*, is the searching out truth, by first laying down some simple and easy principles, and pursuing the consequences till we come at the conclusion. This method begins at the most simple

 and

and easy things, and proceeds to the more compound and general. It is also called the method of composition, and is contrary to the Analytic method; as this proceeds from known principles to an unknown conclusion; whilst the other goes in a retrograde order from the thing sought, as if it was known, to some known principles. And therefore when any truth has been found out by the Analytic method; it may be demonstrated in a backward order, by Synthesis.

The Signification and Explanation of the Signs, or Characters, used in this Treatise.

CHARACTERS.	SIGNIFICATION.
$+$	MORE, or to be added, being an affirmative sign. Thus $9+6$ signifies 6 added to 9; and $a+b$ denotes a added to b, or the sum of a and b. The like must be understood when several numbers are connected with the sign $+$ as $7+4+63+72$. Letters with a line drawn over them, thus $\overline{a+b+c+d}$ signifies the Sum of a, b, c, and d.
$-$	LESS, or to be subtracted, being a negative sign. Thus $9-6$ means 6 taken out of 9; and $a-b$ denotes the remainder when b is subtracted from a.
$\times$	MULTIPLIED by, as 9×6 signifies 6 times 9, or 9 multiplied by 6; also $a\times b$, or ab. is the product of a and b, multiplied together where note, if letters stand together like letters in a word, as $abcx$, they denote the product of a, b, c, and x. But when a number is prefixed to a letter, which is called its numeral coefficient, as $10\,x$, it denotes ten times x.

DIVI-

÷ Divided by, thus $9 \div 3$ ſignifies 9 divided by 3, alſo $3)9($ or $\frac{9}{3}$ ſignifies 9 divided by 3, and in general $a \div b$, or $b)a($, or $\frac{a}{b}$ is the quotient of a divided by b.

= Equality, or equal, as $9+6=15$, that is 9 and 6 equal 15; alſo $9-6=3$, that is 9 leſs by 6 equals 3; alſo $4 \times 3=12$, or 4 multiplied by 3 equals 12; alſo $12 \div 3=4$, or 12 divided by 3 is equal to 4.

: To, or *is to*, in the rule of *Three* or *Proportion*.

:: So is, or *what will*, a note in proportion, thus as $2:3::4:6$, ſignifies as 2 is to 3, ſo is 4 to 6; and as $a:b::c:d$, that is as a is to b, ſo is c to d.

∺ Continual Proportion, $a:b:c:d$ ∺, or a, b, c, d, are in continual proportion.

§ Section, is put to diſtinguiſh the diviſion of a Chapter, as § IV. ſignifies Section Fourth.

Alſo for the ſake of brevity +ed is put for added, —ed for ſubtracted, ×ed for multiplied × for multiply, ÷ for divide, and ÷ed for divided: likewiſe +ing for adding, —ing ſubtracting, ×ing multiplying, ×er multiplier, ÷ing for dividing, and ÷ſor for diviſor, in the following Treatiſe.

I would adviſe the Learner to make himſelf acquainted with the above Signs or Characters, with their ſignifications, which, being perfectly learnt, will help to contract the work, and is a ſhorter, better, and more ſignificant way of denoting what is to be done in moſt operations than can be expreſſed in words at length.

THE

The Arithmeticians Guide.

ARITHMETIC.

DEFINITIONS.

ARITHMETIC is the art of numbering, or of performing calculations by a right management of numbers.

2. An *Unit* is the beginning of number, and is that by which every thing is called one.

3. *Number* is a collection of units: by this every thing is computed.

4. An *Integer* is a whole thing.

5. A *whole Number* is a precise number without any parts annexed.

6. A *mixt Number* is a whole number with some part of an unit annext; which part is called a fraction.

7. One number is said to *measure* another, when it so exactly divides the other, that nothing remains: but if there be a remainder it is said to divide the other, but not to measure it.

Corrollary. Any number is a measure to itself. And 1 is a measure to any number.

8. One number is said to be the *multiple* of another when it contains it a precise number of times: as 30 is the multiple of 6, for 6 is contained 5 times in 30.

9. An.

9. An *Aliquot part* is that which is contained a precise number of times in another. So 4 is an aliquot part of the numbers 8, 12, 16, 20, 24, 28.

Cor. Hence 1 is an aliquot part of any number: but a number cannot be called an aliquot part of itself.

10. An *aliquant* part is such as is contained in another some number of times, with some part or parts over; as 3 and 4 with respect to 10.

11. An *even number* is that which may be divided into two equal parts; as 8 into 4 and 4.

12. An *odd number* is that which cannot be divided into two equal parts; as 3, 5, 7, 9, 11, &c.

Cor. The numbers 1, 2, 3, 4, &c. are alternately odd and even for ever.

13. A *Prime Number* is that which can only be measured by an Unit.

14. *Coprimes*, are such numbers as are prime to each other, when only an Unit can measure them both.

Cor. Therefore 1 is prime to every number.

15. A *plain Number* is the product of two other numbers.

16. A *solid Number* is the product of three numbers.

17. A *square Number* is the product of a number by it self.

18. A *cube number* is the product of a number and its square.

19. A *perfect Number* is that which is equal to the sum of all its aliquot parts: as 6 is a perfect number, because its aliquot parts 1, 2, 3, being added restore the same.

20. A *composite Number* is produced by multiplying several other numbers together, called Factors or Multipliers. Or it is that which other numbers besides unity will measure, as 4 measures 8 by 2, and 2 measures 8 by 4.

21. *Denomination* is the name of any integer or thing. Thus pounds, shillings, pence, and farthings, are several denominations; where shillings are of a lower denomination than pounds, and higher than pence.

NOTATION.

THE *Characters* by which numbers are expressed, are either the ten *numerical Figures* (sometimes called digits) of the *Arabians*, or the seven numerical Let-

ters of the *Romans*. The Names of those Characters, and the Numbers they stand for are here set down.

Arabian Figures.	Roman Letters
0 nothing, or Cypher.	
1 one, or an unit.	I one
Then $1+1=2$ two.	V five.
$2+1=3$ three.	X ten.
$3+1=4$ four.	L fifty.
$4+1=5$ five.	C hundred.
$5+1=6$ six.	D five hundred.
$6+1=7$ seven.	M thousand.
$7+1=8$ eight.	
$8+1=9$ nine.	

Then $9+1=$ ten, which has no single character; and thus by the continual addition of 1, all numbers are generated.

The whole art of Arithmetic is performed by these ten figures, and their various application in the five fundamental *Rules*, viz. *Numeration*, *Addition*, *Subtraction*, *Multiplication*, and *Division*.

AXIOMS.

1. IF to or from equal numbers, equal numbers be added or subtracted their sum or remainder will be equal.

2. If equal numbers be $\times$ed or $\div$ed by equal numbers, their product or quotient will be equal.

3. Unity or 1 neither multiplies nor divides; i. e. the product or quotient is still the same.

4. If a number be composed of two numbers $\times$ed together; either of them measures it by the other.

5. If a number measures several other numbers, it likewise measures the sum, or difference, of these numbers.

6. If a number measures another, it also measures every number which that other measures.

7. If a number measures the whole, and a part taken away, it also measures the residue.

8. The whole is equal to the sum of all its parts taken together.

9. Those things which are equal amongst themselves, are also equal to one another.

Practical

Practical ARITHMETIC.

CHAP. I.

The fundamental RULES in Common ARITHMETIC.

§ I. Of NUMERATION.

NUMERATION teacheth to write any number named, and to read any number written. Observe therefore the places, names, and value of the figures in the Table.

NUMERATION TABLE.

&c.	Hundred Thousands of Millions	Ten Thousands of Millions	Thousands of Millions	Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units
	7	3	2	3	8	9	4	6	7	5	5	5

1. The value of any number does not depend on the figure or figures alone, but upon the figure or figures where they stand jointly. The order of places is backward from the right hand to the left: therefore any character

is

is increased in its value in a tenfold proportion as it is removed to the left of the other figures with which it is connected: as in the figures 555, the first 5 is five ones but the second is 5 tens, or fifty, and the third is 5 five hundreds and so on. Therefore the numbers in the Table are read as follows.

Seven hundred and thirty two thousands, three hundred and eighty nine millions, four hundred and sixty seven thousands, five hundred and fifty five.

2. A cypher of itself is nothing; yet when put on the right hand of other figures, it increases their value in the same tenfold proportion as above described. Thus 4 signifies 4 ones, but 40 signifies four tens or forty, 400 signifies 4 hundred, and 4000 four thousand, &c. Also 1 signifies one, 10 ten, 100 a hundred, 1000 a thousand, and so on; and in general ten units make one ten, ten tens make one hundred, ten hundreds make 1 thousand, &c.

3. To be able to read twelve figures is as much as any common business requires, but you may read as many as you please by the following Rule.

RULE. Begin at the units place and distinguish your figures into periods of six figures each, and half periods of three figures each. The first period to the right is units, the 2d millions, the 3d bi-millions, the 4th tri-millions, the 5th quadri-millions, the 6th, 7th, &c. quinti-millions, sexti-millions, septi-millions, octi-millions, noni-millions, deci-millions, &c. The first half of any period is so many ones of it, but the last half is so many thousands of it.

Example 1. Let it be required to read the following numbers,

Sexti-mil.		quinti-mil.		quadri-mil.		tri-mil.		bi-mil.		Millions	units.
th.	un.	th.	un.	th.	un.	th.	un.	th.	un.	th.	un. cxtcxu
673,	874.	231,	043	756,	131.	736,	730	567,	311.	726,	734. 967,361.

Six hundred seventy three thousand, eight hundred seventy four sexti-millions;

Two hundred thirty one thousand, forty three quinti-mil.

Seven hundred fifty six thousand, one hundred and thirty one quadri-millions;

Seven hundred thirty six thousand, seven hundred and thirty tri-millions;

Five

Five hundred ſixty ſeven thouſand, three hundred and eleven bi-millions;
Seven hundred twenty ſix thouſand, ſeven hundred thirty four millions;
Nine hundred ſixty ſeven thouſand, three hundred and ſixty one.

2. It is required to expreſs in words, 3700474.

3. Write down in words 72010346746701$_3$.

4. Write down in figures, thirty ſeven thouſand bi-millions, three hundred forty thouſand, nine hundred and thirty four millions, two hundred five thouſand, five hundred twenty four.

§ II. SIMPLE ADDITION.

SIMPLE ADDITION is a rule by which ſeveral numbers of the ſame denomination are collected together. The number ariſing from thoſe collections is called their Sum.

RULE.

1. Place the ſeveral numbers under each other ſo, that units may ſtand under units, tens under tens, hundreds under hundreds, &c. and draw a line underneath.

2. Begin at the place of units, add up all the figures in that row, and if their ſum be leſs than ten, ſet down that ſum ſtrait below; if their ſum be even ten or tens, ſet down a cypher; if above ten or tens, ſet down the overplus, and for every ten carry an unit to the next row.

3. Add up in like manner all the figures in the tens place together with the units you carried; ſet down the overplus above the even tens as before, carry the tens to the next row; and ſo on to the laſt; and the figures below the line will be the whole Sum.

To prove addition, begin at the top, add all the figures downwards, by the ſame rule as you added them upwards; if the ſum is the ſame the work is right.

Example

Example 1. Let the numbers, 8967, 890, 103, and 76, be added together. When placed down according as the rule directs, they will ſtand thus ;

	8 9 6 7
	8 9 0
	1 0 3
	7 6
Sum	1 0 0 3 6

Beginning at 6, ſay 6 and 3 is 9 and 7 is 16, ſet down 6 and carry 1. Then ſay 1 and 7 is 8 and 9 is 17 and 6 is 23, ſet down 3, and carry 2. Then 2 and 1 is 3 and 8 is 11 and 9 is 20, ſet down 0, and carry 2, laſtly, 2 and 8 is 10, which being the laſt, ſet down.

It is very eaſy to conceive the reaſon of carrying the tens to the next place ; for the ſum of 6, 3 and 7 being 16, the 6 belongs to the units, and the 1 to the tens. Again, the ſum of 1, 7, 9 and 6 being 23, which are tens, the three belongs to the tens, and the 2 to the hundreds. Then the ſum of 2, 1, 8, and 9 being 20, which are 20 hundreds, the 0 belongs to that place, and the 2 to the next place which is thouſands. Laſtly the ſum of 2 and 8 is 10 i. e. 10 thouſand, that is 0 is in the place of thouſands, and 1 in the place of ten thouſands. Or in ſhort, thus.

The ſum of the row of units	1 6
The ſum of the row of tens	2 2 0
The ſum of the row of hundreds	1 8 0 0
The ſum of the row of thouſands	8 0 0 0
	1 0 0 3 6 by axm 9

Ex. 2.	Ex. 3.	Ex. 4.
674	407461	26
3723	78956	437
7673	4073	4188
91010	219	7610
701	31	10137
103781	490720	22398

5 A perſon dying left to his *Widow* 5000 pounds. He bequeathed to a *Charity* 864 pounds ; to each of his five

Nephews

nephews 1765 pounds: to each of his ſeven *nieces* 1053 pounds : to 10 poor Houſe-keepers 6 pounds each, and 563 pounds to his Executor ; how much did he die poſſeſſed of ? Anſwer 22683 pounds.

6. From the *Creation* to the *Flood* was 1656 years ; thence to the building of *Solomon's Temple* 1336 years ; thence to the birth of *Chriſt* 1008 years ; in what year of the world was *Chriſt* born ? Anſwer Anno M. 4000.

---

§ III. Of *Simple* SUBTRACTION.

SUBTRACTION is the taking a leſs number, called the *ſubducend*, from a greater, called the *minuend*, in order to find a third number, called the *remainder*, or *difference*.

RULE.

1ſt. Place the *minuend* uppermoſt, and the *ſubducend* under, ſo as units may ſtand under units, tens under tens, &c. and under them draw a line.

2. Begin at the place of units, and take each lower figure from that which ſtands over it, ſetting the remainder ſtrait under them below the line, and all thoſe remainders together will be the required difference.

3. When the lower figure is greater than that which ſtands over it, conceive ten to be added to the upper, and take the lower from the ſum, ſet down the remainder, carrying 1 to be added to the next lower figure, take the ſum from the upper, ſet down the remainder ; and ſo proceed from one row to another.

To prove ſubtraction, add the remainder to the *ſubducend*, and if the ſum be equal to the *minuend*, the work is right.

EXAMPLES.

1. From	38042678	the Minuend.
Take	5127435	the Subducend.
Rem.	32915243	or difference.

In operations of this ſort when the lower figure is greater than the upper, 10 is added to the upper; as for inſtance 7 is greater than 2, therefore one is borrowed from 4 to make 12; then 7 from 12, and there remains 5; then the next figure 2 ought to be taken from 3, inſtead of 4; but the reſult will be the ſame, whether you take 2 from 3, or add the 1 borrowed to 2, and take the ſum 3 from 4, in either caſe 1 remains. Again 1 from 0 you cannot take, but borrow 1 from 8 to make 10, from which take 1 and there remains 9, then one and 5 is 6, which taken from 8 leaves 2; laſtly nothing from 3 and 3 remains.

2. From	5 0 4 0 4 6 1 2 3	Minuend
Take	1 7 1 1 5 8 0 1 7	Subducend.
Rem.	3 3 2 8 8 8 1 0 6	

3. Ralph was born in 1689, and died in 1765, how old was he? Anſwer 76 years.

4. Ralph died in 1765, his age was 76 years, in what year of the Lord was he born? Anſwer 1689.

5. In the year 1174, Henry II. conquered Ireland, and annexed it to the Title of the crown of England: how long is that ago, this year being 1765? Anſw. 591 years.

6. In what year of our Lord, did Henry II. conquer Ireland, it being 591 years ſince, and the preſent year 1765? Anſwer 1174 years.

7. What number added to 704376, will make it 800000? Anſwer 95624.

8. I have 3 Debtors A, B, and C. A, owes me 120000 pounds and B, 50000 pounds, A's debt exceeds C's by 94600 pounds, how much then is C's debt leſs than B's? Anſwer 24600 pounds.

9. Your *Grandfather* is 86 years of age, your *Father* is 64, you are not ſo old as your *Grandſire* by 68 years; what is the difference in years between your *Father* and *you*? Anſwer 46 years.

10. The building of *Soloman's Temple* was in the year of the World 3000, Troy was built 443 years before the

temple

temple, and 260 years before London : Carthage was built 113 years before Rome, which was founded 744 years before Chriſt, who was born Anno Mundi 4000; is London or Carthage the ancienter city, and how much ? Anſwer London by 326 years.

11. A is 13 years younger than B, and 17 years older than C, who in the year 1743 was known to be 24 years of age, how old was each of theſe perſons in 1765. Anſwer A, 63, B, 76, C, 46.

12. The ſemidiameter of the *Earth's Orbit*, or annual path round the *Sun*, in the *center* of the ſyſtem is about 81,000,000 of miles; that of *Venus*, 59,000,000; when they are both on the ſame ſide the *Sun*, they are in *Perigæo*; when on different ſides, in Apogæo: what is the difference of their *diſtances* in both thoſe *circumſtances*. Anſwer 118 millions of miles.

13. A is 19 years older than B. who was 27 years of age in the year of our Lord 1745: how old is C in 1765, who in the year 1763, was within 24 years of being as old as both, A and B together ? Anſwer 87.

14. The mean diſtance between the Earth and Sun being 81 millions of miles, and between the Earth and Moon 240000 : how far are theſe two luminaries aſunder in an eclipſe of the Sun, when the Moon is lineally between the Earth and Sun ; and in another of the moon, when the Earth is in a line between her and him ? Anſwer of the Sun 80760000 miles. Of the Moon 81240000.

§ IV. Simple MULTIPLICATION.

MULTIPLICATION is the repeating a given number or quantity, called the multiplicand, ſo many times or parts of a time, as there are units in another given number, called the multiplier, and the reſult is called the product. Both multiplicand and multiplier are called Factors. Multiplication is a compendious method

of addition, and is performed by help of the following Table, which must be got by heart.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3		9	12	15	18	21	24	27	30	33	36
4			16	20	24	28	32	36	40	44	48
5				25	30	35	40	45	50	55	60
6					36	42	48	54	60	66	72
7						49	56	63	70	77	84
8							64	72	80	88	96
9								81	90	99	108
10									100	110	120
11										121	132
12											144

The use of the Table is this; find one figure in the top column of the table, the other in the left hand column, and the square where these two columns meet is the product. For instance, suppose you would multiply 8 by 9, find the 9 in the top, and 8 in the side column, and in the square where they meet stands 72; thus the product of 6 and 7 is 42; of 8 by 8 is 64; of 6 times 12 is 72; of 3 times 9 is 27; of 12 times 12 is 144, and so of any other.

RULE.

1. Place the Multiplier under the Multiplicand, so that units may stand under units, &c. under them draw a line.

2. Multiply from the right hand to the left, thus: begin with the units in the Multiplier, by which multiply the units or first figure of the Multiplicand, and set down the overplus above the tens, carrying the tens in your mind. Then multiply the 2d figure of the Multiplicand by the same Multiplier, adding so many units, as you had tens to carry; set down the overplus, and carry the tens as before.

Do thus till you come to the laſt figure in the multiplicand, whoſe product ſet down intire.

3. Then take the ſecond figure of the Multiplier, and multiply by it as you did by the firſt; ſetting the firſt figure of the product under the figure you multiply with; and in like manner take each figure in its order, till all the figures of the multiplicand are multiplied by all the figures in the Multiplier; obſerving always to ſet the firſt figure of each product ſo many places towards the left hand, as the multiplying figure is diſtant from the place of units.

4. Laſtly, add all theſe products together, and their ſum will be the product of the two numbers given.

Multiplication may be proved, by making the multiplicand the multiplier, for then if your product comes out the ſame as before the work is right.

Some prove multiplication by caſting out the nines, but as that method is not infallible, I omit it.

The beſt and ſureſt way of proving multiplication, is by Diviſion.

EXAMPLES.

1. Multiply 8 multiplicand,
   by 4 multiplier

Product 32. i. e. $8+8+8+8=8\times4=4\times8=32$

2. Multiply	7 0 5 3 4	3. Mult.	9 6 0 8 9
by	8	by	7
Product	5 6 4 2 7 2.	Product	6 7 2 6 2 3.

Explanation of the ſecond Example.

Say 8 times 4 is 32; ſet down 2 and carry 3; 8 times 3 is 24, and 3 you carry is 27; ſet down 7 and carry 2. 8 times 5 is 40 and 2 carried is 42; ſet down 2 and carry 4. 8 times 0 is 0 but 4 is 4; ſet down 4 and carry 0. 8 times 7 is 56 which ſet down, and you have 564272 for the Product.

of addition, and is performed by help of the following Table, which must be got by heart.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3		9	12	15	18	21	24	27	30	33	36
4			16	20	24	28	32	36	40	44	48
5				25	30	35	40	45	50	55	60
6					36	42	48	54	60	66	72
7						49	56	63	70	77	84
8							64	72	80	88	96
9								81	90	99	108
10									100	110	120
11										121	132
12											144

The use of the Table is this; find one figure in the top column of the table, the other in the left hand column, and the square where these two columns meet is the product. For instance, suppose you would multiply 8 by 9, find the 9 in the top, and 8 in the side column, and in the square where they meet stands 72; thus the product of 6 and 7 is 42; of 8 by 8 is 64; of 6 times 12 is 72; of 3 times 9 is 27; of 12 times 12 is 144, and so of any other.

RULE.

1. Place the Multiplier under the Multiplicand, so that units may stand under units, &c. under them draw a line.

2. Multiply from the right hand to the left, thus: begin with the units in the Multiplier, by which multiply the units or first figure of the Multiplicand, and set down the overplus above the tens, carrying the tens in your mind. Then multiply the 2d figure of the Multiplicand by the same Multiplier, adding so many units, as you had tens to carry; set down the overplus, and carry the tens as before.

Do

Do thus till you come to the laſt figure in the multiplicand, whoſe product ſet down intire.

3. Then take the ſecond figure of the Multiplier, and multiply by it as you did by the firſt; ſetting the firſt figure of the product under the figure you multiply with; and in like manner take each figure in its order, till all the figures of the multiplicand are multiplied by all the figures in the Multiplier; obſerving always to ſet the firſt figure of each product ſo many places towards the left hand, as the multiplying figure is diſtant from the place of units.

4. Laſtly, add all theſe products together, and their ſum will be the product of the two numbers given.

Multiplication may be proved, by making the multiplicand the multiplier, for then if your product comes out the ſame as before the work is right.

Some prove multiplication by caſting out the nines, but as that method is not infallible, I omit it.

The beſt and ſureſt way of proving multiplication, is by Diviſion.

EXAMPLES.

1. Multiply 8 multiplicand,
   by 4 multiplier

Product 32. i. e. $8+8+8+8=8\times4=4\times8=32$

2. Multiply	70534	3. Mult.	96089
by	8	by	7
Product	564272.	Product	672623.

Explanation of the ſecond Example.

Say 8 times 4 is 32; ſet down 2 and carry 3; 8 times 3 is 24, and 3 you carry is 27; ſet down 7 and carry 2. 8 times 5 is 40 and 2 carried is 42; ſet down 2 and carry 4. 8 times 0 is 0 but 4 is 4; ſet down 4 and carry 0. 8 times 7 is 56 which ſet down, and you have 564272 for the Product.

4. Multiply 70534.
by 5428.

564272
141068
282136
352670

Product 382858552

Demonstration of the Rule.

In Ex. 2d. 8 multiplying 4 produces 32, of which the 2 belongs to the place of units, and the three to the tens. Then 8 multiplying 3 in the 2d place, or place of tens, produces 24, of which 4 belongs to the tens, to which the three carried, which are alſo tens, being added, makes 7 tens, and the 2 belongs to the 3d place, or hundreds. Then 8 multiplying 5 in the 3d place, produces 40, the 0 belongs to the 3d place to which add 2, which alſo belongs to the 3d place, and the ſum is 42, of which 2 belongs to the 3d place and 4 to the 4th place. Then 8 times 0 is 0, in the 4th place, but 4 added is 4. Laſtly 8 multiplying 7 produces 56, the 6 belongs to the 5th place, and the 5 to the 6th.

This may perhaps more evidently appear by ſetting down the particular products thus:

$$\left.\begin{array}{r} 4 \\ 30 \\ 500 \\ 70000 \end{array}\right\} \times 8 = \left\{\begin{array}{r} 32 \\ 240 \\ 4000 \\ 560000 \end{array}\right.$$

$$70534 \quad \times 8 = \quad 564272 \quad \text{(axiom 8)}$$

And in Ex. 4. $8 \times 4 = 32$, the 2 falls in the place of units, and ſo on as in Ex. 2. Again 2 ×ing $4 = 8$, which falls in the 2d place, becauſe the ×er is really 20. Again 4 ×ing $4 = 16$, the 6 falls in the 3d place, becauſe the ×er is 400. Again 5 ×ing $4 = 20$, the 0 falls in the 4th place, becauſe the ×er is 5000. Each figure being thus proceeded with as in Ex. 2d. the ſum of all their products is that required. (ax. 8)

Ex. 5th	36704674852		96745		3550993768556740
Ex. 6th.	8706543		54086		470902084698
Ex. 7th.	759837	×	90705	=	68921015085
Ex. 8th.	16358724		704006		11516639848344
Ex. 9th.	38015732		4007000065		152329062834222580

C O N-

CONTRACTIONS.

I. When either the multiplicand, or multiplier, or both, have cyphers towards the right hand, reject the cyphers, multiply the significant figures as before, and to the product, annex as many cyphers as are in both factors.

Ex. 10th.	5968000	Ex. 11th.	5683
	6		9000
	35808000		51147000

Ex 12. 97638000×467000=45596946000000

II. When the multiplier is an unit with cyphers to the right, as 10, 100, 1000, &c. the product will be equal to the multiplicand with so many cyphers annext as are in the multiplier.

Ex. 13.	3674	×	10	=	36740
Ex. 14.	3674		10000		36740000

III. When the multiplier is a composite number, multiply continually by the factors; i. e. multiply by one factor, and that product by the other &c. and the last product will give the answer.

Ex. 15th. Multiply 3604670 by 35,=5×7, 3604670×7=25232690,×5=126163450. Answer.

Ex. 16th. Multiply 3704604040 by 56,=8×7, 3704604040×8=29636832320,×7=207457826240.

§ V. Simple DIVISION.

DIVISION is a compendious Subtraction; or the taking of one number, called the *Divisor*, out of another, called the *Dividend*, as often as possible, in order to find a third number, called the *Quote* or *Quotient*; because it shews (quotics) how often the Divisor is contained in the Dividend, thus

Divisor Dividend
4) 20 (5 Quote.

A General RULE.

1. Set down the Divisor and Dividend in the form above, consider if the Divisor be less or equal to the same number of the left hand figures of the Dividend; if it is less or equal, set the figure in the quote, expressing the number of times it is contained in that part of the Dividend: but if not, take one place more of the dividend figures than are in the Divisor, and set in the quote the number of times the Divisor is contained in this part of the Dividend, as before.

2. Multiply the Divisor by the quotient figure.

3. Subtract the product from the said Dividend figures; and set down the remainder.

4. Make a prick under the next figure of the Dividend, in order to mark it, and bring it down to the right of the remainder, then this number call the Dividual.

5. Seek how oft the Divisor can be had in the dividual, and set the figure expressing the number of times in the quote, multiply the Divisor thereby, subtract the product as before, and in this manner the operation must be continued till all the figures in the dividend are brought down one by one. And *note*, for every figure brought down, a figure must be placed in the quote, except when the Dividual is less than the Divisor, and then write a cypher in the quote.

Note 1. If the product of any quotient figure and divisor exceeds the dividual, the quotient figure belonging to such product must be lessened till the product is equal to, or less, than its dividual: again, if after subtracting the product from its dividual, the remainder is equal to, or exceeds the Divisor; the quotient figure must be increased till the remainder be less than the Divisor.

2d. If there be a *Remainder* after division is finished, annex it to the quote with the divisor under it, with a small line drawn between: then the quote will be a mixt number.

To *prove Division*, multiply the Divisor and quote together, adding the remainder, if any; and the *Product* will be equal to the *Dividend* if the work is right.

EX-

EXAMPLES.

```
    Divisor Dividend  Quote
1st.  3) 8 1 2 4 ( 2 7 0 8.
         6 . . .
        ----
         21
         21
        ----
         . . 24
             24
        ----
           . .
```

EXPLANATION.

First ask how oft 3 in 8, which is 2 times, then place 2 in the Quotient, and multiply 3, (the divisor) by 2, and the product is 6; which subducted from 8 leaves 2; then prick under 1 and bring it down to the right of 2. which is then 21 for a dividual; then ask how oft 3 in 21, the answer is 7, which place in the quotient; then 7 times 3 is 21, which subducted from 21 the dividual, leaves 0. Then prick and bring down the next figure 2, and ask how oft 3 in two, the answer is 0, which place in the quotient; then prick and bring down 4 to the right of 2, and the dividual is 24, then ask how oft 3 in 24, the answer is 8, which place in the quotient; then 8 times 3 is 24; which subducted from 24, (the dividual) leaves nothing.

Then 2708 is the quote. Proved thus: $2708 \times 3 = 8124$ the dividend.

Ex. 2d. Divide 18972584 by 6023.

```
6023) 18972584 (3150 134/6023 quote
      18069 . . .
      -----
        9035
        6023
      -----
        30128
        30115
      -----
          134 the remainder.
```

Proof $3150 \times 6023, + 134, = 18972584$.

Demonſtration of the RULE.

In Ex. 1. Since 3 is contained twice in 8 therefore it is contained 2000 times in 8124, i. e. 2 muſt be in the 4th place. And ſince 3 is contained 7 times in 21, it is contained 700 times in the whole remainder 2124, and therefore 7 muſt occupy the third place in the quote; and as 3 cannot be had once in 2, it cannot be had 10 times in the whole remainder 24, therefore a cypher muſt occupy the tens place of the quote; but ſince 3 is contained 8 times in 24, and 4 poſſeſſes the units place of the Dividend, 8 muſt conſequently poſſeſs the units place of the quote. Therefore the Diviſor is contained in the whole dividend 2708 times.

And in Ex. 2. ſince 6023 is contained 3 times in 18972 it is contained 3000 times in 18972584; and 100 times in the remainder 903584; and 50 times in the next remainder 301284; and 0 times in the laſt remainder 134. Or the Diviſor is contained in the whole Dividend 3150 times.

CONTRACTIONS.

I. When your Diviſor is 12 or leſs than 12, diviſion may be expeditiouſly performed by multiplying and ſubtracting mentally, and writing down only the quote below the Dividend.

EXAMPLES.

5)7367347
Quote 1473469 $\frac{2}{5}$

7)1967469
Quote 281067

12)1646749
Quote 137229 $\frac{1}{12}$

8)7674674
Quote 959334 $\frac{2}{8}$

II. If the Diviſor hath cyphers to the right of it, cut them off, then cut off ſo many of the right hand places of the Dividend as there are cyphers in the Diviſor, which annex to the remainder, when the operation is finiſhed.

EX-

EXAMPLES.

1. Divide 9850716 by 8300

Contracted	At Large.
83 | 00)98507 | 16(1186 $\frac{6916}{8300}$	8300)9850716(1166 $\frac{6916}{8300}$
83 . . .	8300 . . .
155	15507
83	8300
720	72071
664	66400
567	56716
498	49800
6916 Rem.	6916 Rem.

III. To divide by an unit with cyphers as 10, 100, 1000, &c. cut off from the Dividend ſo many places as the Diviſor has cyphers, and the figures ſo cut off on the left is the quote, and thoſe on the right the remainder.

Ex. Divide 7217367 by 100. The Quote is 72173 and 67 remainder.

IV. When the Diviſor is a compoſite number, i. e. when it is the product of two or more ſmall numbers, it is much eaſier to divide continually by thoſe numbers, than by the whole diviſor at once, i. e. divide the dividend by one of thoſe numbers, and that quotient by the other, and ſo proceed.

Note. If there be any remainders after ſuch diviſions, multiply the laſt remainder by the preceding Diviſor, and to the product add the remainder belonging to the ſame Diviſor; then multiply the ſum by the next preceding Diviſor, and to the product add its correſponding remainder: and ſo proceed through all the Diviſors and Remainders, and the laſt ſum will be the true remainder as if the Diviſion had been performed at once.

716
85
631

EX.

EXAMPLES.

1st. Divide 563746673 by $35=7\times5$.

$$
\begin{array}{l}
\quad\quad\quad\quad\quad\quad\quad 7) \\
5)563746673(112749334(16107047\tfrac{28}{35}\text{ quote} \\
\quad\text{1st rem. } 3 \quad \text{2d rem } 5
\end{array}
$$

then $5\times5=25,+3=28$ the true remainder.

2d. Divide 67467367 by $72,=8\times9$

$$
\begin{array}{l}
\quad\quad\quad\quad\quad\quad 9) \\
8)67467367(8433420(937046\tfrac{55}{72}\text{ quote} \\
\quad\text{rem. } 7 \quad\quad\quad \text{rem. } 6
\end{array}
$$

then $6\times8,+7=55$ the true remainder.

3d. Divide 7386989 by $144=6\times6\times4$

$$
\begin{array}{l}
\quad\quad\quad\quad\quad 6) \quad\quad 6) \\
4)7386989(1846747(307791(51298\tfrac{77}{144}\text{ quote} \\
\quad\text{rem. } 1 \quad \text{rem. } 1 \quad \text{rem. } 3
\end{array}
$$

then $3\times6,=18,+1=19,\times4=76,+1=77$ true remainder.

V. When you have a large Dividend, and your Divisor is often repeated; make a table of all the products of the Divisor and the 9 digits. By this table Division may be wrought by inspection, by help of Addition and Subtraction only. For you have no more to do, but only to take out of the table the number always next less than (or equal to) each dividual, and subtract it as in the general rule, remembering always to place in the quote the figure in the small column, which is opposite the Product made use of.

EX

EXAMPLES.

1. Divide 403779820571 by 35016.

TABLE.

1	35016
2	70032
3	105048
4	140064
5	175080
6	210196
7	245112
8	280128
9	315144

```
35016)403779820571)11531294.
       35016 .......
       -----
        53619
        35016
        ------
        186038
        175080
        ------
         109582
         105048
         ------
          45340
          35016
          ------
          103245
           70032
          ------
           332137
           315144
           ------
            169931
            140064
            ------
             29867 remain.
```

2d. 355099376855 6740	÷	96745	=	3670467485 2
3d. 3550993768556740		3670467485 2		96745
4th. 47090208469 8		54086		8706543
5th. 1151663984 8344		704006		16358724
6th. 115166398 48344		16358724		704006
7th. 1523290628342 2580		400700065		38015732
8th. 15232906283422580		38015732		400700065

§ VI. *Tables of Money, Weights, and Measures.*

1. *Of* MONEY.

q.	d	s.	£.
4 =	1		
48	12 =	1	
960	240	20 =	1.

Note 1. L. denotes pounds, S. shillings, D. pence, and Q. farthings. Which are the first letters of their latin names libræ, solidi, denarii, quadrantes.

2. The letters at top denote the names of all the numbers strait below them. All the numbers also on the same line, from left to right are of equal value: thus in the last line of this table, 960 farth. 240 pence, 20 shill. and 1 pound are all equal to each other: and so in all the following tables.

3. $\frac{1}{4}$ denotes 1 farthing, or 1 quarter of any thing.
$\frac{1}{2}$ - - - - - a half-penny, or a half of any thing.
$\frac{3}{4}$ - - - - - - 3 farthings, or three quarters of any thing.

2. *Of* TROY WEIGHT.

Grains gr.	penny weights dwt.	ounces oz.	pound lb.
24 =	1		
480	20 =	1	
5760	240	12 =	1

Note. By this Weight gold, silver, jewels, amber. bread, corn, liquors, &c. are weighed. By this also the proportion of Gravity in Philosophical Experiments, which any two bodies have to each other, are tryed; as suppose gold to silver, &c.

3. *Of Apothecaries* WEIGHT.

Grains	Scruples.	Drams.	Ounces.	Pound.
20 =	1			
60	3 =	1		
480	24	8 =	1	
5760	288	96	12 =	1

Note. By this weight Apothecaries compound all their medicines; tho' they buy and sell their drugs by Avoirdupois Weight. Apothecaries is the same as Troy Weight, having only some different divisions.

4. Of

4 *Of Avoirdupois* WEIGHT.

Drams dr.		Ounces oz.								
16	=	1		pounds lb.						
256		16	=	1		Quarters qr.				
7168		448		28	=	1	Hundreds C.			
28672		1792		112		4	=	1	Ton	
573440		35840		2240		80		20	=	1

Note. By this weight are weighed all phyſical drugs, butter, cheeſe, fleſh, grocery, roſin, wax, pitch, tar, tallow, ſoap, hemp, &c. i. e. all things of a droſſy nature, and all metals, except gold and ſilver.

5. *Of* Long Measure.

Inches		Feet									
12	=	1		Yards							
36		3	=	1		Poles					
198		$16\frac{1}{2}$		$5\frac{1}{2}$	=	1	Furlongs				
7920		660		220		40	=	1	Mile		
63360		5280		1760		320		8	=	1	

Note. An inch is ſuppoſed equal to 3 barley corns in length. 4 inches equal to 1 hand. 6 feet equal to 1 fathom. 3 miles equal to 1 league. 60 geographical miles equal to 1 degree, or 69 and a half ſtatute miles nearly.

Alſo 360 degrees, or 25000 miles nearly, is the circumference of the Earth.

6. *Of Cloth* MEASURE.

Inches		Nails				
$2\frac{1}{4}$	=	1	Quar.			
9		4	=	1	Yard	
36		16		4	=	1

Note. 3 qrs. are equal to an ell Flemiſh.
5 ——— ——— Engliſh.
6 ——— ——— French.
4 qr. 1 $\frac{1}{5}$th of an inch —— Scots.

7. *Of Square or Land* MEASURE.

Square Inch	ſqr. feet				
144 =	1	ſqr. Yds.			
1296	9 =	1	ſqr Poles		
39204	$272\frac{1}{4}$	$30\frac{1}{4}$ =	1	Roods	
1568160	10890	1210	40	= 1	Acre
6272640	43560	4840	160	4 =	1

Note. The Land Chain now in uſe is 22 yards long, and is divided into 100 equal parts called Links; 10 ſuch chains in length and 1 in breadth make 100000 ſquare Links (that is 100×100×10) or 1 ſquare acre; therefore if any number of ſquare Links be divided by 100000 the quotient will be acres.

8. *Of Wine* MEASURE.

Pints	Gallons					
8 =	1	Tierces				
336	42 =	1	Hogſh.			
504	63	$1\frac{1}{2}$ =	1	Punch.		
672	84	2	$1\frac{1}{3}$ =	1	Pipe or But	
1008	126	3	2	$1\frac{1}{2}$ =	1	Tun
2016	252	6	4	3	2 =	1

Note. 231 cubic, or ſolid inches are equal to 1 gallon, 10 gallons equal to 1 anchor. 18 gall. equal to 1 rundlet. 31 gall. and a half equal to 1 barrel.

By this meaſure all wines, brandies, ſpirits, ſtrong waters, mead, perry, cyder, vinegar, oil, honey, &c. are meaſured.

9. *Of Ale and Beer* MEASURE.

Pints	Gall.				
8 =	1	Firkins			
68	$8\frac{1}{2}$ =	1	Kilderkins		
136	17	2 =	1	Barrels	
272	34	4	2 =	1	Hhds
408	51	6	3	$1\frac{1}{2}$ =	1

Note. The Ale gall. contains 282 cubic inches. In London, the ale firkin contains 8 gall. and the beer firkin 9; the other meaſures above it decreaſe and increaſe in the ſame proportion.

Of

10. *Of Dry* MEASURE.

Pints	Galls.	Pecks	Bushels	Combs	Quar.	Weys	Last
8	= 1						
16	2	= 1					
64	8	4	= 1				
256	32	16	4	= 1			
512	64	32	8	2	= 1		
2560	320	160	40	10	5	= 1	
5120	640	320	80	20	10	2	= 1

Note. The gall. dry measure, contains $268\frac{4}{5}$ cubic inches. At London 36 bushels of coals make a chaldron. A bushel of water measure is 5 pecks.

By this measure all dry wares, such as corn, seeds, fruits, roots sand, salt, coals, oisters, muscles, cockles, &c. are measured.

11. *Of* TIME.

Minutes	Hours	Days	Weeks	Month
60	= 1			
1440	24	= 1		
10080	168	7	= 1	
40320	672	28	4	= 1

Note. 1. The minute is divided into 60 equal parts called seconds, the seconds into 60 equal parts called thirds, and these again into 60 fourths, &c.

2. The Julian year consists of 13 months 1 day 6 hours, or 52 weeks 1 day 6 hours, or 365 days 6 hours, nearly.

3. An equal month (mentioned above) is so called because it always consists of 28 days. On the contrary, a kalendar month is unequal, because it consists of an unequal number of days. The year is divided into 12 kalendar months, viz. January February, March, April, May, June, July, August, September, October, November, December.

To know the days in each month observe.

Thirty days hath September,
April, June, and November,
February hath twenty-eight alone,
And all the rest have thirty-one;
Except in leap-year, and then's the time
February's days are twenty and nine.

4. The exact solar year consists but of 365 days, 5 hours, 48 minutes, and 57 seconds; tho' the common or Julian year is said to contain 365 days 6 hours: which odd 6 hours every fourth year making a day, that day is added to February, which has then 29 days; and the year is called Bissextile or Leap-year. Therefore all that are born on the 29th of February, have their birth day but every fourth year.

12. *Of a Circle's Circumference, how Divided.*

Minutes		Degrees		Signs		Circles Circum.
60	=	1				
1800		30	=	1		
21600		360		12	=	1

Note 1. The Minute is divided into 60 Seconds, &c. as in time.

2. By this Astronomers measure distances in the Heavens, Geographers measure this Terraqueous Globe; and Mathematicians measure any proposed angle.

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§ VII. *ADDITION of different Denominations.*

RULE.

1. PLACE all numbers, of the same denomination, directly under each other, and under them draw a line.

2. Begin at the least denomination, and add up all the figures in that row, as in simple addition.

3. Find how many ones of the next superior denomination are contained in the sum, which you may do, by dividing it by so many of this name as makes one of the next, or any other way.

4. Set down the remainder or overplus under the added row, and carry the quote to the figures of the next denomination, whose sum you must find and proceed with as before; and so of the rest. In the last denomination, add

them

them up as integers, whose sum set down, which, together with the several remainders, will express the required sum.

Note. Addition of money may be performed either by the above rule, or by help of the following tables.

PENCE TABLE.

d.		s.	d.	
20	—	1	8	
30	—	2	6	is half a Crown.
40	—	3	4	
50	—	4	2	
60	—	5	0	a Crown.
70	—	5	10	
80	—	6	8	a Noble.
90	—	7	6	
100	—	8	4	
110	—	9	2	
120	—	10	0	an Angel.
160	—	13	4	a Mark.

SHILLING TABLE.

s.		l.	s.	
21	—	1	1	is a Guinea.
23	—	1	3	a Jacobus.
25	—	1	5	a Carolus.
27	—	1	7	a Moidore.
40	—	2	0	
60	—	3	0	
80	—	4	0	
100	—	5	0	
120	—	6	0	
140	—	7	0	
160	—	8	0	
180	—	9	0	

Examples of MONEY.

EXAMPLE 1.

l.	s.	d.
56	14	$7\frac{1}{2}$
67	15	$6\frac{1}{2}$
45	13	$5\frac{3}{4}$
74	12	$8\frac{1}{4}$
67	14	$9\frac{1}{4}$
89	11	$4\frac{1}{4}$
93	10	$2\frac{1}{4}$
495	12	$7\frac{3}{4}$

EXPLANATION.

Begin at the farthings, add them up and their sum is 11, or 2 pence 3 farthings; put down $\frac{3}{4}$ under the farthings, and carry 2 to the pence place, and the sum of the pence is 43 or 3 shillings and 7 pence; place the 7 under the pence and carry 3 to the shillings, and the sum is 92 shillings, or 4 pounds 12 shillings; place down the 12 shillings; and carry 4 to the first row of the pounds, and the sum of it will be 45; then place down the 5, and carry 4 to the last row, and the sum is 49, which

place

place down, and the work is done. The answer being 495l. 12s. 7¼d.

2d.			3d.			4th.		
l.	*s.*	*d.*	*l.*	*s.*	*d.*	*l.*	*s.*	*d.*
87	16	7¾	74	14	0½	7	17	3½
96	18	8	7	9	4	76	13	4
77	17	6½	19	10	11¾	67	10	0¾
65	15	5	80	14	9½	49	8	7
31	11	3½	73	19	0	76	12	11¼
43	13	4	8	0	1¼	47	16	1
24	14	9½	2	3	10	76	0	0½
428	7	8¼						

5th. A of Rotherham is debtor to B of Sheffield. For 367 groſs of penknives, 467l. 17s. 6d. For 264 groſs of table knives and forks, 567l. 13s. 9½d. For 120 groſs of razors, 96l. 14s. 7¾d. For 8 dozen of ſciſſors, 3l. 0s. 10½d. For 10 tons of iron, 179l. 17s. 6¾d. For 48 tons of copper, 1176l. 9s. 3½d. For 10 tons of ſteel, 250l 10s. 6d. For his acceptance of a bill drawn, 76l. 19s. 6¼d. For carriage, and incidental charges 4l. 13s. 6½d. For what ſum muſt B draw to clear the accompt?

Anſ. 2823l. 17s. 2¾d.

		l.	*s.*	*d.*
1	Penknives, value,	467	17	6
2	Table knives and forks, ditto.	567	13	9½
3	Razors, ditto.	96	14	7¾
4	Sciſſors, ditto.	3	0	10½
5	Iron, ditto.	179	17	6¾
6	Copper, ditto.	1176	9	3½
7	Steel, ditto.	250	10	6
8	A Bill, ditto.	76	19	6¼
9	Incidental expences, ditto.	4	13	6½

6th. Sup-

6th. Suppose A is indebted to B 57l. 6s. 8d. To C 127l. 14s. To D 9s. 6½d. To E 17l. 0s. 3¾d. To F 1044l. 19s. and to G 1l. 7s. 2d. What is A's whole debt? Ans. 1248l. 16s 8¼d.

7th. A Nobleman, going out of Town, is informed by his Steward, that his corn chandler's bill comes to 123l. 19s. His brewer's to 41l. 10s. His butcher's to 212l. 0s. 6d. That to his Lordship's baker is owing 24l. To his tallow chandler, 13l. 8s. To his taylor, 137l. 9s. 9d. To his draper, 74l. 13s. 6d. That his coach-maker's demand was 214l. 16s. 6d. His wine merchant's 68l. 12s. His confectioner's 16l. 2s. That his rent was 82 guineas, and his servants wages 46l. 5s. What money must he send to his banker for, in case he would carry with him 50l. to defray his expences on the Road? Ans. 1108l. 18s. 3d.

Examples of Weights and Measures.

1. Troy Weight.

	lb.	*oz.*	*dwts.*	*grs.*
	47	11	15	14
	14	9	13	15
	71	10	10	10
	67	10	15	17
	81	9	10	18
Sum	284	4	6	2

2. Apothecaries Wt.

lb.	*oz.*	*dr.*	*sc.*	*gr.*
27	5	7	1	19
43	6	5	2	18
34	4	6	2	17
75	3	6	1	16
36	2	4	2	17
217	0	0	0	7

In the first Exam. the Grains are 74, or 3 penny weights and 2 grains; put down 2 under the grains, and carry 3 to the penny weights row, and the sum of the penny weights is 66, or 3 ounces and 6 dwts; place the 6 under the dwts, and carry 3 to the ounces, and the sum is 52 ounces, or 4 pounds 4 ounces; place down the 4 ounces, and carry 4 to the pounds; then proceed as in the examples of integers, and you will find the sum to be 284 lb. 4 oz. 6 dwts. 2 grs.

3d. Avour-

3d. Avoirdupoise Wt.

T.	*c.*	*q.*	*lb.*	*oz.*	*dr.*
12	15	2	24	12	15
17	10	3	21	15	10
0	18	1	14	11	5
1	19	3	27	15	12
Sum 33	5	0	5	7	10

4th. Long Measure.

deg.	*l.*	*m.*	*f.*	*yd.*	*ft.*	*i.*	*bc.*
76	19	2	7	84	2	11	2
60	10	1	6	76	1	10	1
30	3	0	2	61	0	6	0
10	0	2	5	30	1	3	2
Sum 177	14	1	5	33	0	7	2

5. Cloth Measure.

Ells Eng.	*Q.*	*N.*
70	4	3
61	3	2
16	2	1
10	1	0
3	0	2
162	2	0

6. Land Measure.

A.	*R.*	*P.*
3	3	37
72	1	4
21	2	16
76	1	0
9	0	3
183	0	20

7. Wine Measure.

Hhds.	*Gall.*	*Q.*	*P.*
614	14	2	1
721	37	1	0
361	56	3	1
123	45	0	1
102	17	1	0
1923	45	0	1

8. Ale & Beer Measure.

hhds.	*k.*	*g.*	*p.*
29	2	16	7
17	1	11	2
12	2	14	5
9	0	12	6
69	2	4	4

9. Dry Measure.

bu.	*p.*	*g.*	*p.*
7	3	1	7
19	1	0	6
8	2	1	2
6	1	1	4
42	1	1	3

10. Time.

yrs.	*m.*	*w.*	*d.*	*h.*	*m.*
70	12	3	6	23	59
16	9	1	6	16	16
10	6	2	1	[illegible]	30
2	0	1	6	10	10
100	3	2	0	19	55

11. In a Gentleman's service of plate there are dishes 12, weighing 16lb. 1 oz. 13 dwts: plates 32, weighing 35lb. 1 oz. 11 dwts: spoons 5 dozen, weighing 18lb. 8 oz. 6 dwts: salts six, weighing 2lb. 8 oz: knives and forks weighing 11lb. 9 oz. 6 dwts: presenters six, weighing 9lb. 5 oz. 4 dwts: mugs, tumblers, beakers, and other odd pieces, weighing 23lb. 8 oz 18 dwts: a silver tea kettle and

 John

and lamp, weighing 10lb. 6 oz. 9 dwts: and the rest of that equipage 7lb. 9 oz. 2 dwts. What quantity of plate had the Butler under his care? Ans. 125. 0 oz. 9dwts.

12. An Apothecary made a composition of six ingredients, the 1st of which weighed 17lb 9 oz. 6d. 2 scr. 19 gr.; the 2d, 9lb. 7 oz. 5dr. 1scr. 14gr.; the 3d 3lb 6dr. 1scr. 12gr.; the 4th 24lb. 3 oz. 1 dr 2 scr. the 5th 11 oz. 15gr.; and the 6th 1lb. 5 oz. 4dr. 1scr. 19gr. What was the weight of the whole?

Answ. 57lb. 2 oz. 1dr. 1scr. 19gr.

13. A Grocer bought of a Merchant, sugars weighing 7cwt. 3qr 7lb. teas, 1cwt. 2qr. 17lb. 13 oz. coffee 3qr. 7lb 15 oz, spices, 4cwt. 3qr. 27lb. 9 oz six casks of currants, each 3cwt. 2qr. 17lb. 12 oz. five casks of raisins, each 3cwt. 3qr. 20lb. 14 oz and three bag of hops, each 2cwt. 1qr. 13 oz What is the weight of the whole? Answ. 64cwt. 5lb. 10 oz.

14 From the city A to B is 8 miles, 6 fur. 36 poles; from B to C is 5m. 7fur. 23p. from C to D is 36m. 19p from D to E 163m. 2fur. and from E to F is 7fur. 21p. What is the distance between A and F. Answ. 215 miles, 19 poles.

15. A Mercer bought five pieces of cloth, the first of which contained 49 yds. 2qrs. the second, 34 yds. 3qrs. 3 nls. the third, 17 yds. 3 nls. the fourth, 19 yds. and the fifth, 143 yds. 2 nls. How many yards did he buy in all? Ans. 263 yards 3 qr.

16. There are 4 pieces of land, the 1st of which measures 4[illegible]ac. 2r. 17p. the 2d 53ac. 3r. the 3d 9[illegible]ac. [illegible]p. and the 4th 6[illegible]ac. 3r. What is the sum of their measures?

Answ. 266ac. 34 poles.

17. A Gentleman bought of a Wine Merchant, of port wine 6 tuns, 3hhds. 46gal. of Bourdeaux claret 2hhds. 56 gal. 7pts. of Madeira 3hhds. 24gal. 5pts. and of Mountain 1 tun, 37gal. What quantity did he buy in all?

Answ. 9 tuns, 2hhds, 38gal. 4pts.

18. An Inn-keeper bought ale of a wholesale Brewer as follows, viz. at one time 4hhds. 34gal, at another [illegible]hhds. 19gal. 2qrts. at another 47gal. 1pt. and at another [illegible]hhds. How much did he buy in all?

Answ. 16hhds. 49gal. 2qrts. 1pt.

19. A

19. A Corn Merchant bought, of wheat 15 lasts, 1 wey, 3qr. 6 bush. of rye 4qrs. 7bush 3pks. of oats 30 lasts, 3qr. of beans 1 wey, 2qr. 2pks. and of peas 9qr. How much did he buy in all? Ans. 48 lasts, 2qr. 6bush. 1pk.

20. When B was born, A was 4 ys. 11 ms. 3 ws. old; when C was born, B was 7 ys. 9 ms. 2 ws. 5 ds. old; when D was born, C's age was 13 ys. 2 ws. 6 ds. 17 hrs. and when E was born D's age was 9 years. What was A's age when E was born? Ans. 34 years 9 months, 4 days, 17 hours, reckoning 13 months to the year.

21. England was conquered by *William* I. Oct. 4, 1066; his son *William* II. came to the Crown Sept. 9, 1087, and left it Aug. 2, 1100; *William* III. received it February 3, 1689, and died March 8, 1701: How many Days did each of these Princes govern separately, and how many together, respect being had to the intercalary Days (added to February, every leap year) as they rose in the course of time? Ans. Will. I. 7645 Days, Will. II. 4710 Days, Will. III. 4416 Days, and altogether 16771 Days

Note. Every fourth year being Leap-Year, to find which are such, divide the year of our Lord by 4, and when nothing remains, those are the Leap-Years; to such therefore you add one Day to 365.

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§ VIII. *SUBTRACTION of different Denominations.*

RULE.

1. PLACE the less number under the greater, as directed in addition of different denominations, and under them draw a line.

2. Begin at the least denomination, take the under numbers from the upper, and set down their respective remainders below them.

3. But if the under number in any denomination happens to be greater than the upper, borrow one, i. e. add as many to the upper number as makes one of the next superior

uperior denomination, and ſubtract as before, and ſo on o the laſt or greateſt denomination, which ſubtract as ntegers.

EXAMPLES of MONEY.

Ex. 1.	L.	s.	d	Ex. 2.	L.	s.	d.	
From	767	14	$7\frac{1}{2}$		772	12	$3\frac{1}{4}$	Minuend
Take	213	7	$2\frac{1}{4}$		189	13	$7\frac{3}{4}$	Subducend
Rem.	554	7	$5\frac{1}{4}$		582	18	$7\frac{1}{2}$	
Proof	767	14	$7\frac{1}{2}$		772	12	$3\frac{1}{4}$	

3. What is the difference between 136l. 13s. $1\frac{1}{2}$d. and 98l. 16s. $9\frac{3}{4}$d. ? Anſ. 37l. 16s. $3\frac{3}{4}$d.

4. A Gentleman bought an eſtate, valued at 34761l. 13s. 4d. his caſh only amounted to 21967l. 17s. $7\frac{1}{2}$d. how much muſt he ſend for to his Banker, to enable him to pay for it ?

	£	s.	d.
Value of the Eſtate —	34761	13	4
His ready money — —	21967	17	$7\frac{1}{2}$
He muſt ſend to his Banker for	12793	15	$8\frac{1}{2}$

5. What ſum when added to 1367l. 14s. $7\frac{3}{4}$ will make 2000l. ? Anſ. 632l. 5s $4\frac{1}{4}$d.

6 What 5 different ſums of money added together will make 3674l. 13s. $9\frac{3}{4}$. ?

To ſolve this Queſtion, ſet down at pleaſure any four of the numbers, take their ſum for a ſubducend, make the required number a minuend, and the remainder will be the 5th number ?

	L.	s.	d.		L	s.	d.	
The 4 Nos. taken at pleaſure.	267	13	$4\frac{1}{2}$	Minuend	3674	13	$9\frac{3}{4}$	
	300	19	$10\frac{3}{4}$	Subducend	1490	19	$9\frac{1}{2}$	
	401	14	$7\frac{1}{4}$					
	520	11	11	Rem.	2183	14	$0\frac{1}{4}$	5th No
their ſum	1490	19	$9\frac{1}{2}$	Proof	3674	13	$9\frac{3}{4}$	

7. A lent B 404l. 13s, 6d. and ſome time after 260l. 17s. more; he alſo ſold him a horſe on truſt valued at 10 guineas. B paid him at three ſeveral times 80 guineas each, at another time gave him a note upon C, for 100l. and ſold him goods to the value of 69l. 17s. 4$\frac{3}{4}$d. I demand how the balance ſtands between them?

A's BILL againſt B,

	L.	s.	d.
1. Caſh lent	404	13	6
2. Ditto	260	17	0
3. Horſe on Truſt	10	10	0
B's whole debt,	676	0	6
A's whole debt,	421	17	4$\frac{3}{4}$
Balance due to A	254	3	1$\frac{1}{4}$

B's BILL againſt A.

	L.	s.	d.
1. Guineas at three ſeveral payments 80 each.	252	0	0
2. Note upon C, value	100	0	0
3. Goods valued at	69	17	4$\frac{3}{4}$
A's whole debt	421	17	4$\frac{3}{4}$
Balance due to A	254	3	1$\frac{1}{4}$
Proof	676	0	6

8. A Steward received Rents from ſeveral perſons as follow: from A 120l. 10s. 7$\frac{1}{2}$d. from B 210l. 13s. 2d. from C 314l. 14s. 11d. and from D 96l. 12s. 7$\frac{1}{4}$d. He remitted to his Lord in caſh 200l. in Bank notes 156l. he paid for repairs done to the eſtate 26l. 10s. 6d. his own ſalary for a quarter was 20l. 10s. What remains in his hands due to the Lord? Anſ. 339l. 10s. 9$\frac{3}{4}$d.

A

1606:11:3 $\frac{3}{4}$
502 0:6
1104:10:9 $\frac{3}{4}$

9. A Trader failing was indebted to A 71l. 12s. 6d. To B 34l. 9s. 9d. To C 16l. 8s. 8d. To D 44l. To E 66l. 7s, 6d. To F 11l 2s. 3d. To G 19l. 19s, and to H a fine of thirty marks. At the time of this disaster he had by him in cash 3l. 13s, 6d. in *commodities* 23l. 10s. in *houshold furniture* 13l. 8s. 6d. in plate 7l. 18s. 5d. in a *tenement* 56l. 15s. in recoverable book debts, 87l. 13s. 10d. Supposing these things faithfully surrendered to his creditors, what will they then lose by him? Ans. 91l 5d.

10. A *chaise*, *horse*, and *harness*, were together valued at 100l. the horse and harness was worth 36l. 14s. 6d. the *chaise* and harness were esteemed at 79l. 19s. 9d. Their several values are required. Ans 79l. 19s 9 + 36l. 14s. 6d. —100 = 16l. 14s. 3d. value of the harness; hence the value of the *horse* is 20l. 3d. and of the *chaise* 63l. 5s. 6d.

11. A gave a *bond* for 114l. 10s. the *interest* came to 19l. He then paid off 40 guineas, and gave a fresh bond for what was behind. By that time there was 13l. 4s 8d due on the second for *interest*, he paid off 37l. 14s. 2d. more, took up the *old bond*, and signed a *new one* for the *residue*. The *principal* again ran on till there was 9l. 11s 3d. more due, and then he determined to take the new bond up. What money had his *creditor* to receive? Ans. 76l. 11s. 9d.

12. A *Merchant* at his out-setting in *trade* owed 280l. He had in cash 1764l. 14s. 6d. in wares 567l. He cleared the first year 316l. What at the years end was his neat *balance*? Ans. 2367l. 14s. 6d.

Examples of Weights, Measures, &c.

	1. *Troy Weight.*				2. *Apothecaries Weight*				
	lb.	oz.	dwts.	gr.	lb.	oz.	dr.	sc.	gr.
From	74	7	17	14	96	7	2	1	13
Take	35	9	13	15	69	9	5	0	14
Rem.	38	10	3	23	26	9	5	0	19
Proof	74	7	17	14	96	7	2	1	13

 3 Avoir-

3. *Avoirdupoise Weight.*

	tons	c.	qr.	lb.	oz.	dr.
From	27	12	1	15	13	14
Take	18	16	2	13	15	15
Rem.	8	15	3	1	13	15

4 *Cloth Measure.*

	yds.	qr.	nls.
From	371	1	2
Take	186	3	3
Rem.	184	1	3

5. *Land Measure.*

	A.	R.	P.
From	261	1	21
Take	192	2	39
Rem.	68	2	32

6. *Long Measure.*

	deg.	lea	m.	fur.	pl.	yds.	ft.	in.	b.c.
From	39	14	1	3	17	2	1	9	1
Take	19	17	1	5	21	3	1	9	2
Rem.	19	16	2	5	35	3½	2	11	2

7. *Wine Measure.*

	tons	hhds.	gal.	pts.
From	36	2	32	3
Take	29	3	47	7
Rem.	6	2	47	4

8. *Ale and Beer Measure.*

	hhds	k.	gal.	pts.
From	76	1	16	5
Take	67	2	16	6
Rem.	8	1	16	7

9. *Dry Measure.*

	last	qr.	bush.	p.	g.
From	7	5	5	3	0
Take	6	7	6	3	1
Rem.	0	7	6	3	1

10 *Time.*

	mon.	ws.	ds.	hrs.	m.
From	11	2	3	13	1[illegible]
Take	9	3	5	16	3[illegible]
Rem.	1	2	4	20	4[illegible]

11. A *Silver-smith* bought 5 *ingots* of silver each weighing 5 lb. 9 oz. 17 dwts. 19 gr. Out of which he made and sold 3 *tankards*, each weighing 1 lb. 7 oz. 19 dwts. six *cups*, each 9 oz. 16 dwts. 13 gr. eight salts, each 4 oz. 18 dwts. twelve snuff boxes, each 3 oz. 23 gr. and [illegible] spoons, each 1 oz. How much silver had he left? Ans. 11 lb. 6 oz. 17 dwts. 5 gr.

12 An

12. An *Apothecary* made a composition of several drugs weighing 13 lb. 6 oz. 5 dr. He at one time sold 7 lb. 7 dr. 1 scr. 14 gr. and at another time 3 lb. 9 oz. How much of the composition had he left? Ans. 2 lb. 8 oz. 5 dr. 1 scr 6 gr.

13. Received in lieu of two golden repeaters, sent to Jamaica, in 1763, the 5 *chests* of *indigo* following; and on a like adventure, in 1765, the subsequent five chests: The *question* is, how much indigo I had less the second time than the first?

Anno 1763.	C.	qr.	lb.		lb.	Anno 1765.	C.	qr.	lb.		lb.
No. 1	2	3	16	Tare	24	No. 1	1	3	17	Tare	19
2	4	1	17	—	27	2	2	1	27	—	26
3	7	2	10	—	25	3	3	0	18	—	25
4	3	0	16	—	19	4	5	1	0	—	23
5	6	1	0	—	26	5	2	3	11	—	18

Ans. 8 c. 2 qr. 4 lb. neat.

14. The distance of the Towns A and B is 19 miles 7 furlongs, 126 yards; also of A and C, 100 miles, What is the distance of B and C? Ans. 80 miles, 94 yards.

15. There is 39 gal. 3 qts. 1 pt. of wine, in a hogshead; how much put to it will fill up the hogshead? Ans. 23 gal. 1 pt.

16. A Mercer bought 3 pieces of cloth each containing 27 yds. 3 qr. 2 nls. he at one time sold 44 yds. 1 qr. 3 nls. and at another 13 yds. 3 qr. 2 nls. How much has he left? Ans. 25 yds. 1 qr. 1 nl.

17. There is a field whose content is 18 ac. 1 r. 17 pls. of which 9 ac. 2 r. 27 pls is sown with wheat and the rest with rye. What quantity then is sown with rye? Ans 8 ac. 2 r. 30 p.

18. What quantity of Ale when added to 17 hhds. 17 gal. 7 pts. will make it 20 hogsheads? Ans. 2 hhds. 33 gal 1 pt.

19 There are two quantities of wheat which when added together make 16 lasts, 7 qr. 7 bush. 3 pks. 1 gal. the less quantity is 6 lasts, 5 qr. 6 bush. What is the greater? Ans. 10 lasts, 2 qr. 1 bush. 3 pks. 1 gal.

§ IX.

§ IX. MULTIPLICATION *of different Denominatonis.*

RULE.

1. PLACE the ×er under the least denomination of the multiplicand.

2. Multiply the least denomination by the ×er, see how many ones of the next superior denomination are contained in the product, as in addition of different denominations.

3. Set down the odds, and carry the ones to the product of the next superior denomination, with it proceed as before; and in like manner with all the other denominations to the greatest.

Note, Multiplication and Division of different denominations, must be learned together, because they reciprocally prove each other: therefore the learner is desired to turn to § X. and learn it along with § IX.

EXAMPLES of MONEY.

1. what is the value of 9 yds. of velvet

	l.	s.	d.
at	2	12	$7\frac{1}{2}$
per yd.			9
Ans	23	13	$7\frac{1}{2}$

2. Tea, 3lb at

		8	$9\frac{3}{4}$
			3
Ans.	£ 1	6	$5\frac{1}{4}$

3. Knives 5 doz. at

		3	$6\frac{1}{2}$
			5
Ans.	£	17	$8\frac{1}{2}$

4. Sugar 7 lb. at $8\frac{3}{4}$d. ans. 5s. $1\frac{1}{4}$d.

5. Tobacco 8lb. at 1s. $7\frac{1}{2}$d ans. 13s.

6. Wine 10 gal. at 9s. 6d. ans. 4l. 15s.

7 Sugar 9C. at 4l. 17s. $11\frac{3}{4}$d ans. 44l. 1s. $9\frac{3}{4}$d.

8. Beef 7 stone at 3s $4\frac{1}{2}$d ans. 1l. 3s. $7\frac{1}{2}$d

If

If the ×er be a large composite number made up of several factors ×ed together, it is commonly the best way to × successively by the factors instead of the whole ×er. But note, for the sake of ease, never × by any number above 10.

9. What is the value of 45lb. of tea at

	l.	s.	d.
at		7	$8\frac{1}{2}$
			9
price of 9lb. £	3	9	$4\frac{1}{2}$
			by 5
price of 45lb.	17	6	$10\frac{1}{2}$

10. Raisins 100 lb. at

Raisins 100 lb. at			$7\frac{1}{2}$
			10
price of 10lb.		6	3
			10
of 100lb.	3	2	6

11. Tobacco, 80lb. at 1s. $8\frac{1}{2}$d. ans. 6l. 16s 8d

12. Cloth, 35 yds. at 8s. 6d ans. 14l. 7s. 6d.

13. Silver 81 oz. at 6s. $8\frac{1}{2}$d. ans. 27l. 3s. $4\frac{1}{2}$d.

14. Razors 90 gross at 3l. 13s. $7\frac{1}{2}$. ans. 331l. 6s. 3d.

15 Knives 70 gross at 5l. 10s. $5\frac{1}{2}$d. ans. 386l. 12s. 1d.

If the ×er is not composed of small numbers ×ed together; find two or more numbers whose product comes nearest: then × as before, and add what is wanting, or subtract what is over.

When the given ×er consists of $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$; take $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$, of the multiplicand and add it to the product. But in proving it by division, subtract the value of the $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$ from the given sum, and divide the remainder by the integers.

16. Knives 37 doz.

	l.	s	d
at	2	5	$7\frac{3}{4}$
			6
price of 6 doz. is	13	13	$10\frac{1}{2}$
			6
price of 36 doz. is	82	3	3
+ price of 1 doz	2	5	$7\frac{3}{4}$
price of 37 doz.	84	8	$10\frac{3}{4}$

17. Sciffars 47 grofs

at	3	10	$5\frac{1}{4}$
			8
price of 8 grofs is	28	3	6
			6
price of 48 grofs	169	1	0
—price of 1 grofs	3	10	$5\frac{1}{4}$
price of 47 grofs	165	10	$6\frac{3}{4}$

In the following queftions x is put to reprefent the given price.

	l.	s.	d.
18 Gold $67\frac{1}{2}$ oz. at	3	10	$7\frac{1}{2}$
			8
$x \times x = 8x$	28	5	0
			8
$8x \times 8 = 64x =$	226	0	0
$x \times 3 = 3x =$	10	11	$10\frac{1}{2}$
$x \div 2 = \frac{1}{2}x =$	1	15	$3\frac{3}{4}$
Anf. $67\frac{1}{2}x$	238	7	$2\frac{1}{4}$

19. Coffee $134\frac{3}{4}$ oz. at

$x =$	0		$5\frac{3}{4}$
			10
$x \times 10 = 10x =$		4	$9\frac{1}{2}$
			10
$10x \times 10 = 100x =$	2	7	11
$10x \times 3 = 30x =$	0	14	$4\frac{1}{2}$
$x \times 4 = 4x =$	0	1	11
$x \div 2 = \frac{1}{2}x =$	0	0	$2\frac{3}{4}$
$\frac{1}{2}x \div 2 = \frac{1}{4}x =$	0	0	$1\frac{1}{4}$
Anf. $134\frac{3}{4}x =$	3	4	$6\frac{1}{2}$

20. 6742 at $x =$		0	6
			10
$x \times 10 = 10x =$	4	5	0
$10x \times 10 = 100x =$	42	10	0
			10
$100x 10 = 1000x =$	425	0	0
			6
$1000x \times 6$ $6000x =$	2550	0	0
$100x \times 7 = 700x =$	297	10	0
$10x \times 4 = 40x =$	17	0	0
$x \times 2 = 2x =$	0	17	0
Anf. $6742x =$	2865	7	0

21. Cloth $789\frac{1}{2}$ yds at 1s $7\frac{1}{2}$d Anf. 64l. 2s. $11\frac{1}{4}$d.

22. Snuff-Boxes $103\frac{1}{2}$ doz. at 17s. $6\frac{1}{2}$d. Anf. 90l. 15s. $6\frac{3}{4}$d.

23. Salts $376\frac{1}{2}$ doz at 1l. 17s $8\frac{3}{4}$d. Anf. 710l. 5s. $0\frac{1}{4}$d.

24. $1367\frac{3}{4}$ at 12s. $9\frac{1}{2}$d. Anf. 874l. 15s. $9\frac{1}{2}$d.

25. 7098 at $1\frac{3}{4}$d. Anf. 51l. 15s. $1\frac{1}{2}$d.

26. 14087 at 16s $8\frac{1}{2}$d. Anf 11768l. 10s. $3\frac{1}{2}$d.

Examples

Examples of Weights, Measures, &c.

	lb.	oz	dwts	gr.	
1. Multply	36	11	16	13	by 496
product	18344	10	4	16	

	lb.	oz.	dr.	sc.	gr.	
2. mul·	16,	5	6	1	12	by 45
prod.	741	9	6			

	C.	qr.	lb.	
3. mult.	15	3	12	by 17
prod.	269	2	8	

	miles	fur:	pols	
4. mult:	23	5	25	by 9
prod.	213	2	25	

	yd:	qrs	nls.	
5 mult:	48	3	3	by 27.
prod.	1321	1	1	

	Tuns	hhds:	gal	pts.	
6. mul.	16	3	15	5	by 18
prod.	302	2	29	2	

	acres	roods	p:	
7. mult.	34	2	16	by 5
prod.	173 acres.			

	hhds.	gal.	pts.	
8 mult.	45	32	7	by 27
prod.	1232	20	5	

	c.	qr.	lb.	oz.	drs.	
9	76	3	17	14	13	by 464
prod.	35686 c.	1 qr	1l	9 oz		

	la:	w:	qr.	bush:	
10. mul	1	1	3	5	by 32
prod.	59	1	1		

	m.	w.	d.	h·	m.	
11. mul	7	3	4	22	14	by 7
prod.	55	1	6	11	38	

	Degrees				
12. mult:	5	17	10	20	by 41
prod.	216	44	3	40	

---

§ X. DIVISION *of different Denominations.*

RULE.

1. PLACE the Divisor and Dividend as in simple Division.

2. Begin at the highest denomination and divide each by the Divisor, placing the answer in the Quote, which answer, will be of the same denomination with that divided:

3. If

3. If there be a remainder, after ÷ing any of the denominations except the laſt, × that remainder by ſo many ones of that denomination as makes one of the next leſs; adding thoſe of the ſaid leſs denomination to the product, if ſuch there be, and divide as before.

Thus proceed thro' all the denominations.

EXAMPLES.

1. If 9 yards of velvet coſt 23l. 13s. $7\frac{1}{2}$ what will a yard coſt?

```
        l.   s.   d.
    9)  23   13   7½  (2l.
        18
       ----
Rem.    5l.
        20 , + 13s.
       ----
    9) 113      (12s.
         9
       ----
        23
        18
       ----
rem.     5s:
        12  +7d.
       -------
      9( 67      (7d:
         63
        ----
         •4
         4, + 2 qr.
        -------
      9) 18      (2 q.
         18
        ----
          0
```

EXPLANATION.

Say how oft 9 in 23 2 times, which ſet in the quote; then twice 9 is 18, which —ed from 23, leaves 5l. which × by

20

20, +ing 13s. to the product, and the sum is 113s. Then say how oft 9 in 113? Ans. 12 times, with a remainder of 5. This remainder × by 12 +ing 7 to the product and the sum is 67d. in which 9 may be had 7 times, with a remainder of 4. This remainder × by 4, +ing two to the product and the sum is 18, in which 9 may be had twice without a remainder. So the quote is 2l. 12. $7\frac{1}{2}$ which proves the first sum in compound multiplication. And so of all the rest.

2. What is tea per lb. if 3lb. cost 1. 6s. $5\frac{1}{4}$d.? Ans. 8s. $9\frac{3}{4}$d.

3. What is the 5th part of 17s. $8\frac{1}{2}$d. Ans. 3s. $6\frac{1}{2}$d.

4. What is the 7th part of 5s. $1\frac{1}{2}$d.? Ans. $8\frac{3}{4}$d.

5. What is the 8th part of 13s.? Ans. 1s. $7\frac{1}{2}$d.

6. What is wine per gal. if 10 gals. cost 4l. 15s.? Ans. 9s. 6d.

7. If 44l. 1s $9\frac{3}{4}$d. be equally ÷ed amongst 9 persons, what is the share of each? Ans. 4l. 17s. $11\frac{3}{4}$d.

8. What is beef per stone, if 7 stone cost 1l. 3s. $7\frac{1}{2}$d Ans. 3s. $4\frac{1}{2}$d.

If the ÷or exceed 12, and is a composite number, it is best to divide continually by the factors, and set the quotes under the dividends, as in simple division.

But if the ÷or be not a composite number, you must divide by it after the manner of long division.

9. Divide 17l. 6s. $10\frac{1}{2}$d. by 45

	l.	s.	d.
9)	17	6	$10\frac{1}{2}$
5)	1	18	$6\frac{1}{2}$
Ans.	0	7	$8\frac{1}{2}$

10. Divide 3l. 2s. 6d. by 100.

	l.	s.	d.
10)	2	2	6
10)	0	6	3
Ans.	0	0	$7\frac{1}{2}$

11. Divide

11. If 6l. 16s. 8d. be divided amongst 80 persons what is each man's share? Ans. 1s. $8\frac{1}{2}$d.

12. Divide 14l. 17. 6d. by 35.

	l.	s.	d.
7)	14	17	6
5)	2	2	6
Ans.	0	8	6

13. Divide 27l. 3s. $4\frac{1}{2}$d. by 81.

	l.	s.	d.
9)	27	3	$4\frac{1}{2}$
9)	3	0	$4\frac{1}{2}$
Ans.	0	6	$8\frac{1}{2}$

14. Divide 331l. 6s. 3d. by 90. Ans. 3l. 13s. $7\frac{1}{2}$d
15. Divide 386l. 12s. 1d. by 70. Ans. 5l. 10s. $5\frac{1}{2}$d.
16. Divide 84l. 8s. $10\frac{3}{4}$d. by 37. Ans. 2l. 5s. $7\frac{3}{4}$d.
17. Divide 165l. 10s $6\frac{3}{4}$d. by 47. Ans. 3l. 10s. $5\frac{1}{4}$d.
18. If 67 oz. of silver cost 236l. 11s. $10\frac{1}{2}$d what is it per oz.? Ans. 3l. 10s. $7\frac{1}{2}$d.
19. If 134 oz. of coffee cost 3l. 4s. $2\frac{1}{2}$d. what is it per oz.? Ans. $5\frac{3}{4}$d.
20. Divide 2865l. 7s. by 6742. Ans. 8s. 6d.
21. Divide 64l. 2s. $1\frac{1}{2}$d. by 789. Ans. 1s. $7\frac{1}{2}$d.
22. Divide 90l. 6s. $9\frac{1}{2}$ by 103. Ans. 17s. $6\frac{1}{2}$d.
23. Divide 709l. 6s. 2d. by 376. Ans. 1l. 17s. $8\frac{3}{4}$d
24. Divide 174l. 6s. $2\frac{1}{2}$d. by 1367. Ans. 12s. $9\frac{1}{2}$d.
25. Divide 51l. 15s. $1\frac{1}{2}$d. by 7098. Ans. $1\frac{3}{4}$.d.
26. Divide 11768l. 10s. $3\frac{1}{2}$d. by 14087. Ans. 16s. $8\frac{1}{2}$

Ex.

Examples of Weights, Measures, &c.

1. Divide 18344 lb. 10 oz. 4 dwts. 16 gr. by 496.

```
          lb.      oz.  dwts.  gr.
496)    18344      10    4     16   (36lb.
        1488
        ----
        3464
        2976
        ----
Rem.     488
          12 + 10
        -----------
496)    5866   (11
        5456
        ----
Rem.     410
          20, + 4
        -----------
496)    8204  (16
         496
        ----
        3244
        2976
        ----
Rem.     268
          24, + 16
        -----------
496)    6448   (13
         496
        ----
        1488
        1488   Ans. 36 lb. 11 oz. 16 dwts. 13 gr.
        ----
        ....
```

2. Divide 741 lb. 9 oz. 6 dr. by 45.

	lb.	oz.	dr.	sc.	gr.
9)	741	9	6	0	0
5)	82	5	0	2	0
Anf.	16	5	6	1	12

3. Divide 269 C. 2 qr. 12 lb. by 17. Anf. 15 C. 3 qr. 12 lb.

4. Divide 213 m. 2 f. 25 p. by 9. Anf. 23 m. 5 f. 25 p.

5. Divide 1321 yds. 1 qr. 1 nl. by 27. Anf. 48 yds. 3 qr. 3 nls.

6. Divide 302 tuns, 2 hhds. 29 gal. 2 pts. by 18. Anf. 16 tuns, 3 hhds. 15 gal. 5 pts.

7. Divide 173 acres, by 5. Anf. 34 acr. 2 r. 16 poles.

8. Divide 1232 hhds. 20 gal. 5 pts. by 27. Anf. 45 hhds. 32 gal. 7 pts.

9. Divide 35686 C. 1 qr. 1 lb. 9 oz. by 464. Anf. 76 C. 3 qr. 17 lb. 14 oz. 13 dr.

10. Divide 59 la. 1 w. 1 qr. by 32. Anf. 1 la. 1 w. 3 qr. 5 bush.

11. Divide 55 m. 1 w. 6 dys. 11 h. 38 min. by 7. Anf. 7 m. 3 w. 4 dys. 22 hrs. 14 m.

12. Divide 216 degrees 44 m. 3 f. 40 t. by 41. Anf 5 degrees 17 m. 10 f. 20 t.

13. My purfe and my Money, quoth Jack, is worth 17l. 6s. 10½d. but the money is worth 44 of the purfe: pray what was there in it? Anf. 16l. 19s. 2d.

14. A Gentleman bought a horfe and harnifh for 52l. 8s. 6d. the horfe coft 8 times as much as the harnifh. I demand the value of the horfe and harnifh feparately?

Anf. { The horfe coft 46 l. 12's.
The harnifh 5 l. 16 s. 6 d.

The

The ſum and difference of any two numbers being given, the numbers themſelves may be found thus: Add the given ſum and difference together, and half that ſum will be the greater number; then from the given ſum ſubtract the given difference, and half that remainder will be the leſs number.

EXAMPLES.

1. There are two different ſums of money, which both together make 7642l. 16s $4\frac{1}{2}$d. the leſs quantity taken from the greater leaves 762l. 9s. 4d. What are the ſums ſeparately?

		l.	s.	d.			l.	s.	d.
First		7642	16	$4\frac{1}{2}$	Secondly from		7642	16	$4\frac{1}{2}$
+ed to		762	9	4	take		762	9	4
	2)	8405	5	$8\frac{1}{2}$		2)	6880	7	$0\frac{1}{2}$
Greater ſum		4202	12	$10\frac{1}{4}$	Leſs ſum		3440	3	$6\frac{1}{4}$

2. A Dealer bought two lots of ſnuff, that together weighed 9 cwt. 100 lb. for 97l. 17s. 6d. Their difference in point of weight was 1 cwt. 72 lb. and of price 8l. 13s. 3d. Their reſpective weights and values are required.

		cwt.	lb.		l.	s.	d.
Anſ.	Lot 1ſt.	5	86	Coſt	53	5	$4\frac{1}{2}$
	Lot 2d	4	14		44	12	$1\frac{1}{2}$

3. Says A to B, give me your money and I ſhall have 787l. 19s. $11\frac{1}{2}$d. nay replied B, but if you give me ſo much of your money as I now have, you will but have 246l 10s. left. The queſtion is, how much money had each?

		l.	s.	d.
Anſ	A's money	517	4	$11\frac{3}{4}$
	B's money	270	14	$11\frac{3}{4}$

§ XI. *Of* REDUCTION.

REDUCTION is not properly any Rule of Arithmetic different from Multiplication and Division already taught, but rather an application of these two rules, in converting numbers of one denomination into another, but still retaining the same value; and is generally performed by three *cases* in the following rule.

RULE.

CASE I. If numbers of a greater denomination are to be reduced to their equivalents of a less, multiply continually by all the denominations from the given one to that sought; adding to each product, by the way, those of the same denomination with itself, if such there be, and the last product will be the answer. This is called *reduction descending*.

CASE II. If numbers of a less denomination are to be reduced to their equivalents of a greater, divide continually by all the denominations from the given one to that sought, and the last quote, with the several remainders (if any), will be the answer. This is called *reduction ascending*.

CASE III. If numbers of any proposed *denomination* cannot be reduced to that required by Multiplication or Division alone, both rules are to be used promiscuously; for numbers are frequently reduced to some convenient denomination by the one rule, in order to be reduced to the required one by the other.

Note, When in reduction ascending, you have any remainders after ÷ing, they will always be of the same denomination with their respective dividends.

Ex-

Examples of MONEY.

1. In 36l. how many farthings?

	L.
By case 1.	36
	20
	720 s.
	12
	8640 d.
	4
Anſ.	34560 farthings.

2. In 34560 farthings how many pounds?

By caſe ſecond.

qr. 12) 2 | 0s) anſ.
4) 34560 (8640 d. (72 | 0 (36l.

3. In 32l. 14s. 6¾d. how many farthings?

	l.	s.	d.
	32	14	6¾
	20		
	654 s. + ing 14.		
	12		
	7854 d. + ing 6.		
	4		
Anſ.	31419 qr. + ing 3.		

4. In 31419 farthings how many pounds?

qr. 12) d. 2 | 0)s.
4) 31419 (7854 (65 | 4 (32l.
rem. 3 rem. 6 rem. 14.

Anſ. 32l 14s. 6¾d

By this time the young Student is supposed to be well acquainted with the characters in the Introduction, in which characters will now be inserted the method of working most of the questions, that are solved.

5. In 460 guineas, and 15s. how many crowns? By case 3d. 460 guineas $\times 21, + 15s. = 9675s. \div 5 = 1935$ crowns.

The above operation is to be thus understood; 460 multiplied by 21, and added to 15 is equal to 9675s. which divided by 5 is equal to 1935 crowns.

6. In 1935 crowns how many guineas? By case 3d. $1935 \times 5 = 9675s. \div 21 = 460$ guineas, with a remainder of 15s.

7. In 460 guineas, how many farthings? Ans. 463680 qr.

8. In 540 dollars, at 4s. 4d. per dollar, how many pounds? Ans. 117l.

9. In 460 marks, how many nobles, groats, and farthings? Ans. 920 nobles, 18400 groats, and 294400 farthings.

10. In 340l. how many angels, crowns, half crowns, sixpences, and pence? $340 \times 2 = 680$ ang. $\times 2 = 1360$ cr. $\times 2 = 2720$ half cr. $\times 5 = 13600$ sixpences, $\times 6 = 81600$ pence.

11. In 81600 d. how many six-pences, half crowns, crowns, angels, and pounds? $81600d. \div 6 = 13600$ sixpences $\div 5 = 2720$ half cr. $\div 2 = 1360$ cr. $\div 2 = 680$ ang. $\div 2 = 340l.$

12. If A lends B 1296 guineas when they were valued at 1l. 1s. 6d. a piece: how many must B pay A when they are valued at 1l. 1s. a piece? Ans. 1326 guineas, 18 shill.

13. Reduce 987654320 farthings into pounds, shillings, and pence. Ans. 987654320 q. $=$ 246913580 d. $=$ 20576131s. 8d. $=$ 1028806l. 11s. 8d.

14 Reduce 270 moidores into seven-pence, half-penny pieces. Ans. 270 moi. $=$ 7290s. $=$ 174960 halfpences $=$ 11664 sevenpence, halfpences.

15. How

15. How many pieces of 4d. of 6d. and of 8d. each of an equal number may be had in exchange for 360l. 16s 6d? By case 3 thus, $4+6+8=18$d. the divisor, and 360l. 16s. 6d.$=86598$d. the dividend ; then $86598 \div 18 = 4811$, the number of pieces of each sort, viz. of 4d. of 6d. and 8d. which may be proved, thus ;

		l.	s.	d.
4 pences	$4811 \div 3 \div 20 =$	80	3	8
6 pences	$4811 \div 2 \div 20 =$	120	5	6
8 pences	$4811 \times 2 \div 3 \div 20 =$	160	7	4
Proof		360	16	6

Note, this question is no more than if I had said, in 360l. 16s. 6d. how many 18d. pieces : since the sum of all the parts is equal to the whole. (Ax. 8)

16. Seven persons viz. A. B. C. D. E. F. and G. have 99999l. 2s. amongst them, and to be thus divided, viz. as often as A takes a guinea, B is to take a mark, C an Angel, D a noble, E a crown, F half a crown. and G a six-pence : Each persons particular share is required.

First, a guinea, a mark, an angel, a noble, a crown, a half crown, and a six pence, when + ed together, make 59s. for a divisor, and 99999l 2s.$=1999982$s. for a dividend, whence $1999982 \div 59 = 33898$ the number each takes of his respective piece ; which is proved thus

		l.	s.	d.	
33898 guineas	=	35592	18	0	A's share.
33898 marks	=	22598	13	4	B's share.
33898 angels	=	16949	0	0	C's share.
33898 nobles	=	11299	6	8	D's share.
33898 crowns	=	8474	10	0	E's share.
33898 half crowns	=	4237	5	0	F's share.
33898 six-pences	=	847	9	0	G's share
Proof		99999	2	0	(axiom 8)

17. A General of an army distributes 47l. 17s. $7\frac{1}{2}$d. among 4 captains, 5 lieutenants, and 60 common soldiers,

in the manner following; every captain is to have 3 times as much as a lieutenant, and every lieutenant twice as much as a common soldier: I demand their several shares.

The share of a common soldier,	10s.		$2\frac{1}{4}$d
of a lieutenant	1l.	0	$4\frac{1}{2}$
of a captain	3	1	$1\frac{1}{2}$

18. In 254l. 11s. $2\frac{1}{2}$d. how many French crowns, at 4s. $6\frac{1}{2}$d. per crown? Ans. 1121 crowns.

19. In 1121 French crowns, at 4s. $6\frac{1}{2}$d each, how much sterling money? Ans. 254l. 11s. $2\frac{1}{2}$d.

20. In 4260 dollars, at 4s. 6d. each, how many milreas each 6s. 6d.? Ans. $2949\frac{1}{7}\frac{8}{8}$ milreas.

21. In $2949\frac{1}{7}\frac{8}{8}$ milreas, each 6s. 6d. how many dollars, each 4s. 6d.? 6s. 6d. =78d. the ×er, and 4s. 6d.= 54d. the ÷for, hence 2949×78,+18=230040d. ÷54= 4260 dollars. The Ans.

Examples of Weights, Measures, &c.

1. In 326lb. 11 oz. 17 dwt. 23 gr. Troy wt. how many grains? 326 lb.×12+11=3923 oz.×20+17=78477 dwts. ×24+23=1883471 gr. The Ans.

2. In 1883471 grains troy, how many pounds, ounces and pennyweights? 1883471 gr.=78477 dwts. 23 gr.= 3923oz. 17 dwts. 23 gr.=326lb. 11 oz. 17 dwts. 23 gr. Ans.

3. Reduce 367 lb. 11 oz. 7 dr. 2 scr. 17 gr. into ounces, drams, scruples and grains? 367 lb. 11 oz.=4415 oz,× 8+7=35327 dr.×3+2=105983 scr.×20+17=2119677 grains.

4. Reduce 2119677 gr. apothecaries weight into pounds. 2119677 gr.=105783 scr. 17 gr.=35327 dr. 2 scr. 17 gr. =4415 oz. 7 dr. 2 scr. 17 gr.=367lb. 11 oz. 7 dr. 2 scr. 17 gr.

5. In 36 tons, 17 c. 2 qr. 23 lb. 13 oz. 14 dr. how many drams? Anf. 21151710 drams.

6. In 21151710 drams, how many tons?

21151710 ÷ 16 = 1338981 oz, ÷ 16 = 82623 lb. ÷ 28 =

rem. 14 rem. 13 rem. 23

2950 qr ÷ 4 = 737 c. ÷ 20 = 36 tons.

rem. 2 rem. 17

The Anf. 36 tons 17 c. 2 qrs 23 lb. 13 oz. 14 dr.

7. How many barley corns will reach from Sheffield to London, the diftance being 162 miles?

162 m × 8 = 1296 fur. × 40 = 51840 poles, × 11 = 570240 $\frac{1}{2}$ yds. × 18 = 10264320 in. × 3 = 30792960 barley corns.

8. If 30792960 barley corns will reach from Sheffield to London, how many miles are thefe places afunder?

30792960 ÷ 3 = 10264320 inches, ÷ 18 = 570240, $\frac{1}{2}$ yds. ÷ 11 = 51840 poles, ÷ 40 = 1296 fur. ÷ 8 = 162 miles. Anf.

9. If the circumference of the earth contains 360 degrees, and each degree 70 Englifh miles; how many barley corns will reach round the earth? Anf. 4790016000.

10. In 3 pieces of cloth, each containing 46 yds. 3 qr. 2 nls. how many nls.? Anf. 2250.

11 In 35364 ells Englifh, how many ells Flemifh?

35364 × 5 = 176820 qr. ÷ 3 = 58940 ells Flemifh. Anf.

12. In 370 ac. 2 r. 30 p. of land, how many poles? Anf. 59310.

13. In 32 tuns, 3 hhds. 54 gal. 7 pts of wine, how many pts? 32 × 4 + 3 = 131 hhds. × 63 + 54 = 8307 gal. × 8 + 7 = 66463 pints. Anf.

14. In 66463 pints of wine, how many tuns?

66463 ÷ 8 = 8307 gal. ÷ 63 = 131 hhds. ÷ 4 = 32 tuns.

rem. 7 rem. 54 rem. 3

Anf. 32 tuns, 3 hhds. 54 gal. 7 pts.

15. In

15. In 36 hhds of ale, how many gallons, pints and cubic inches? Anſ. 36 hhds=1836 gal.=14688 pts. and 1836 gal. ×282=517752 cubic inches.

16. In 517752 cubic inches, how many hogſheads, gallons, and quarts of ale? 517752 c in.=1836 gal.=7344 qrts.=36 hogſheads.

17. In 36 chald. 3 qr. 7 buſh. 3 p. 1 gal. of wheat, how many pints? 36×4+3=147 qr.×8+7=1183 buſh.×4+3=4735 pks.×2+1=9471 gal, ×8=75768 pints.

18. In 75768 pints, how many chaldrons of wheat? Anſ. 36 chal. 3 qr. 7 buſh. 3 pks. 1 gal.

19. How many days, hours, and minutes, in 1765 Julian years?

Firſt, 1765÷4=441 days 6 hours, produced by the odd hours in every year. Therefore 1765×365,+441=644666 days, ×24+6=15471990 hrs, ×60=928319400 minutes. The Anſ.

20. In 1765 months how many ſeconds? 1765×28×24×60×60=4269888000 ſeconds.

21. In 4269888000 ſeconds, how many months, weeks, and days? 4269888000 ſeconds=49420 days, ÷7=7060 weeks÷4=1765, months.

22. How many ſeconds in a ſolar year? Anſ 31556937 ſeconds.

23. How many thirds in 29 days 12 hours 44 minutes 1 ſecond, 45 thirds, or one lunar month? Anſ. 153086505 thirds.

24. The mean time of a lunation, i. e. from new moon to new moon, is 29 days, 12 hours 44 minutes, 1 ſecond, and 45 thirds: I demand then how many lunations are contained in 1765 Julian Years? Firſt 1765 Julian years =334194984000 thirds, which ÷ed by 153086505 (the thirds in 1 lunation,) is=21830 lunations, 13 days, 18 hours, 43 minutes, 17 ſeconds 30 thirds. Anſ.

25 How

25. How many minutes, ſeconds, and thirds, in the 12 ſigns of the Zodiac? Anſ. 12×30=360 degrees,= 21600 min.=1296000 ſec.=77760000 thirds.

26. A cubic foot of water weighs 76lb. troy; and air is 860 times lighter then water: What then is the weight of a cubic foot of air? Anſ. 1 oz. 1 dwt $5\frac{70}{860}$ grs.

27. There is a ſquare field, the length of whoſe ſide is 760 links, how many acres doth it contain? Anſ. 5 acres, 3 roods, 4 poles.

```
      760
      760
    -----
5 | 77600
        4
    -----
3 | 10400
       40
    -----
4 | 16000
```

Note. to find the area of a ſquare, 1ſt multiply the ſide into itſelf.

2d. Having multiplied the ſide into itſelf, cut off five figures from the right of the product.

3d. The five figures ſo cut off multiply by 4 and from the right of the product cut off five figures more.

4th. Multiply the five figures laſt cut off by 40, and cut off other five figures as before: thoſe figures to the left are acres, roods, and poles.

The reaſon for cutting off five figures from the right hand of the product is obvious; becauſe your diviſor is 100000, Vide table 7 § VI.

28. There is an oblong field whoſe length is 4620, and breadth 370 yards, what is its area in acres?

Note, to find the area of an oblong, × the length into the breadth. 4620×370=1709400 ſquare yards, which ÷ed by 4840, the ſquare yards in one acre, gives 353 acres, and 880 yards remain.

29. In a triangular field, whoſe baſe is 9 chains, 64 links, or 964 links, and the perpendicular 5 chains, 56 links, how many acres are contained?

Note. To find the area of a triangle, multiply half the baſe into the perpendicular, or half the perpendicular into the baſe, or multiply the baſe and perpendicular together and take half the product; any of theſe ways equally gives the area.

964

964÷2=482
556

2 | 67992
4

2 | 71968
40

28 | 78720

	A.	R.	P.
Anſ.	2	2	28

30. In a Trapezium whoſe Diagonal is 1660 links, one perpendicular is 702 links, and the other perpendicular is 712 links, what is the area in acres ?

Note, To find the area of a Trapezium, multiply half the ſum of the perpendiculars into the Diagonal, or half the Diagonal into the ſum of the perpendiculars, or multiply the ſum of the perpendiculars into the Diagonal and take half the product, any of theſe ways gives the area.

702+712÷2=1660
707

11620
116200

11 | 73620
4

2 | 94480
40

37 | 79200

	A.	R.	P.
Anſ.	11	2	37.

31. Suppoſe a curve whoſe length is 340 links, with breadths taken at five places as follows, viz. the 1ſt 33 links, the 2d. 48, the 3d 62, the 4th 43, and the 5th 24: What is the area ?

Note, To find the area of a curve, or ſegment in a field, meaſure in a right line from one corner of the figure to the

the other, and at right angles to this line take as many breadths as you chuſe, but eſpecially the greateſt and leaſt breadths, add all theſe breadths together and divide the ſum by the number of them which gives the mean breadth, this $\times$ed by the length gives the area.
$33+48+62+43+24=210$ the ſum of the breadths, then $210 \div 5=42$ the mean breadth, and $340 \times 42=14280$ ſquare links the area.

N. B. The ſquare, the oblong, the triangle, the trapezium, and curve, are all the figures that are uſed in the practice of Land Meaſure.

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§ XII. The RULE of THREE DIRECT.

THIS Rule is called the Rule of Three, becauſe it teaches by three numbers given, called terms, to find a fourth that ſhall have the ſame proportion to ſome one of the given numbers, as is expreſſed by the other two. For this reaſon it is ſometimes named the *Rule of Proportion*. And becauſe of its excellent and extenſive uſe, it is often called the *Golden Rule*.

RULE.

In this Rule three things are uſually required, viz. *preparation*, *diſpoſition*, and *operation*.

1. As to the preparation, it muſt be obſerved that of the three numbers given in the queſtion, 2 will always be of the ſame kind, and muſt be reduced to the ſame denomination, if they be not ſo already; and if the remaining number conſiſts of ſeveral denominations, that alſo muſt be reduced to ſome ſimple one.

2. To ſtate the queſtion, diſpoſe the prepared numbers ſo, that the firſt and third may be thoſe of the ſame denomination; place that number the third which aſks the queſtion, or contains the demand, and the remaining number will of courſe be the ſecond term.

3. Having thus ſtated the queſtion, for the operation, multiply the ſecond and third terms together; divide the product by the firſt, and the quote thence ariſing will be the fourth term ſought; which fourth term will always be of the ſame name, kind, or ſpecies with the ſecond.

Note 1. When it can conveniently be done, multiply or divide as in multiplication and diviſion of different denominations.

2. When there happens to be a remainder, it will either make a fractional part, or it muſt be reduced to the name next below the laſt quotient, and divided as before; ſo ſhall the quote be ſo many of the ſaid next name: do this ſo long as there is any remainder, till you have reduced it to the leaſt name, and all the quotes together will be the anſwer.

EXAMPLES.

1. If 3lb. of tea coſt 18 ſhillings, what will 12lb. coſt at the ſame rate?

```
        lb.   s.    lb.
If      3  :  18 :: 12
              12
        ----------------   s.    l.    s.
        3)    216      (  72  = 3     12 the Anſ.
              21
        ----------------
               6
               6
             -----
               0
             -----
```

Note. 12lb. being the term which contains the demand, it muſt occupy the third place, 3lb. being of the ſame kind and denomination, muſt be put in the firſt place; and 18s. which is of the ſame kind with the anſwer, muſt be in the ſecond place.

2. If 12lb. of tea coſt 3l. 12s. what will 3lb. coſt? If 12lb.: 72s.::3lb. $72 \times 3 \div 12 = 18$s. the Anſ.

3.

3. If 18s. will buy 3lb. of tea, how much will 72s. buy at that rate? If 18s. : 3 lb. :: 72s. : $72 \times 3 \div 18 = 12$lb. the Anſ.

4. If 72s. will buy 12lb. of tea, how much will 18s. buy at the ſame rate? If 72s. : 12lb. :: 18s : $18 \times 12 \div 72 =$ 3lb. the Anſ.

Note. The three laſt queſtions are only the firſt varied: by ſuch variations may all the queſtions in the Rule of Three be proved. Or thus, in the Rule of Three Direct, the product of the firſt and fourth terms, will always be equal to the product of the ſecond and third, when the work is right.

5. What will 3 quarters of a yard of velvet coſt, when $21\frac{1}{2}$ yards is worth 22l. 10s. 6d?

Here 22l. 10s. 6d. = 901 ſixpences; and $21\frac{1}{2}$yds = 86 quarters.

```
            qr.    ſix d.    qr.
Then if     86  :   901  ::   3
                      3
                  -------   ſix d.   s.  d.
            86)    2703    (31   =  15 : 6
                   258
                  -----
                    123
                     86
                  -----
                     37
                      6
                  -----
            86)     222 ( 2d.
                    172
                   -----
                     50
                      4
                   -----
            86)     200 ( 2
                    172
                   -----
                     28
                   -----
                     s.   d.   qr.
            Anſ.    15    8    2 28/86
```

Or

	qr.	l	s.	d.		qr.
Or thus, if	86 :	22	10	6	::	3
				3		

86) 67 11 6 (0 l.
20

86) 1351 (15s.
86

491
430

61 Rem.
12

86) 738 (8d.
688

50 Rem.
4

86) 200 (2 qr.
172

28 Rem.

6. What will $21\frac{1}{2}$ yards of velvet cost, when 3 quarters is worth 15s. 8d. $2\frac{28}{86}$q ?

Here 15s. 8d. $2\frac{28}{86} = 754\frac{28}{86}$q and $21\frac{1}{2}$=86qr. Then

3qr. : $754\frac{28}{86}$q : : 86 : $754 \times 86 + 28 \div 3 = 21624$ q. = 2[...]
10s. 6d. the Anf.

7. If $21\frac{1}{2}$ yds.=(86qr.) of cloth cost 22l. 10s. 6d.[...]
21624q.) how many yards may be bought for 15s. 8d. 2[...]

$= 754\frac{28}{86}$q. ?

If 21624 q. : 86 :: 754 $\frac{28}{86}$q : $754 \times 86 + 28 \div$ 21624[...]
3 qr. Anf.

8. Silv[...]

8. Silver at 5s. 8d. per ounce; What's the value of 4 tankards whose weights are as follows? viz.

	lb.	oz.	dwts	grs.
1st weighs	2	11	19	23
2d ditto	3	7	17	7
3d ditto	3	8	14	4
4th ditto	4	0	7	9
The weight of them all	14	4	18	19 =

83011 grains. 1 oz. = 480 grains; and 5s. 8d. = 68d.

Then as 480 gr. : 68d. :: 83011 gr. 48l. 19s. 11d. 3 q. $\frac{272}{480}$ the Ans.

Note. When four terms are placed as in the form above, they are to be understood and read thus: as the 1st term is to the 2d, so is the 3d to the 4th. That is the product of the 2d and 3d terms divided by the 1st produces the 4th term, of the same denomination with the 2d, which 4th term, when reduced to the required denomination is wrote in the 4th place for the answer.

9. If 83011 grains of silver cost 48l. 19s. 11d. 3q. $\frac{272}{480}$ =(47039 $\frac{272}{480}$ q what is valued at per ounce?

As 13011 gr. : 47039 $\frac{272}{480}$ q :: 480 gr. : 5s. 8d. the Ans.

10. A Merchant bought an hhd. of wine for 32l. how much per gallon must he sell it for to gain 4l. 10s. by it? It must all be sold for 32l. + 4l. 10s. = 36l. 10s. = 730s.

Then as 63 gal. : 730s. :: 1 gal. : 11s. 7 $\frac{3}{63}$ d. the Ans.

11. If 1 gallon of wine cost 11s. 7 $\frac{3}{63}$ d. what will a hhd. cost at that rate? Ans. 36l. 10s.

12. If 1 be advanced to 2, and 2 be advanced to 3,
To how much will 20 in such proportion advanced be?

1+2=3 the sum before advanced; 2+3=5 their advanced sum: then as 3 : 5 :: 20 : 33⅓ the Ans.

 13. If

13. If 2 be diminiſhed to 1, and 3 be diminiſhed to 2; what will $33\frac{1}{3}$ be diminiſhed to in the ſame proportion?

As $5 : 3 :: 33\frac{1}{3} : 33\times3+1\div5=20$ the Anſ.

14. There is a ciſtern with two evacuating cocks viz. A and B; the cock A can empty it in 56 hours, and B in 42; in how many hours can they empty the ſame, if they run together?

Firſt $56+42=98$; then ſay, as 98hrs : 56hrs :: 42hrs : 24h. the Anſ.

Note. The work of ſome ſtatings in the Rule of Three Direct, may be much ſhortened when they can be done by any of the three following contractions.

I. Divide the 2d term by the 1ſt, $\times$ing the quote into the third, and the product is the anſwer.

15. If 7 yards coſt 56s. what will 20 coſt?
As 7 yds. : 56s. :: 20yds : 160s $=$8l. the Anſ.
Thus $56\div7=8$, and $20\times8=160$s.

II. Divide the 3d term by the 1ſt, $\times$ing the quote into the 2d, and the product is the anſwer.

16. If 8lb. of ſugar coſt 6s. what will 1cwt. coſt?
As 8lb. : 6s. :: 112lb. : 84s.$=$4l. 4s. the Anſ.
Thus $112\div8=14$, and $14\times6=84$s.

III. When the firſt term and either of the other two, can be exactly $\div$ed by ſome common $\div$ſor; then $\div$ them, and take the quotes inſtead of theſe terms; proceeding thus as oft as you can.

17. If 9 yards coſt 12s. what will 24 yds. coſt?

	yds.	s.	yds.
3)	9 :	12 ::	24
3)	3 :	12 ::	8 contracted by $\div$ing the 1ſt and 3d by 3.
	1 :	4 ::	8 con. by $\div$ing the 1ſt and 2d by 3,
			4
			32s. the Anſ.

18. If a hhd. of wine cost 49l. 14s. what will 72 gallons cost? Ans. 5l. 16s.

	gal.	l.	gal.	
	63 :	49 14 ::	72	
Contr.	7 :	49 14 ::	8	by ÷ing the 1st and 3d terms by 9.
Contr.	1 :	7 2 ::	8	by ÷ing the 1st and 2d by 7.
		8		
		56 16		

19. If $\frac{4}{11}$ of an ounce of gold cost 28s what will $\frac{7}{11}$ oz. cost?

oz. s. oz.

$\frac{4}{11} : 28 :: \frac{7}{11}$

4 : 28 : : 7 cont. by rejecting the ÷ for 11

1 : 7 : : 7 : 49s. the Ans

20. A certain steeple projected upon level ground a shadow to the distance of 57 yards, when a four foot staff perpendicularly erected cast a shadow of 5 feet 4 inches: what was the height of the steeple? Here 5 feet 4 inches =64 inches, and 57 yards=2052 inches Then as 64 in. : 4 feet : : 2052 : 42 yds. 2 f. 3 inches the Ans.

21. If a wall 6 feet thick, 9 feet high, and 432 feet long, cost 720l. in building, what will be the price of a wall of the same materials, that is 12 feet thick, 18 feet high, and 576 feet long?
9×6×432=23328 cubic feet: and 12×18×576=124416 cubic feet. Then as 23328 cu. feet : 720l. : : 124416 cu. feet : 3840l. Ans.

In the mint of England a pound of gold, that is 11 ounces fine and 1 allay, is at this time coined into 44 guineas and a half: I demand how much sterling a pound of pure gold is worth observing that the allay is valued at nothing? Ans. 50l. 19s. 5d. $1\frac{9}{11}$q.

23 What

23. What is the annual interest of 987l. 6s. 5d. at the rate of 6 per cent.? Anf. 59l. 4s. $9\frac{1}{4}$d.

24. If 1000 French feet be equivalent to 1068 English feet, and the circumference of the earth according to the French menfuration be 123249600 French feet: I demand the fame in English miles. Anf. 24930 miles, 57 yards 1 foot 9 inches $\frac{18}{10}$ barley corns.

25. Suppofe all things as in the foregoing queftion, I demand how long a found will be in paffing from pole to pole, upon a fuppofition that a found paffes over 1142 feet in a fecond of time? Anf. 16 hours, 32 feconds 28 thirds.

26. A Draper bought four bales of cloth each containing 6 pieces, and each piece 27 yards, at 16l. 4s a piece; I demand the price of the whole and the rate per yard? Anf. the whole coft 388l. 16s. and 12s. per yard.

27. If the daily expences of all the people in London be 65660l. fterling: how much will keep them a year and a half? Anf. 35948850l.

28. If I give 9d. for a fkin, how much muft I pay for 19 fcore, 6 doz. and 8 skins? Anf. 17l. 5s.

29. A Trader bought 92lb. of tea at 12s. 10d. per lb. and not anfwering in goodnefs, is willing to lofe 40s. by the whole, at what rate muft he fell it by the ounce? Anf. 9d. 1 $\frac{288}{1472}$ qr.

30. A Gentleman fpends 1l. 6s. 8d. a day and lays up at the years end 598l. what is his yearly income? Anf. 1084l. 13s. 4d.

31. I have 820l. 12s. 6d. to diftribute among 32 poor men at $8\frac{1}{2}$d. and 70 poor women at $5\frac{1}{2}$d. and 40 children at $3\frac{3}{4}$ each per day; how long will the fame adminifter to their relief. Anf. 244 $\frac{168}{3228}$ days.

32. Bought

32. Bought 30 pieces of broad cloth for 130l. and sold them again for 150l. Now, if they had cost 150l. how must they have been sold to have yielded gain at the same rate? Ans. 173l. 1s. 6d. $1\frac{11}{13}$ q.

33. A ship sailed 48 leagues in 24 hours, how many leagues will she sail in a week at the same rate? Ans. 336 leagues.

34. A Merchant hath owing him 968l. and his debtor agrees to pay him 13s. for every pound; the question is what must he pay him for the whole debt, and what must the Merchant lose by composition or abatement?

Ans. The debtor must pay 629l. 4s. The Merchant must lose 338l. 16s.

35. As I was beating on the forest grounds,
Up starts a hare before my two greyhounds:
The dogs being light of foot, did fairly run
Unto her fifteen rods just twenty-one:
The distance that she started up before
Was four score, sixteen rods just, and no more.
Now this I'd have you unto me declare,
How far they ran before they caught the hare.

Ans. 336 rods. For $21-15=6$ rods the dogs gain at the hare in running 21 rods; but the question is how far must they run to gain 96 rods at her, i. e. to overtake her, which is thus solved. As $6 : 21 :: 96 : 336$.

XIII. *The Rule of Three Inverse.*

INVERSE, or reciprocal, proportion is, when of four numbers the 3d bears the same ratio or proportion to the first as the 2d. does to the 4th.

Therefore the less the 3d. term is, in respect to the first, the greater will the 4th term be, in respect to the second.

The

The *preparation* and *diſpoſition* of the terms differ nothing here from the Rule of Three Direct, only the *operation* is *inverſe*. The only difficulty is in knowing when a queſtion belongs to one rule, and when to the other; in order to which obſerve the following directions.

Having ſtated the queſtion, obſerve whether the 3d term, being more than the 1ſt, requires more, or being leſs requires leſs; if ſo, the queſtion belongs to the direct rule: but if the 3d term being more than the 1ſt, requires leſs, or being leſs, requires more, it then belongs to the Rule of Three Inverſe, and in ſuch caſe, the fourth term muſt be found by an *inverſe operation*, i. e. by ×ing the firſt and ſecond terms together, and ÷ing the product by the third term.

EXAMPLES.

1. If 10 yards of cloth, half yard wide, will make a garment, how many yards 5 quarters wide, will make a lining for the ſaid garment?

Here it is manifeſt, that by how much more this lining has of breadth, ſo much leſs muſt it have in length: therefore the third term being more than the firſt, and requiring the fourth to be leſs, this is more requires leſs.

	qr.		yds.		qr.		
If	2	:	10	: :	5	:	10×2÷5=4 yds. the Anſ.

2: If 4 yards of cloth 5 quarters wide, will make a garment; how many yards, half yard wide, will line the ſame garment?

Here it is obvious, that by how much leſs the lining has of breadth, ſo much more it muſt have of length: therefore this is leſs requires more.

	qr.		yds.		qr.		
If	5	:	4ds	: :	2	:	5×4÷2=10yds. the Anſ.

This rule may be proved by inverting this queſtion as above, or thus, the product of the firſt and ſecond terms, will always be equal to the product of the third and fourth, when the work is right. Thus in queſt. 1ſt 10×2=5×4.

CON-

CONTRACTIONS.

I. Divide the second term by the third, ×ing the quote into the first, and the product is the answer.

3. If 24 men in 27 days do a piece of work, in how long time will 9 men do the like ?

	men		days		men		days	
If	24	:	27	::	9	:	72	the Ans.

Thus 27÷9=3, and 24×3=72 the Ans. as before.

II. Divide the first term by the third, ×ing the second by the quote and the product is the Answer.

4. If a friend lend me 2800l for 8 months, how long should I lend him 700l. to requite him ?

	l.		m.		l.		m.	
If	2800	:	8	::	700	:	32	the Ans.

Thus 2800÷700=4 and 8×4=32 months the Ans.

III. When the third term and either of the other two can be exactly ÷ed by some common ÷for ; then ÷ them and take the quotes instead of these terms ; proceeding thus as oft as you can.

5. If 378 oxen will eat up the grass of a field in 36 days, how long will 18 oxen be eating up the same ?

	oxen		days		oxen		days
	378	:	36	::	18	:	756 the Ans.
Contracted	63	:	36	::	3		by ÷ing the 1st and 3d by 6
Contracted	63	:	12	::	1		by ÷ing the 2d and 3d by 3.
	12						

Ans. 756 days, by ×ing 1st and 2d terms together.

6. If the penny white loaf ought to weigh 9 ounces troy, when wheat is at 4s. 6d. a bushel ; what ought it to weigh when wheat is at 6s. 9d. a bushel. Ans. 6 ounces.

7. Suppose

7. Suppose 3 cocks, or 4 hens, will eat up a certain quantity of corn in 56 days, how long will 1 cock and 1 hen be in eating up the same?

Because of 3 cocks, or 4 hens, some number must be found that is divisible by 3 and by 4 without remainder, such is the number 12; (for $3 \times 4 = 12$) make then 3 cocks or 4 hens equivalent to 12 chickens, and you will have 1 cock equivalent to 4 chickens, 1 hen to 3 chickens, and 1 cock and 1 hen to 7 chickens; in which case the question will stand thus: If *12 chickens will eat up a certain quantity of corn in 56 days, how long will 7 chickens be in eating up the same?* Ans. 96 days.

8. There are 2400 soldiers in a garrison in a besieged town, who have only provisions for 9 months; how many must be disbanded that the provisions may serve 16 months? Ans. 1050.

9. At what price per bushel is wheat, when the penny white loaf weighs 5 ounces, 8 dwts. if it weighs 9 ounces when wheat is at 4s. 6d. a bushel? Ans. 7s. 6d.

10. If 1l. 2s. worth of wine will suffice a club of 12 men when the wine is sold after the rate of 25l. 4s a hhd; how many men will 1l. 2s. worth serve, when the wine is sold after the rate of 18l. 18s. a hhd? Ans. 16 men.

Here follows a collection of questions put down promiscuously in both rules; wherein the learner is left to himself to distinguish the particular rule, whether Direct or Inverse, which each question belongs to.

1. If the expence in house-keeping, during 6 weeks amount to 9l. 3s. 6d. how long will 100l. last at that rate

Ans. 65 weeks, 2 days, $18\frac{138}{367}$ hours,

2. If 736 dollars, at 4s. 6d. each, were given in exchange for 144 Jacobuses, what was the value of 1 Jacobus Ans. 23s.

3. If

3. If 5 oxen, or 7 colts, will eat up a close in 87 days, in what time will 2 oxen and 3 colts eat up the same? (vide Saunderson's Algebra)

Because of the 5 oxen, or 7 colts, some number must be found that can be $\div$ed by 5 and by 7 without remainder, such is the number $35 = 7 \times 5$; make then 5 oxen or 7 colts equivalent to 35 heifers, and you'll have 1 ox equivalent to 7 heifers, 1 colt to 5 heifers, 2 oxen $= 14$ heifers, 3 colts $= 15$ heifers, and 2 oxen $+$ 3 colts $= 14 + 15 = 29$ heifers, and the question will stand thus: *If 35 heifers will eat up a close in 87 days, in what time will 29 heifers eat up the same?* Ans. in 105 days.

4 If 4lb of tea cost 1l. 13s. 4d. what will 4 parcels cost, each weighing 2 c. 3 qr. 21 lb? Ans. 548l. 6s. 8d.

5. If a field will graze 18 horses for seven weeks, how long will it graze 42 horses? Ans. 3 weeks.

6. If 2 c. 3 qr 21 lb. of sugar cost 6l. 1s. 8d. what will 12 c. 2 qr. cost at the same rate? Ans. 25l. 17s. 8d. $3\frac{3}{47}$q.

7. If 60 gallons of water in one hour's time fall into a cistern, containing 200 gallons, and by a pipe in the same cistern there runs out 45 gallons in one hour; in how many hours will the cistern be filled, if both the cocks keep running? Ans. 13 hours 20 min.

8. If a piece of ordnance shoot $14\frac{3}{4}$lb. of powder at a time; how many pounds will discharge it 68 times? and what will the powder come to, at $11\frac{1}{4}$d. per lb? Ans. 1003lb. of powder. Value 47l. $3\frac{3}{4}$d.

9. If a square pipe $4\frac{1}{2}$ inches wide, will discharge a certain quantity of water in one hours time; in what time will another square pipe, $2\frac{1}{2}$ inches wide, discharge the same quantity from the same current?

The discharge of water through pipes, all other things being the same, is as the area of their orifices. The orifice

fice of a square pipe $4\frac{1}{2}$ inches wide contains 81 square half inches, and the orifice of a pipe $2\frac{1}{2}$ inches wide contains 25 square half inches. Say then: *If an orifice of 81 square half inches will discharge a certain quantity of water in 1 hour, in what time will an orifice of 25 square half inches discharge the same?* Anf. 3 hours, 14 seconds, 24 thirds.

10. A Butcher bought an ox for 10l. 10s. which weighed, besides skin and legs, 108 stone. There were in him $8\frac{1}{2}$ stone of fat, which he sold for $4\frac{3}{4}$d. per lb. the skin and legs he sold for 1l. 5s. 9d how much does 1 stone of beef stand him in? Anf. 1s. 4d. $2\frac{32}{199}$q.

11. a Plummer bought 134 fother of lead at $1\frac{1}{4}$d. per pound, what did the whole cost him? Anf. 1524l. 5s.

Note, A fother of lead is 19 cwt. 2 qr.

12. A man bought 120 eggs at 2 a penny, and 120 more at 3 a penny, he mixt both parcels together and sold them out at 5 for two-pence, whether did he gain or lose by the bargain, and how much? Anf. he lost 4d.

13. A Draper laid out 120l. as follows; viz. one third of it in callicoes, at 20s. 5d. per piece, one third in cambricks at 32s. 9d per piece, and the rest in holland at 5s. 9d: per ell, how much had he of each sort?

Anf. { Callicoes $39\frac{45}{245}$ pieces.
Cambricks $24\frac{168}{393}$ pieces.
Hollands $139\frac{9}{69}$ ells.

§ Compound

§ XIV. *Compound Proportion.*

COMPOUND Proportion (or the double Rule of Three) is, when five numbers are given to find a sixth proportional, and is commonly performed by two operations in the single *Rule of Three*, either *Direct* or *Inverse* as the question requires.

The three first terms contain a supposition, and the two last a demand.

RULE.

1. Of the two numbers which contain the demand, one must be the third term in the first stating, and the other the third term in the second stating.

2. The fourth term, found by the first operation, must always be put for the middle term in the second stating, and the fourth term found by this second operation, is the answer required.

EXAMPLES.

1. If 100l. principal in 12 months, gain 6l. interest; what will 250l. principal gain in 9 months.

If 100l. : 6l. :: 250l. : 250l × 6 ÷ 100 = 15l.

This 4th number, viz. 15l. is the interest of 250l. for 12 months, and the question is reduced to this: If 12 months gain 15l. what will 9 months gain?

m. l. m l. s.

If 12 : 15 :: 9 : 15 × 9 ÷ 12 = 11. 5 the Ans.

2. If 250l. principal in 9 months gain 11l. 5s. = 225s. interest, what will 100l. principal gain in 12 months?

l. s. l. s.

If 250 : 225 :: 100 : 90 fourth term.

m. s m s.

If 9 : 90 :: 12 : 120 = 6l. the Ans.

3. If 100l. principal in 12 months gain 6l. in what time will 250l. gain 11l. 5s.=225s ?

	s.		m.		s.		m.	
6l.=120s. If	120	:	12	::	225	:	$22\frac{1}{2}$	This is direct.

$22\frac{1}{2}$m.=45 half months : then say,

	l.		$\frac{1}{2}$m.		l.		$\frac{1}{2}$m.	
If	100	:	45	::	250	:	18=9 mon.	This is inverse.

4. If the carriage of 100lb. weight 30 miles cost 2s; what will the carriage of 500lb. cost for 200 miles ?

	lb.		s.		lb.		s.
If	100	:	2	::	500	:	10
If	30	:	10	::	200	:	66s. 8d. the Answer.

All questions in Compound Proportion, may be solved with greater expedition by the following Rule.

RULE.

1. Here as in the single Rule of Three, put that term in the second place which is of the same kind and denomination with the answer, or term sought; and the terms of supposition one above another in the first place; all the terms of demand in the same order, one above another in the third place. But note, the first and third term of every row must be of one name or denomination.

2. Take the three terms in the first row, and of them form a question in your mind as in the single Rule of Three, marking which term would be the ×er, viz. the first or the third. Proceed thus with each row as so many separate questions in the single Rule of Three, using the second term in common with each of them, and always marking which extream would be the ×er. That is, in any row say: If the first term gives the second, does the third require more, or less? If more, mark the greater extream, if less, the lesser, for a ×er.

3 Mul-

3. Multiply the middle term and all these ×ers together for a dividend ; and the rest of the terms together for a divisor : the quote thence arising will be the answer, and of the same name with the middle number.

CONTRACTIONS.

I. When the same numbers are concerned in both the dividend and ÷sor, throw them out of both.

II. When it can be done, ÷ any numbers by some common ÷sor, and take the quotes instead of them.

Ex. 5. If a regiment of 936 soldiers eat 351 quarters of wheat in 168 days, how many quarters will suffice an army of 11232 soldiers 56 days?

Soldiers qrs. Soldiers contracted

12) 936 : 351 :: 11232 ×er 78 : 351 :: 936 ×er.
days days
7) 168 : —— :: 56 ×er 24 : —— :: 8 ×er.

by contraction

Dividend $\dfrac{351\times936\times8}{78\times24}=\dfrac{351\times936\times1}{78\times3}=1404$ qr. the Ans.
Divisor

6. If a regiment of 936 soldiers eat 351 quarters of wheat in 168 days, how many soldiers will 1404 quarters suffice 56 days?

qr. Soldiers qr.

351 : 936 :: 1404 ×
× 168 days : — :: 56 days.

Then $\dfrac{936\times1404\times168}{351\times56}=11232$ soldiers the Ans.

EXPLANATION.

Ex. 5th. Say, if 936 soldiers eat up 351 quarters, 11232 soldiers will eat up more quarters, therefore mark the greater extream 11232 for a ×er.

Again, say if 168 days require 351 quarters of wheat to suffice an army; 56 days will require fewer quarters to suffice the same army; so mark the less extream 56 for a ×er.

Then contract the terms by first ÷ing by 12 and then by 7. Again you have 351×936×8 for a dividend, and 78×24 for a ÷sor; which being contracted further by ÷ing both dividend and ÷sor by 8, you have 351×936= 328536 for a dividend, and 78×3=234 for a ÷sor, and the quote 1404 quarters for the Answer.

Ex. 6. Say if 351 quarters serve 936 men, 1404 quarters will serve more; therefore 1404 must be a ×er.

Again say, if 168 days require 936 men to eat up any quantity, 56 days will require more men to eat up the same, mark therefore the greater extream 168 for a ×er. Then having 936×1404×168 for a dividend, and 351×56 for a ÷sor, the quote arising is 11232 men the Answer.

7. If the carriage of 240 feet of wood, that weighs 7 stone a foot, comes to 6l. for 50 miles, how much will the carriage of 70 feet of free stone, that weighs 10st. a foot, cost for 300 miles?

240 ft.	:	120s.	::	70 ft. ×
7 st.	:	———	::	10 st. ×
50 m.	:	———	::	300 m. ×

$$\text{Then } \frac{120\times70\times10\times300}{240\times7\times50}=300=15\text{l. the Ans.}$$

8. If

8. If 240 men, in 6 days of 12 hours each, dig a trench of 8 degrees of hardneſs, and 230 yards long, 3 wide, and 2 deep; in how many days of 10 hours, will 24 men dig a trench, of 5 degrees of hardneſs, 320 yards long, 5 wide, and 3 deep?

$$\begin{array}{rcccl}
\times\ 240 \text{ m.} & : & 6 \text{ days} & :: & 24 \text{ m.} \\
\times\ 12 \text{ hrs} & : & \text{———} & :: & 10 \text{ hrs.} \\
8 \text{ deg} & : & \text{———} & :: & 5 \text{ deg.} \times \\
230 \text{ long} & : & \text{———} & :: & 320 \text{ long} \times \\
3 \text{ w.} & : & \text{———} & :: & 5 \text{ w.} \times \\
2 \text{ deep} & : & \text{———} & :: & 3 \text{ deep} \times
\end{array}$$

$$\text{Dividend } \frac{6\times240\times12\times5\times320\times5\times3}{24\times10\times8\times230\times3\times2} = \quad \div\text{ſor}$$

$$\text{(by contraction)} \frac{6\times12\times5\times32\times5}{8\times23\times2} =$$

$$\frac{3\times3\times5\times16\times5}{23} = 156\tfrac{12}{23} \text{ days the Anſ.}$$

9. If 54 men can build a wall in 18 days, when the day is 17 hours long, in how many days will 68 men build the same when the day is but 9 hours long?

$$\left.\begin{array}{l}
\text{men days men} \\
\times 54 : 18 :: 68 \\
\times 17\text{hr} : \text{—} :: 9\text{hr}
\end{array}\right\} \frac{54\times17\times18}{68\times9} = \frac{27\times17\times2}{34} = 27$$

days the Anſwer.

10. If there muſt be 5400 bricks, 6 inches long, and 3 broad, to pave a hall; how many bricks will it require to pave the same hall that are 9 inches long and 4 broad?

$$\left.\begin{array}{l}
\quad\ \text{bricks} \\
6 \text{ l.} : 5400 :: 9 \text{ l.} \\
3 \text{ b.} : \text{———} :: 4 \text{ b.}
\end{array}\right\} \frac{6\times3\times5400}{9\times4} = 2700 \text{ bricks Anſ.}$$

Note.

Note, The two ſingle compoſing analogies whereby the two laſt queſtions are ſolved, are both of them Inverſe; though it hath been aſſerted by ſome, that it never happens that both analogies are Inverſe.

11. A Scrivener lent 800l. upon intereſt the 17th of Auguſt 1760; and took it in on the 13th of January 1764, what did it amount to at 5l. per cent. per annum?

From the 17th of Auguſt 1760 to the 12th of January 1764 are 1244 days. The day on which money is lent out muſt be reckoned, but not the day on which it is taken in.

$$\begin{array}{lcccc} & \text{p.} & \text{int.} & & \text{p.} \\ & 100\text{l.} : & 5\text{l.} & :: & 800\text{l.} \times \\ \text{days } 365 & : & — & :: & 1244 \text{ days.} \times \end{array}$$

$$\text{Theorem} \frac{800 \times 5 \times 1244}{365 \times 100} = 136\text{l. } 6\text{s. } 6\text{d. } 3\tfrac{225}{365}\text{q Int.}$$

800l. + 136l. 6s. $6\frac{3}{4}$d. = 936l. 6s. $6\frac{3}{4}$d. the Amount.

The above Theorem furniſheth us with a general rule for finding the intereſt of any ſum of money for any number of days, which will be better explained hereafter.

12. If a regiment of 936 ſoldiers eat 351 quarters of wheat in 168 days, how many days will 1404 quarters ſuffice an army of 11232 ſoldiers? Anſ. 56 days.

13. If the carriage of 5 c. 3qr. weight 150 miles, cost 3l. 7s. 4d. what muſt be paid for the carriage of 7 c. 2qr. 25 lb. weight 64 miles at the ſame rate? Anſ. 1l. 18s. 8d.

14. A Gentleman lent a certain ſum of money at per cent per annum for 5 years and 6 months, and the

the end of that term, received for interest thereof 478l. 10s. what was the sum of money lent? Ans. 1450l.

15. If 8 men in 6 days dig 24 cubic fathoms of earth, how many fathoms will 12 men dig in 3 days at the same rate of working? Ans. 18 fathoms.

16. If the carriage of 60 cwt. 20 miles cost 4l. 10s. what weight ought to be carried 30 miles, for 5l. 8s. 9d. at that rate? Ans. 15 cwt.

17. If 2 penny white loaves will suffice 3 men, when wheat is at 4s. 6d. a bushel, how many such loaves will suffice 9 men, when wheat is at 9s a bushel? Ans. 12 penny loaves.

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§ XV. PRACTICE.

THE Rule of Practice is a certain compendious way of working such questions in the Rule of Three as have an unit for the first term, and is of great use among Merchants, Tradesmen, &c. in the quick dispatch of business. It is performed by considering what aliquot part of a pound sterling the given price is, or is reduceable to; and taking such part or parts accordingly.

A TABLE of the aliquot parts of a pound and a shilling sterling, to be got by heart.

s.	d.		d.		
10	0	is half a l.	6	is the 40th of a L.	or $=\frac{1}{2}$ of a shill.
6	8	—— 3d	5	—— 48th	
5	0	—— 4th	4	—— 60th	=3d
4	0	—— 5th	$3\frac{3}{4}$	—— 64th	
3	4	—— 6th	3	—— 80th	=4th
2	6	—— 8th	$2\frac{1}{2}$	—— 96th	
2	0	—— 10th	2	—— 120th	=6th
1	8	—— 12th	$1\frac{1}{2}$	—— 160th	=8th
1	4	—— 15th	$1\frac{1}{4}$	—— 192d	
1	3	—— 16th	1	—— 240th	=12th
1	0	—— 20th	$0\frac{3}{4}$	—— 320th	=16th
0	10	—— 24th	$0\frac{1}{2}$	—— 480th	=24th
0	8	—— 30th	$0\frac{1}{4}$	—— 960th	=48th
0	$7\frac{1}{2}$	—— 32d.			

For

Note, The two single composing analogies whereby the two last questions are solved, are both of them Inverse; though it hath been asserted by some, that it never happens that both analogies are Inverse.

11. A Scrivener lent 800l. upon interest the 17th of August 1760; and took it in on the 13th of January 1764, what did it amount to at 5l. per cent. per annum?

From the 17th of August 1760 to the 12th of January 1764 are 1244 days. The day on which money is lent out must be reckoned, but not the day on which it is taken in.

p.	int.	p.
100l. :	5l. ::	800l. ×
days 365 :	— ::	1244 days. ×

$$\text{Theorem}\ \frac{800 \times 5 \times 1244}{365 \times 100} = 136\text{l. } 6\text{s. } 6\text{d. } 3\tfrac{225}{365}\text{q Int.}$$

$800\text{l.} + 136\text{l. } 6\text{s. } 6\tfrac{3}{4}\text{d.} = 936\text{l. } 6\text{s. } 6\tfrac{3}{4}\text{d.}$ the Amount.

The above Theorem furnisheth us with a general rule for finding the interest of any sum of money for any number of days, which will be better explained hereafter.

12. If a regiment of 936 soldiers eat 351 quarters of wheat in 168 days, how many days will 1404 quarters suffice an army of 11232 soldiers? Ans. 56 days.

13. If the carriage of 5 c. 3qr. weight 150 miles, cost 3l. 7s. 4d. what must be paid for the carriage of 7 c 2qr. 25 lb. weight 64 miles at the same rate? Ans. 1l. 18s. 8

14. A Gentleman lent a certain sum of money at per cent per annum for 5 years and 6 months, and the

the end of that term, received for intereſt thereof 478l. 10s. what was the ſum of money lent? Anſ. 1450l.

15. If 8 men in 6 days dig 24 cubic fathoms of earth, how many fathoms will 12 men dig in 3 days at the ſame rate of working? Anſ. 18 fathoms.

16. If the carriage of 60 cwt. 20 miles coſt 4l. 10s. what weight ought to be carried 30 miles, for 5l. 8s. 9d. at that rate? Anſ. 15 cwt.

17. If 2 penny white loaves will ſuffice 3 men, when wheat is at 4s. 6d. a buſhel, how many ſuch loaves will ſuffice 9 men, when wheat is at 9s a buſhel? Anſ. 12 penny loaves.

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§ XV. PRACTICE.

THE Rule of Practice is a certain compendious way of working ſuch queſtions in the Rule of Three as have an unit for the firſt term, and is of great uſe among Merchants, Tradeſmen, &c. in the quick diſpatch of buſineſs. It is performed by conſidering what aliquot part of a pound ſterling the given price is, or is reduceable to; and taking ſuch part or parts accordingly.

A TABLE of the aliquot parts of a pound and a ſhilling ſterling, to be got by heart.

s.	d.		d.		
10	0	is half a l.	6	is the 40th of a L.	or = $\frac{1}{2}$ of a ſhill.
6	8	—— 3d	5	—— 48th	
5	0	—— 4th	4	—— 60th	= 3d
4	0	—— 5th	$3\frac{3}{4}$	—— 64th	
3	4	—— 6th	3	—— 80th	= 4th
2	6	—— 8th	$2\frac{1}{2}$	—— 96th	
2	0	—— 10th	2	—— 120th	= 6th
1	8	—— 12th	$1\frac{1}{2}$	—— 160th	= 8th
1	4	—— 15th	$1\frac{1}{4}$	—— 192d	
1	3	—— 16th	1	—— 240th	= 12th
1	0	—— 20th	$0\frac{3}{4}$	—— 320th	= 16th
0	10	—— 24th	$0\frac{1}{2}$	—— 480th	= 24th
0	8	—— 30th	$0\frac{1}{4}$	—— 960th	= 48th
0	$7\frac{1}{2}$	—— 32d.			

For

For the better delineating the meaning of any operation, put $x =$ the quantity given, or number propoſed,
$a =$ the firſt operation,
$b =$ the ſecond,
$c =$ the third, and ſo on alphabetically,
$r =$ the rate or price of C. wt. yd. &c.

R U L E 1.

When the given price of 1 is an aliquot part of a penny, ſhilling, or pound, take the ſame part of x by $\div$ing it by ſo many as the given price of 1 is continued in a penny, ſhilling, or pound, for the anſwer in pence, ſhillings, or pounds reſpectively.

E X A M P L E S.

Yards per yd.
$x = 3672$ at 10s.
$x \div 2 =$ 1836l. Anſ.

$x = 4361$ at 6s. 8d.
$x \div 3 =$ 1453l. 13s. 4d. Anſ.

$x = 566$ at 5s.
$x \div 4 =$ 141l. 10s. Anſ.

$x = 6732$ at 4s.
$x \div 5 =$ 1346l. 8s. Anſ.

$x = 5121$ at 3s. 4d.
$x \div 6 =$ 853l. 10s. Anſ.

Yards per yd.
$x = 8347$ at 2s. 6d.
$x \div 8 =$ 1043l. 7s. 6d. Anſ.

$x = 4267$ at 2s.
$x \div 10 =$ 426l. 14s. Anſ.

$x = 5283$ at 1s. 8d.
$x \div 12 =$ 440l. 5s. Anſ.

$x = 6473$ at 1s. 4d.
$x \div 5 = 1294$ 12 $= a$
$a \div 3 =$ 431l. 10s 8d.

$x = 7386$ at 6d.
$x \div 40 =$ 184l. 13s. Anſ.
$x =$

lb. per lb.

$x = 7284$ at 1s. 3d.

$x \div 8 = 910\ 10 = a$

$a \div 2 = 455$l. 5s. Ans.

$x = 3267$ at 1s.

$x \div 20 = 163$l. 7s. Ans.

$x = 7413$ at 10d.

$x \div 4 = 1853\ 5 = a$

$a \div 6 = 308$l. 17s. 6d. Ans.

$x = 5432$ at 8d.

$x \div 30 = 181$l. 1s. 4d.

$x = 6215$ at $7\frac{1}{2}$d.

$x \div 8 = 776\ 17\ 6\text{d} = a$

$a \div 4 = 194$l. 4s. $4\frac{1}{2}$ Ans.

$x = 8437$ at 5d.

$x \div 6 = 1406$l. 3s. 4d. $= a$

$a \div 8 = 175$l. 15s. 5d Ans.

$x = 8347$ at 4d.

$x \div 60 = 139$l. 2s. 4d. Ans.

lb. per lb.

$x = 4915$ at $3\frac{1}{4}$d

$x \div 8 = 614$l. 7s. 6d. $= a$

$a \div 8 = 76$l. 15s. $11\frac{1}{4}$d Ans.

$x = 7284$ at 3d.

$x \div 80 = 91$l. 1s Ans.

$x = 5473$ at $2\frac{1}{2}$d.

$x \div 12 = 456$l. 1s. 8d. $= a$

$a \div 8 = 57$l 0s. $2\frac{1}{2}$d. Ans.

$x = 7432$ at 2d.

$x \div 120 = 61$l. 18s 8d. Ans.

$x = 5743$ at $1\frac{1}{2}$d.

$x \div 80 = 71$l. 15s. 9d. $= a$

$a \div 2 = 35$l. 17s $10\frac{1}{2}$d Ans.

$x = 6578$ at $1\frac{1}{4}$d.

$x \div 12 = 548$l 3s. 4d. $= a$

$a \div 4 = 137$l. 0s. 10d. $= b$

$b \div 4 = 34$l 5s. $2\frac{1}{2}$d. Ans.

$x = 4327$ at 1d.

$x \div 120 = 36$l. 1s. 2d $= a$

$a \div 2 = 18$l. 0s. 7d. Ans.

x =

lb. per lb.

$x = 8274$ at $\frac{3}{4}$d.

$x \div 80 =$ 103l. 8s. 6 $= a$

$a \div 4 =$ 25l. 17s. $1\frac{1}{2}$d. Anſ.

$x = 7438$ at $\frac{1}{2}$d.

$x \div 120 =$ 61l. 19s. 8d. $= a$

$a \div 4 =$ 15l. 9s. 11d. Anſ.

lb. per lb.

$x = 8347$ at $\frac{1}{4}$d.

$x \div 120 =$ 69l. 11s. 2d. $= a$

$a \div 8 =$ 8l. 13s. $10\frac{3}{4}$d. Anſ.

proof of the laſt example.

$x = 8347$ at $\frac{1}{4}$d.

$x \div 80 =$ 104l. 6s. 9d $= a$

$a \div 12 =$ 8l. 13s. $10\frac{3}{4}$d Anſ.

The proof of this rule is to work the queſtion by different methods, or by the Rule of Three.

RULE II.

When the given price of 1 is not an aliquot part of a penny, ſhilling, or pound, divide it into ſeveral aliquot parts; then work for each by Rule I. and their ſum will be the anſwer. Or it may often be ſo divided, that the leſs will be aliquot parts of the greater; then take the ſame parts of the prices found for the greater.

EXAMPLES.

lb. per lb.

$x = 3987$ at $2\frac{1}{4}$d.

$x \div 120 =$ 33l. 4s. 6d. $= a$

$a \div 8 =$ 4l. 3s. $0\frac{3}{4}$ $= b$

$a + b =$ 37l. 7s. $6\frac{3}{4}$d. Anſ.

lb. per lb.

$x = 4713$ at $4\frac{1}{4}$d.

$x \div 120 =$ 39l. 5s. 6d. $= a$

$x \div 120 =$ 39l. 5s. 6d $= a$

$a \div 8 =$ 4l 18s. $2\frac{1}{4}$d $= b$

$2a + b =$ 3l. 9s. $2\frac{1}{4}$d. Anſ.

lb.	per lb.

$x = 6395$ at $3\frac{1}{2}$d.

$x \div 80$ is 79l. 18s. 9d $= a$
$a \div 6$ is 13l. 6s. $5\frac{1}{2}$d is b

$a + b$ is 93l. 5s. $2\frac{1}{2}$d. Anſ.

$x = 4325$ at $4\frac{1}{2}$d.

$x \div 80$ is 54l. 1s. 3d. $= a$
$a \div 2$ is 27l. 0s. $7\frac{1}{2}$d is b

$a + b$ is 81l. 1s. $10\frac{1}{2}$d.

$x = 3891$ at $4\frac{3}{4}$d.

$x \div 80$ is 48l. 12s. 9d $= a$
$a \div 2$ is 24l. 6s: $4\frac{1}{2}$ is b
$b \div 6$ is 4l. 1s $0\frac{3}{4}$d is c

$a + b + c$ is 77l. 0s. $2\frac{1}{4}$d Anſ

$x = 6793$ at $5\frac{3}{4}$d.

$x \div 80$ is 84l. 18s. 3d. $= a$
$x \div 120$ is 56l: 12s. 2d. is b
$a \div 4$ is 21l. 4s. $6\frac{3}{4}$d. is c

$a + b + c$ is 162l. 14s. $11\frac{3}{4}$d. Anſ.

$x = 534$ at $7\frac{1}{4}$d

$x \div 40$ is 13l 7s. 0d. $= a$
$a \div 6$ is 2l. 4s. 6d is b
$b \div 4$ is 0l. 11s. $1\frac{1}{2}$d is c

$a + b + c$ is 16l. 2s. $7\frac{1}{2}$d Anſ.

lb.	per lb.

$x = 4443$ at $7\frac{3}{4}$d.

$x \div 40$ is 111l. 1s. 6d. $= a$
$a \div 4$ is 27l. 15s. $4\frac{1}{2}$d. is b
$b \div 6$ is 4l. 12s. $6\frac{3}{4}$d. is c

$a + b + c$ is 143l. 9s. $5\frac{1}{4}$d. Anſ.

Ells.	per Ell.

$x = 4846$ at $8\frac{1}{2}$d.

$x \div 60$ is 80l. 15s. 4d. $= a$
$x \div 60$ is 80l. 15s. 4d. is a
$a \div 8$ is 10l. 1s. 11d. is b

$2a + b$ is 171l. 12s. 7d.

$x = 7501$ at $8\frac{3}{4}$d.

$x \div 120$ is 62l. 10s. 2d. $= a$
$a \times 3$ is 187l. 10s. 6d. is b
$b \div 8$ is 23l. 8s $9\frac{3}{4}$d. is c

$a + b + c$ is 273l. 9s $5\frac{3}{4}$d. d

$x = 4591$ at $9\frac{3}{4}$d.

$x \div 40$ is 114l. 15s. 6d. $= a$
$a \div 2$ is 57l. 7s. 9d $= b$
$b \div 12$ is 4l. 15s. $7\frac{3}{4}$d. $= c$

$a + b + c$ is 176l. 18s. $10\frac{3}{4}$d Anſ.

Ells per Ell.

x =4065 at 9d.

x $\div$ 80 is 50l. 16s. 3d.=a

a $\times$ 3 is 152l. 8s. 9d. Ans.

x =4416 at $9\frac{1}{2}$d.

x $\div$ 40= 110l. 8s. =a
a $\div$ 2 is 55l. 4s. is b
b $\div$ 6 is 9l. 4s. is c

a+b+c is 174l. 16s. Ans.

x =5559 at $11\frac{3}{4}$d.

x $\div$ 30= 185l. 6s. 0d.=a
x $\div$ 80 is 69l. 9s. 9d. is b
b $\div$ 4 is 17l. 7s. $5\frac{1}{4}$d is c

a+b+c is 272l. 3s. $2\frac{1}{4}$d. Ans.

x =3956 at 1s. $0\frac{3}{4}$d.

x $\div$ 40= 98l. 18s. 0d=a
x $\div$ 40 is 98l. 18s. 0d. is a
a $\div$ 8 is 12l. 7s. 3d. is b

2a+b is 210l. 3s. 3d. Ans.

x =6756 at 1s. $1\frac{1}{4}$d.

x $\div$ 20= 337l 16s 0d=a.
a $\div$ 12 is 28l. 3s. 0d. is b
b $\div$ 4 is 7l. 0s. 9d. is c

a+b+c is 372l 19s 9d Ans

Ells per Ell.

x =9479 at 1s. $1\frac{3}{4}$d.

x $\div$ 20 is 473l. 19s. 0d.=a
a $\div$ 8 is 59l. 4s. $10\frac{1}{2}$ is b
b $\div$ 6 is 9l. 17s. $5\frac{3}{4}$d is c

a+b+c is 543l 1s $4\frac{1}{4}$d Ans.

x =7128 at 1s. $2\frac{1}{4}$d.

x $\div$ 20 is 356l. 8s. 0d.=a
a $\div$ 6 is 59l. 8s. 0d. is b
b $\div$ 8 is 7l. 8s. 6d. is c

a+b+c is 423l. 4s. 6d.

x =9124 at 1s. $3\frac{1}{4}$d.

x $\div$ 80 is 114l. 1s. 0d.=a

a $\times$ 5 is 570l. 5s. 0d. is b
a $\div$ 12 is 9l. 10s. 1d is c

b+c is 579l. 15s. 1d. Ans.

x =8397 at 1s. $3\frac{3}{4}$d.

x $\div$ 20 is 419l. 17s. 0d.=a
a $\div$ 4 is 104l. 19s. 3d. is b
b $\div$ 4 is 26l. 4s. $9\frac{3}{4}$d. is c

a+b+c is 551l 1s $0\frac{3}{4}$d Ans

x =4597 at 1s. 4d.

x $\div$ 30 is 153l. 4s. 8d=a

a $\times$ 2 is 306l. 9s. 4d. Ans

Ells

Ells per Ell.

x =2597 at 1s. $4\frac{1}{4}$d.

x÷120 is 21l. 12s. 10d.=a

a×8 is 173l. 2s. 8 is b
a÷8 is 2l. 14s. $1\frac{1}{4}$ is c

b+c is 175l. 16s. $9\frac{1}{4}$d.

Ells per Ell.

x =7948 at 1s. $4\frac{3}{4}$d

x÷20 is 397l. 8s. od.=a
a÷4 is 99l. 7s. od. is b
b÷2 is 49l. 13s. 6d. is c
c÷6 is 8l. 5s. 7d is d

a+b+c+d=554l 14s 1d.
Anſ

x =8090 at 1s. $4\frac{1}{2}$d.

x÷20 is 404l. 10s. od=a

a÷8 is 50l. 11s. 3d. is b

b×11 is 556l. 3s. 9d. Anſ.

x =3798 at 1s. $6\frac{3}{4}$d.

x÷20 is 189l. 18s. od=a
a÷2 is 94l. 19s. od. is b
b÷8 is 11l. 17s. $4\frac{1}{2}$d is c

a+b+c 296l 1,s $4\frac{1}{2}$d Anſ.

RULE III.

Sometimes the value may be eaſily found by reckoning the price of ſome even number above what is given, and then taking ſome aliquot part for what is above, and ſubtracting it from the former.

EXAMPLES.

lb. per lb.

x =3987 at $1\frac{3}{4}$d

x÷120 is 33l. 4s. 6d. =a
a÷8 is 4l. 3s. $0\frac{3}{4}$d is b

a−b is 29l. 1s. $5\frac{1}{4}$d. Anſ.

lb. per lb.

x =112 at 11d.

x÷20 is 5l. 12s. od=a
a÷12 is 0l. 9s. 4d. is b

a−b is 5l. 2s. 8d. Anſ.

lb. per lb.
x =2958 at 1s. $5\frac{1}{4}$d.

x÷40 is 73l. 19s. 0d. = a

a×3 is 221l. 17s 0d is b
a÷8 is 9l. 4s. $10\frac{1}{2}$d. is c

b—c is 212l. 12s. $1\frac{1}{2}$d Anſ

x =4721 at 1s. 5d.

x÷12 is 393l. 8s. 4d. = a
x÷80 is 59l. 0s. 3d. is b

a—b is 334l. 8s. 1d. Anſ.

dozens per doz.
x =3219 at $5\frac{1}{2}$d.

x÷40 is 80l. 9s. 6d. = a
a÷12 is 6l. 14s. $1\frac{1}{2}$d is b

a—b is 73l. 15s. $4\frac{1}{2}$d. Anſ.

dozens per doz.
x =8746 at $5\frac{1}{4}$d

x÷40 is 218l. 13s. 0d. = a
a÷8 is 27l. 6s. $7\frac{1}{2}$d is b

a—b is 191l. 6s. $4\frac{1}{2}$d Anſ.

x =7698 at 1s. $2\frac{3}{4}$d

x÷80 is 96l. 4s. 6d = a

a×5 is 481l. 2s. 6d. is b
a÷12 is 8l 0s. $4\frac{1}{2}$d is c

b—c is 473l. 2s. $1\frac{1}{2}$d. Anſ.

x =4793 at 1s. $5\frac{1}{2}$d

x÷12 is 399l. 8s. 4d = a
a÷8 is 49l. 18s. $6\frac{1}{2}$d is b

a—b is 349l 9s. $9\frac{1}{2}$d Anſ.

Queſtions to exercise the foregoing RULES.

Queſtions lb.		s.	d.		Anſwers. l.	s.	d.
4350	at	1	$7\frac{1}{4}$	is	348	18	$1\frac{1}{2}$
6987	at	1	$7\frac{3}{4}$	is	574	19	$5\frac{1}{4}$
4329	at	1	$8\frac{3}{4}$	is	374	5	$6\frac{3}{4}$
4859	at	1	$9\frac{3}{4}$	is	440	6	$11\frac{1}{4}$
8696	at	0	7	is	253	12	8
6748	at	0	$9\frac{3}{4}$	is	274	2	9
5349	at	0	$10\frac{1}{2}$	is	234	0	$4\frac{1}{2}$
8107	at	0	$11\frac{1}{2}$	is	388	9	$2\frac{1}{2}$

7160

Questions		s.	d.		Answers l.	s.	d.
7100	at	5	10½	is	2085	12	6
702	at	11	4½		399	5	3
853	at	13	9		586	8	9
184	at	15	4		141	1	4
571	at	19	0¾		544	4	8¼
9271	at	1	10½		869	3	1½
3578	at	1	10¾		339	3	3½
6986	at	1	11		669	9	10
3583	at	1	11¾		354	11	4¼
3543	at	0	8¼		121	15	9¾
959	at	0	10¼		40	19	1¾
5056	at	0	10¾		226	9	4
3500	at	0	11¼		164	1	3
579	at	3	2		91	13	6
1504	at	9	2		689	6	8
847	at	12	7½		534	13	4½
714	at	14	11¼		533	5	4½
544	at	18	3¾		498	2	0
970	at	19	8		953	16	8

RULE IV.

If there are Pounds in the Price, multiply the given quantity by the number of them, and if there be also some odd money, find its produce by the former Rules and add them together.

EXAMPLES.

x = 7642 at 6l.
x×6 is 45852l. Ans.

x = 347 at 16l.
x×16 is 5552l. Ans.

x =369 at 3l. 6s. 8d.

x×3 is 1107l. = a
x÷3 is 123l. is b

a+b is 1230l. Anſ.

x =261 at 2l. 10s. 6d

x×2 is 522l. 0s. 0d. =a
x÷2 is 130l. 10s. 0d is b
b÷20 is 6l. 10s. 6d. is c

a+b+c is 659l 0s 6d Anſ

Questions		l.	s.	d.		Anſwers l.	s	d.
407	at	1	13	5		680	0	7
941	at	7	0	4		6602	13	8
950	at	4	17	8	is	4639	3	4
4613	at	1	5	7		5900	15	11
318	at	4	6	10		1380	13	0
563	at	11	14	6		6601	3	6

RULE V.

When the price is an even number of ſhillings, multiply the quantity by half their number, doubling the units, or firſt figure of the product, for ſhillings; the reſt are pounds.

EXAMPLES.

x =7423 at 6s.

x×3 is 2226l. 18s. Anſ.

x =7412 at 18s.

x×9 is 6670l. 16s. Anſ.

x =8324 at 2l. 6s.

x×2 is 16648l. 0 s. =a
x×3 is 2497l. 4s. is b

a+b is 19145l. 4s. Anſ.

x =8327 at 1l. 8s.
x×4 is 3330l. 16s. =a

x+a is 11657l. 16s.

x =8327 at 1[illegible]s.

x×7 is 5828l. 18s. Anſ.

x =83 at 5l. 18s.

x×5 is 415l. 0s. = a
x×9 is 74l. 14s. is b

a+b is 489l. 14s. Anſ.

Queſtions

Questions		Answers		
s.		l.	s.	d.
345 at 12	is	207	0	0
456 at 14		319	4	0
567 at 18		510	6	0
979 at 8		391	12	0

RULE VI.

When the rate is an odd number of ſhillings, work for the greateſt even number contained in it by the laſt Rule; and for the other odd ſhilling, take one 20th of the given quantity. Or × the quantity by the number of ſhillings, and ÷ the product by 20; the quote will be pounds, and the remainder ſhillings.

EXAMPLES.

x =682 at 7s.

x×3 is 204l. 12s. =a
x÷20 is 34l. 2s. is b

a+b is 238l. 14s. Anſ.

x =678 at 7s.

x×7 is 4746s. =a

a÷20 is 237l. 6s. Anſ.

x =238 at 2l. 17s.

x×2 is 476l. 0s. 0d. =a
x×8 is 190l. 8s. is b
x÷20 is 11l. 18s. is c

a+b+c is 678l. 6s. Anſ.

x =7622 at 3s.

x×1 is 762l. 4s. =a
x÷20 is 381l. 2s. is b

a+b is 1143l. 6s. Anſ.

x =789 at 11s.

x×11 is 8679s. =a

a÷20 is 433l. 19s. Anſ.

Questions.		Anſwers	
		l.	s.
898 at 17s.	=	763	6
796 at 13s.		517	8
652 at 15s.		489	0

RULE

RULE VII.

When the quantity, whose price is required, consists, of several subordin ate denominations whereof only the highest is rated; work for the rated denomination singly, by some of the foregoing rules; and for the lower denominations, add such parts of the rate, as those lower denominations are parts of the rated denomination.

A Table of aliquot parts of a cwt.

lb.	cwt.	lb.	cwt.
56	is $\frac{1}{2}$	7	is the 16th
28	— 4th	4	— 28th
16	— 7th	$3\frac{1}{2}$	— 32d
14	— 8th	2	— 56th
8	— 14th	$1\frac{3}{4}$	— 64th
		1	— 112th

Note. Aliquot parts of most other things are easily found.

EXAMPLES.

What's the value of 123 C. 2 qr. 14lb. at 1l. 13s. 4d. =r. per C.

		l.	s.	d.	
123 C. at 1l. 0s. 0d.	=	123	0	0	=a
123 C. at 0 6 8	is	41	0	0	is b
123 C. at Ditto	is	41	0	0	is b
r ÷2, for 2 qrs.	is	0	16	8	is c
c ÷4, for 14lb	is	0	4	2	is d
+2b+c+d	is	206	0	10	Ans.

What's

What's the value of 34 tons 5 C. 2 qr: at 25l. 10s. = r per ton.

Tons l.		l.	s.	d.	
34×25	=	850	0	0	= a
34÷2	is	17	0	0	is b
r÷4, for 5 C.	is	6	7	6	is c
c÷10, for 2 qr.	is	0	12	9	is d
a+b+c+d	is	874	0	3	Anf.

What's the value of 37 C. 3 qrs. 14 lb. at 5l. 17s. $7\frac{1}{2}$d per cwt.? Anf. 218l. 19s. $3\frac{1}{2}$d.

What's the value of $483\frac{3}{4}$ yards at 17s. $10\frac{1}{2}$d per yard? Anf. 432l. 7s. $0\frac{1}{4}$d.

What's the value of 7 C. 2 qr. $15\frac{1}{2}$lb. at 3l. 7d. per cwt.? Anf. 23l. 2s. 9d.

What's the value of 14 acres, 3 ro. 5 pls. at 2l. 12s. 10d. per acre? Anf 39l. $11\frac{3}{4}$d.

What's the value of 3 C. $22\frac{1}{4}$lb. at 13s. $5\frac{1}{4}$d. per C.? Anf. 2l. 3s. 2d.

What come 12 gallons, 5 pints to at 5s. 8d. per gallon? Anf. 3l. 11s. $6\frac{1}{2}$d.

Note 1. If the rated denomination be not the highest denomination of the quantity proposed, reduce the highest to the rated denomination.

What is the value of 4 tons, 5 cwt. 1 qr. 7 lb. at 1l. 5s. = r per C.?

ton	C.	qr.	lb.
4	5	1	7
20			

at 1l.	=	85l.	0s.	0 d.	= a
1	is	85	0	0	is a
a÷4	is	21	5	0	is b
r÷4	is	0	11	3	is c for 1 qr.
c÷4	is	0	2	$9\frac{3}{4}$	is d for 7 lb.
a+b+c+d =		191	19	$0\frac{3}{4}$	Anf.

I ſhall give ſome examples wherein the given quantity is but the part of a integer or whole thing.

3 lb. at 7l. = r per C.

		l.	s.	d.		lb.
$r \div 8$	=	0	17	6	= a, for	14
$a \div 7$	is	0	2	6	is b, for	2
$b \div 2$	is	0	1	3	is c,	1
$b + c$	is	0	3	9	for	3

12lb at 14l. 7s. 6d = r per C.

$r \div 7$	=	2	2	6	= a for	16lb.
$a \div 4$	is	0	10	$7\frac{1}{2}$	is b for	4
$a - b$	is	1	11	$10\frac{1}{2}$	for	12

49lb at 9l. 19s. 10d. = r per C.

$r \div 4$	=	2	9	$11\frac{1}{2}$	= a for	28lb.
$a \div 2$	is	1	4	$11\frac{3}{4}$	is b for	14
$b \div 2$	is	0	12	$5\frac{3}{4}\ \frac{1}{2}$	is c for	7
$a + b + c$	is	4	7	$5 - \frac{1}{2}$	for	49

90lb. at 12l. 17s. 3d. = r per C.

$r \div 2$	=	6l.	8s.	$7\frac{1}{2}$d.	= a for	56lb.
$a \div 2$	is	3	4	$3\frac{3}{4}$	is b	28
$b \div 7$	is	0	9	$2\frac{1}{4}$	is c	4
$c \div 2$	is	0	4	$7 - \frac{1}{2}$	is d	2
$a + b + c + d$	is	10l.	6s.	$8\frac{1}{2}$d. $\frac{1}{2}$	for	90

97lb at 9l. 17s. 11d. = r per C.

r÷2 =	4	18	11½ = a for	56lb.
a÷2 is	2	9	5¾ is b	28
b÷4 is	0	12	4¼ ¾ is c	7
b÷7 is	0	7	0¾ is d	4
d÷2 is	0	3	6¼ ½ is e	2
a+b and c is	8	11	4¼ ¼ for	97

Note 2. Tons, hundreds, and quarters are reduced to the form of l. s. d. by multiplying only the quarters by 3. Then work by some of the foregoing rules.

What's the value of 33 tons, 3 C. 3 qrs. at 33l. 3s. 4d. per ton?

	l.	s.	d.			l.	s.	d.
Let r =	33	3	9 thrice the qrs.					
			10					
r×10 =	331	17	6 = a					
			3					
a×3 is	995	12	6 is b	}	at	33	0	0
r×3 is	99	11	3 is c	}				
r÷6 is	5	10	7½ is d		at	0	3	4
b+c+d is	1100	14	4½		Ans. at	33	3	4

RULE VIII.

TROY WEIGHT.

The ounces and penny weights are in the proportion of l. s. but the grains are as half-pence; therefore take half the grains, and the whole is reduced to the form of l. s. d.

What

What comes 67 oz. 8 dwt. 9 gr. of Gold to, at 4l. 2s. per oz.

	l.	s.	d.		l.	s.
Let r =	67	8	$4\frac{1}{2}$ half the grains.			
			4			
$r \times 4$ is	269	13	6 = a at		4	0
$r \div 10$ is	6	14	10 is b at		0	2
a + b is	276	8	4 Anf. at		4	2

What comes 45 oz. 6 dwt. 7 gr. of plate to, at 5s. 10d. per oz.

	l.	s.	d.		s.	d.
Let r =	45	6	$3\frac{1}{2}$ half the grains.			
$r \div 4$ is	11	6	$6\frac{3}{4}\ \frac{1}{2}$ = a at		5	10
$a \div 6$ is	1	17	$9-\frac{1}{2}\ \frac{1}{6}$ = b at		0	10
a + b is	13	4	4 $\frac{1}{6}$ Anf. at		5	10

What comes 36lb. 11 oz. 12dwts. 12gr. to at 8s 4d per oz.

First $36 \times 12 + 11 = 443$ oz. and 12 grs. $\div 2 = 6$ then we have

	l.	s.	d.			
	443	12	6 = x, at		8s.	4d.
$x \div 3 =$	147	17	6 is a for		6	8
$a \div 4$ is	36	19	$4\frac{1}{2}$ is b for		1	8
a + b is	184	16	$10\frac{1}{2}$ for		8	4 Anf.

What comes 736lb 11 oz. 12 dwts. 13 gr. to at 3l. 13s. 4d. per lb.? Anf. 1744l. 11s. 11½d. nearly.

§ XVI.

§ XVI. TARE and TRET.

GROSS WEIGHT of any commodity, is its own weight together with that of its package, whether cask, chest, or whatever else.

TARE is the weight of the package, or an allowance made instead thereof. What remains after the Tare is taken from the Gross, may be called *Tare-suttle*, if there are more deductions.

TRET is an allowance of 4lb. upon every 104lb. of Tare-suttle on account of dust or other waste. What remains after Tret is deducted, may be called Tret-suttle, if there be any following deductions.

CLOFF is an allowance of 2lb. for every 3 cwt. and some say for every 100lb. of Tret-suttle, to make the weight hold good when sold by retail.

When all the deductions are made, the last remainder is called Neat or Net weight.

Note. The Tret being 4 to 104, or 1 to 26, will be found by taking the 26th part of the Tare-suttle.

CASE I.

If the Tare be at so much the whole; subtract the Tare from the Gross weight, and the remainder is Neat.

EXAMPLE I.

Suppose the Gross weight 261 cwt. 2 qr. 18 lb. 13 oz. Tare to be allowed 9 cwt. 3 qr. 19 lb. 14 oz. What's the Neat weight?

	c.	qr.	lb.	oz.	
From	261	2	18	13	Gross.
Take	9	3	19	14	Tare.
Answ.	251	2	26	15	Neat.

E X A M P L E II.

If the Groſs weight be 323 cwt. 2 qr. 17 lb. Tare 11 cwt. 1 qr. 18 lb. what is the Neat weight ?

Anſw. 312 cwt. 27 lb.

C A S E II.

When the allowance for Tare is at ſo much per frail, cheſt, bag, &c. then × the Tare of one frail by the number of frails, and ſubtract the product from the Groſs weight.

E X A M P L E I.

What is the Neat weight of 4 frails of raiſins, weighing Groſs as follows, Tare 18 lb. per frail ?

	cwt.	qr.	lb.	
No. 1 Groſs weight	3	1	27	
2	3	2	18	
3	3	1	13	
4	3	3	0	
	14	1	2	whole Groſs weight, $=x$
$18 \times 4 = 72$ lb. $=$	0	2	16	whole Tare $=a$
$x - a =$	13	2	14	Neat.

E X A M P L E II.

In 16 hogſheads of tobacco, each 5 cwt. 1 qr. 19 lb. Groſs, Tare per hhd. 100 lb. how much Neat weight ?

	cwt.	qr.	lb.	
	5	1	$19 = x$	
			4	
$x \times 4 =$	21	2	$20 = a$	
			4	
$a \times 4 =$	86	2	$24 = b$	Groſs.
$16 \times 100 = 1600$ lb. $=$	14	1	$4 = c$	Tare.
$b - c =$	72	1	20	Neat.

EXAM-

EXAMPLE III.

In 70 bales of Smyrna silk, each weighing 317lb. Grofs, Tare at 16lb. per bale, what is the Neat weight? Anfw. 21070lb.

EXAMPLE IV.

What is the Neat weight of 30 bales of Cyprus filk, each bale weighing 249lb. Grofs, Tare per bale 14lb? Anfw. 7050lb.

CASE III.

When the Tare is an aliquot part or parts of an hundred weight, work by the rules in Practice, i. e. take fuch part or parts of the Grofs weight as the Tare is of (112lb.) an hundred weight.

EXAMPLE I.

What is the Neat weight of 44 cwt. 1 qr. 17 lb. Grofs, when 16 lb. per cwt is allowed for Tare?

	cwt.	qr.	lb.	
$x=$	44	1	17	Grofs.
$x\div7=$	6	1	$10\frac{3}{7}=a$,	Tare.
$x-a=$	38	0	$6\frac{4}{7}$	Neat.

EXAMPLE II.

What's the Neat of 32 cwt. 2 qr. Tare at 14 lb. per cwt?

	cwt.	qr.	lb.	
$x=$	32	2	0	Grofs.
$x\div8=$	4	0	7	Tare $=b$.
$x-b=$	28	1	21	Neat.

2 cwt.

	cwt.	qr.	lb.	
Grofs x=	124	3	0	Tare 18lb. per cwt.
x÷7=	17	3	8=a,	for 16 lb.
a÷8=	2	0	$25\frac{1}{2}$=b,	for 2 lb.
a+b=	20	0	$5\frac{1}{2}$=c,	for 18lb.
x—c=	104	2	$22\frac{1}{2}$ Neat.	

	cwt.	qr.	lb.	
Grofs x=	289	2	0	Tare 22 per cwt.
x÷8=	36	0	21=a,	for 14lb.
a÷2=	18	0	$10\frac{1}{2}$=b,	for 7lb.
b÷7=	2	2	$9\frac{1}{2}$=c,	for 1lb.
a+b+c=	56	3	13=d	Tare for 22lb.
x—d=	232	2	15 Neat.	

	cwt.	qr.	lb.	
Grofs x=	246	[illegible]	4	Tare 21lb. per cwt.
x÷8=	30	3	4=a,	for 14lb.
a÷2=	15	1	16=b,	for 7lb.
a+b=	46	0	20=c,	Tare for 21lb.
x—c=	200	0	12 Neat.	

What's the Neat of 29 cwt. 3 qr. 24 lb. Tare 12 lb. per cwt? Anfw. 26 cwt. 3 qr. $\frac{1}{2}$lb.

What's the Neat of 39 cwt. 1 qr. 14 lb. Tare at 18 lb. per cwt? Anfw. 33 cwt. 0 qr. $5\frac{1}{4}$ lb.

What's the Neat of 363 cwt. 3 qr 14 lb. Grofs, when 24 lb. per cwt. is allowed for Tare? Anfw. 285 cwt. 3 qr. $15\frac{1}{2}$lb.

CASE

CASE IV.

When Tret is allowed with Tare, find the Tare-futtle according to the foregoing rules; and the Tret will be found by taking the 26th part of the Tare-futtle; alfo when Cloff is allowed, take it from the Tret-futtle, and the remainder is Neat.

EXAMPLE I.

Grofs 17 cwt. 3 qr. 14 lb. Tare 12lb. per cwt. Tret 4 lb. to 104 lb. or 1 to 26; and Cloff 2 to 100, or 1 to 50; how much Neat weight?

	cwt.	qr.	lb.	oz.
x=	17	3	14	0
x÷8 is	2	0	26	4=a, for 14lb.
a÷7 is	0	1	7	12 is b, for 2lb.
a—b is	1	3	18	8 is c, Tare, 12lb.
x—c is	15	3	23	8 is d, Tare-futtle
d÷26 is	0	2	12	12 is e, Tret.
d—e is	15	1	10	12 is f, Tret-futtle
f÷50 is	0	1	6	6 is g, Cloff.
f—g is	15	0	4	6 Neat.

EXAMPLE II.

Bought 3 hhds, of fugar whofe Grofs weights are as [un]der,

		cwt.	qr.	lb.
[T]are of all 1 cwt.	No. 1 Grofs weight	11	2	15
qr. 2 lb. Tret	2	15	1	17
lb. to 104. at	3	17	3	14

16s. per cwt. Neat, what come they to? Anfw. 117l.

EXAMPLE III.

What is the Neat weight of 3 hhds of tobacco, weighing 15 cwt. 3 qr. 20 lb. Groſs, Tare per cent. 7 lb, Tret 4 lb. to 104, Cloff 2 lb. for 3 cwt? Anſw. 14 cwt. 1 qr. 3 lb. nearly.

EXAMPLE IV.

In 7 hhds of tobacco, each weighing 5 cwt. 2 qr. 7 lb. Groſs; how much Neat weight, allowing 8 lb. per cent for Tare, 4 lb per 104 lb. for Tret, and 2 lb. per 3 cwt. for Cloff? Anſw. 34 cwt. 0 qr. 7 lb. 14 oz.

EXAMPLE V.

In 4 frails of raiſins, each 7 cwt. 1 qr. 12 lb. Groſs, Tare 2 qr. 10 lb. per frail, Tret 4 lb. per 104 lb. and Cloff 2 lb. per 100 lb; how much Neat? Anſw. 25 cwt. 2 qr. $1\frac{1}{4}$ lb.

Note. In calculating oil, if you reduce the Neat weight into half pounds, and divide them by 15, the quote will be the Neat gallons; becauſe 7 lb. and a half Neat are allowed to the gallon.

EXAMPLE I.

If 14 lb. per cwt. be allowed for Tare, how many gallons Neat are there in 5 caſks of oil, weighing as follows:

	cwt.	qr.	lb.	
No. 1	4	3	26	
2	7	1	18	
3	6	2	20	
4	5	3	16	
5	7	2	4	
$x=$	32	2	0	Groſs
$x \div 8$ is	4	0	7	$=a$ Tare.
$x-a$ is	28	1	21	is $6370=b$, $\frac{1}{2}$lbs. Neat.

$b \div 15$ is $424\ \frac{10}{15}$ gallons the Anſw.

EXAM-

EXAMPLE II.

In 7 casks of oil, each weighing 3 cwt. 1 qr. Gross; how many Neat gallons, allowing 20lb. per cwt. Tare, and $7\frac{1}{2}$lb. per gallon? Answ. $279\frac{1}{15}$ gallons.

EXAMPLE III.

In 289cwt 2qr. Gross weight of oil, how many gallons Neat at $7\frac{1}{2}$lb. per gallon, allowing 22lb. per cwt. for Tare? Answ. 3474 gallons.

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§ XVII. OF INTEREST.

INTEREST is either *simple* or *compound*: and is the premium, or reward, allowed for the loan of money by the borrower to the lender.

First of Simple Interest: in which the four following things are particularly to be regarded.

1. The money lent, called the *principal*, which put $=p$.
2. The interest of 100l. for one year, called the rate per cent. *per annum*, or *rate* of *interest*: which put $=r$.
3. The *time* for which the said money, or *principal*, is lent: which put $=n$, whether years, half years, quarters, months, weeks, or days.
4. The amount ($=m$,) or the sum of principal and interest: which, when the other three are given, may be found as follows.

A GENERAL RULE.

Multiply the principal by the rate per cent. and that product by the time, whether in years, half years, quarters, months, weeks, or days, for a dividend; then multiply 100 by one year for a divisor, taking the year in the same denomination as the time proposed in the question, and the quotient thence arising will be the interest, in the same denomination with the principal; to which add the principal,

principal, and the sum will be the amount, That is, $\frac{p \times r \times n}{100 \times t}$ = the interest, And $p + \frac{p \times r \times n}{100 + t} = m$, the amount. t being put = one year, or the months, weeks, or days in one year. Vide Theorem to question 11th page 75.

EXAMPLES.

1. If 467l. be put out to interest for 6 years, what will it amount to in that time at 5 per cent. per annum?

First $\frac{467 \times 5 \times 6}{100 \times 1} = \frac{14010 \text{ dividend}}{100 \text{ divisor}}$

14010 ÷ 100 =	140l.	2s	the interest
	467	0	principal
The amount	607	2 = m	

2. What is the amount of 674l. 17s. 6d = p at $4\frac{1}{2}$l. per cent per annum = r, for $5\frac{1}{2}$ years = n?

	l.	s.	d.	qr.	
p =	674	17	6	0	
p × 4 is	2699	10	0	0	= a
p ÷ 2 is	337	8	9	0	is b
a + b is	3036	18	9	0	is p × r
			5		
p × r × 5 is	15184	13	9	0	is c
p × r ÷ 2 is	1518	9	4	2	is d
c + d is	16703	3	1	2	is p × r × n, dividend
e ÷ 100 × t is	167	0	7	$2\frac{30}{100}$	is f, the interest
p + f is	841	18	1	$2\frac{30}{100}$	is m, the amount.

What

3. What is the intereſt of 450l. for 9 months, at 4 per cent per annum?

By the rule $450 \times 9 \times 4 = 16200$ the dividend, and the time given in the queſtion being months, we have $12 \times 100 = 1200$ the diviſor; then $16200 \div 1200 = 13$l. 13s. the intereſt as required.

4. What is the intereſt of 560l. for $3\frac{3}{4}$ years, at $3\frac{1}{2}$ per cent. per annum?

Firſt $3\frac{3}{4}$ years = 15 quarters, then $560 \times 3\frac{1}{2} \times 15 = 29400$, the dividend, and the time in the queſtion being quarters, we have $4 \times 100 = 400$ the diviſor; then $29400 \div 400 = 73$l. 10s. the intereſt required?

5 What will 340l. 10s. 6d amount to in 3 weeks, at 4 per cent. per annum?

Firſt 340l. 10s. 6d. = 13621 ſix-pences,

Then $\dfrac{13621 \times 4 \times 3}{100 \times 52} = \dfrac{163452}{5200} = 30\dfrac{1452}{5200}$ ſix-pences

the intereſt, therefore the amount is 341l. 5s. $7\frac{1}{2}$d nearly.

6. What is the intereſt of 7760 d. for 200 days, at $4\frac{1}{2}$ per cent. per annum?

$$\frac{7760 \times 4\frac{1}{2} \times 200}{100 \times 365} = 191 \text{ d. } 1\frac{135}{365} \text{ qr.} = 15\text{s. } 11\text{d.}$$

$1\frac{135}{365}$ qr. the intereſt required.

7. What is the amount of 19s. $6\frac{1}{2}$d = 469 half pence, at 6 per cent per annum, for 3674 days? Anſw. 1l. 1s. 4d. 0. $\frac{18272}{365co}$ qr.

PARTICULAR RULES.

RULE I.

When the rate is at 5 per cent, × the principal in pence by

by the number of days, dividing the product by 7300, and the quote will be the interest in pence.

Note, 7300 is found by multiplying 365 by 100, and dividing by 5.

8. What's the interest now due on a bond of 320l 10s. commencing interest July the 10th 1764, this being the 25th of March 1765, at 5 per cent per annum?

From July 10th 1764, to March 25th 1765, are 258 days, and 320l. 10s. = 76920d. Therefore $76920 \times 258 \div 7300 = 2718\frac{1}{2}$d. = 11l. 6s. $6\frac{1}{2}$d. the interest, sought.

9. What's the interest of 260l. 14s. 7d. for 760 days, at 5 per cent per annum? Answ. 27l. 2s, 10d. $2\frac{46}{73}$ qr.

10. What will 76l. 13s. 10d. amount to in 3 years 164 days, at 5 per cent. per annum? Answ. 89l. 18s. 4d. $1\frac{4516}{7300}$ qr.

RULE II.

Twenty years interest of any sum of money at 5 per cent. is equal to the principal. Therefore the interest at this rate may be found by taking such proportional part of the principal as the time is of 20 years; and for any other rate, it is but adding or subtracting a proportionable part.

11. What's the interest of 327l. for 6 years, 7 months, and 12 days, at 5 per cent per annum?

		l.	s.	d.	
p	=	327	0	0	= to 20 years interest.
$p \div 4$	is	81	15	0	is a, for 5 years.
$a \div 5$	is	16	7	0	is b, for 1 year.
$b \div 2$	is	8	3	6	is c, for 6 months.
$c \div 6$	is	1	7	3	is d, for 1 month.
$d \div 5$	is	0	5	$1\frac{3}{5}$	is e, for 6 days.
$d \div 5$	is	0	5	$1\frac{3}{5}$	is e, for 6 days.
$a+b+c+d+2e$	=	108	3	7	$3\frac{1}{5}$ the Answ.

Now

Now if the intereſt had been required at 4, or 6 per. cent. it would but be ſubtracting or adding a fifth part;

		l.	s.	d.	
Thus,	x =	108	3	$7\frac{3}{4}$	= intereſt at 5 per cent.
	$x \div 5$ is	21	12	$8\frac{3}{4}$	is a,
	x—a is	86	10	11	= intereſt at 4 per cent.
	x+a is	129	16	$4\frac{1}{2}$	= intereſt at 6 per cent.

Or, having found the intereſt at 5 per cent. ſay, as 5 is the intereſt found :: ſo is any other given rate : to the intereſt required. Thus,

	l.	s.	d.			l.	s.	d.
					4 :	86	10	11
As 5 :	108	3	$7\frac{3}{4}$	::	6 :	129	16	$4\frac{11}{25}$

12. May the 18th, 1762, I put out 624l. 18s. 10d. at intereſt at 5 per cent. per annum; how much does the intereſt amount to the 23d of July 1765?

		y.	m.	d.
From	May 18th 1762 to May 18th 1765 is	3	0	0
	May 18th 1765 to July 18th 1765 is	0	2	0
	July 18th 1765 to July 23d 1765 is	0	0	5

The Anſw. 99l. 7s. $6\frac{1}{2}$d.

RULE III.

To find the intereſt of any ſum of money for a year, × the principal by the rate of intereſt, and ÷ the product by 100.

13. What is the intereſt of 769l. for a year at 6 per cent. per annum?

$769 \times 6 \div 100 = 46$l. 2s. 9d. 2 $\frac{40}{100}$qr. the intereſt.

Note,

Note, If the intereſt of one year be multiplyed by any number of years, the product will be the intereſt for that number of years.

14. What is the intereſt of 769l. for 5 years at 6 per cent. per annum?

	l.	s.	d.	qr.	
769×6÷100=	46	2	9	2 $\frac{40}{100}$	intereſt for 1 year.
				5	
Anſw.	230	14	0	0	intereſt for 5 years.

Note. The above rules ſerve to calculate any thing that's rated at ſo much per cent: as Factorage, Brokerage, Inſurance, purchaſing of Stocks, &c.

OF FACTORAGE.

Factorage, Commiſſion, or Proviſion, is an allowance made by the Merchants to their Factors or Agents, for buying or ſelling of their goods; and is generally at a certain rate per cent. and calculated, as for one year, by the 3d particular rule.

EXAMPLE I.

What is the commiſſion of 862l. 10s. at $3\frac{1}{2}$l. per cent? Anſw. 30l. 3s. 9d.

EXAMPLE II.

What is the factorage of 568l. 15s. 6d. at $2\frac{1}{2}$l. per cent? Anſw. 14l. 4s. 4d. 2 $\frac{60}{100}$ qr.

OF BROKERAGE.

Brokerage is the allowance made to Brokers for aſſiſting others in buying and diſpoſing of their goods: and is

is generally calculated by ÷ing the principal by 100, and taking ſuch a part of the quote as the rate per cent. is of a l.

EXAMPLES.

1. What is the brokerage of 847l. 10s. at 4s per cent?

847l. 10s. ÷ 100 = 8l. 9s. 6d. ÷ 5 = 1l. 13s. 10d. $3\frac{1}{5}$qr. the brokerage required. Or by the Rule of Three, thus,

as 100l. : 4s. :: 847l. 10s. : 1l. 13s. 10d. $3\frac{20}{100}$ qr.

2. What is the brokerage of 650l. at 5s. or one 4th per cent? Anſw. 1l. 12s. 6d.

OF INSURANCE.

Inſurance is a ſecurity given that the value of goods, ſhips, houſes, &c. ſhall be reſtored, in caſe of loſs or damage, by ſtorms, fire, &c. For ſuch ſecurity a premium is paid down, at a certain rate per cent. and may be calculated by ſome of the foregoing rules.

EXAMPLES

1. What is the inſurance of 356l. 3s. for 16 months at $7\frac{1}{2}$ per cent. per annum?

By the general rule in page 98,

$$\frac{356\text{l. } 3\text{s.} \times 7\frac{1}{2} \times 16}{100 \times 12} = 35\text{l. } 12\text{s. } 3\text{d. } 2\frac{480}{1200}\text{qr.}$$ The Anſw.

2. How much muſt be paid yearly for the inſurance of a houſe, whoſe value is 287l. 10s. per annum, at $3\frac{1}{2}$ per cent? Anſw. 10l. 1s. 3d.

OF STOCKS.

Stocks are the public funds of the nation; the ſhares of which being transferable from one perſon to another

 occaſion,

occasion the extensive business called, Stock-jobbing. These are commonly best calculated by the Rule of Three.

E X A M P L E S.

1. What is the purchase of 6490l. 15s. 6d. Bank Stock at 110¾ per cent?

```
                              l.      s.     d.
As 100l. : 110¾l. :: 6490    15      6     | x              |
                                     10    |                |
               ---------------------------  |                |
                     64907   15      0     | x × 10         | a
                             10            |                |
               ---------------------------  = |              = |
                    649077   10      0     | a × 10         | b
                     64907   15      0     |                | a
                      3245    7      9     | x ÷ 2          | c
                      1622   13     10½    | c ÷ 2          | d
               ---------------------------  |
                  7188 | 53   6      7½    | a + b + c + d.
                       | 20
                  ------
                    10 | 66
                       |        Answ. 7188. 10s. 7d 3 98/100 qr.
                  ------
                     7 | 99
                       |  4
                  ------
                     3 | 98
```

2. What is the purchase of 2400l. South-Sea Stock at 123¾ per cent? Answ. 2970l.

3. What is the purchase of 2474l. 16s. Bank Annuities at 96⅓, or 96l. 6s. 8d. per cent? Answ. 2384l. 1s. 1d $3\frac{4}{100}$ qr

What

4. What is the purchase of 575l. 10s. South-Sea Annuities at $131\frac{3}{4}$ per cent? Answ. 758l. 4s. 5d. $\frac{1}{10}$ qr.

5. At $97\frac{1}{4}$l. per cent. what is the purchase of 254 l. 17s. Bank Annuities? Answ. 247l. 16s. 9d. 3 $\frac{96}{100}$ qr.

OF COMPOUND INTEREST.

COmpound Interest is that which arises from any principal and its interest put together, as the interest becomes due; that is, when at every payment, or at the time when the payments become due, the amount is converted into a *new principal*. On this account it is sometimes called interest upon interest.

For instance, were I to put out 100l. for two years at 6l. per cent. per annum, according to the utmost improvement, the interest as it becomes due not being received but forborn, at the end of the first year it would amount to 106l. which 106l. would be my principal for the second year; and at the determination of the second year it would be increased to 112l. 7s. $2\frac{1}{2}$d. or 100l. principal, 12l. for two years simple interest, and 7s. $2\frac{1}{2}$d. the interest of 6l. my first year's interest.

RULE.

Find the amount of the given principal, for the time of the first payment, by simple interest; then make this amount your principal for the second payment, whose amount calculate in the same manner; and so on through all the payments, still accounting the *last amount* as the principal of the next payment.

EXAMPLES.

1. What is the compound interest of 500l. forborn 3 years, at 5 per cent. per annum?

First 500 × 5 ÷ 100 equal to 25l. the 1st year's interest.

Then 500 + 25 equal to 525l. the second years principal.

Secondly 525 × 5 ÷ 100 equal to 26l. 5s. the 2d year's interest.

Then 525 + 26l. 5s. equal to 551l. 5s. the 3d year's principal.

Thirdly 551l. 5s. × 5 ÷ 100 equal to 27l. 11s 3d. the 3d year's interest.

	l.	s.	d.	
The 3d year's principal is	551	5	0	Add.
The 3d year's interest is	27	11	3	
The total amount is	578	16	3	Sub.
The money first lent is	500	0	0	
The total interest is	78	16	3	

2. What will 550l. 10s. amount to in $3\frac{1}{2}$ years, at 6 per cent. per annum, compound interest? Answ. 675l. 6s. $5\frac{3}{4}$d.

3. What is the amount of 650l. for 5 years at $5\frac{1}{2}$ per cent. per annum compound interest? Answ. 849l. 10s. $5\frac{1}{2}$d.

Note. If the payments are not yearly, but half yearly, quarterly, monthly, or any other aliquot part of a year, take such aliquot part of the principal and with it work for the interest as before. Or work by the general Rule in Simple Interest: for the Rule still holds true, provided there be always a complete integer number of times of payments, that is, a certain number of times without a part of another: nor will it be just to calculate for a complete time and then to take the same part of the result as is the part of the time, for the Rule takes no notice of such parts. In this manner some Authors have made erroneous calculations. By Logarithms it is as easy to perform calculations with parts of times of payments as with whole ones; nor is it impossible to perform such calculations without Logarithms, but the trouble in some cases is intolerable.

What

4. What is the amount of 600l. forborn 2 years, at 5 per cent. per annum, ſuppoſing the intereſt payable half yearly? Anſw. 662l. 5s $9\frac{1}{8}$d. For $600 \div 2 \times 5 \div 100$ equal to 15l. the 1ſt half years intereſt. $600 + 15$ equal to 615l the principal for the 2d half year. $615 \div 2 \times 5 \div 100$ equal to 15l. 7s. 6d. the intereſt for the 2d half year. $615 +$ 15l. 7s. 6d. equal to 630l. 7s. 6d. the principal for the 3d half year. 630l. 7s. 6d. $\div 2 \times 5 \div 100$ equal to 15l. 15s. $2\frac{1}{4}$d. the intereſt for the 3d half year. 630l. 7s. 6d. + 15l. 15s. $2\frac{1}{4}$d. equal to 646l. 2s. $8\frac{1}{4}$d. the principal for the 4th half year. 646l. 2s. $8\frac{1}{4}$d. $\div 2 \times 5 \div 100$ equal to 16l. 3s. $0\frac{3}{4}\ \frac{1}{4}$ the 4th half years intereſt. 646l. 2s. $8\frac{1}{4}$d + 16l 3s. $0\frac{3}{4}\ \frac{1}{4}$ equal to 662l. 5s. $9\frac{1}{8}$d. the amount at the end of 2 years, or the 4 half years.

5. What will 600l. amount to in 2 years at 5 per cent. per annum, compound intereſt, ſuppoſing the intereſt payable quarterly?

$600 \times 5 \div 400$ equal to 7l. 10s. the intereſt for the 1ſt quarter. $600 +$ 7l. 10s. equal to 607l 10s. the principal for the d quarter. 607l. 10s. $\times 5 \div 400$ equal to 7l. 11s. $10\frac{1}{2}$d the intereſt for the ſecond quarter, and ſo on to the 8th quarter, whoſe intereſt is 8l. 3s. $7\frac{1}{2}$d. the whole amount being 662l. 13s. $9\frac{3}{4}$d. the Anſwer required.

6. What will 50l. amount to in 5 years at 5 per cent per annum compound intereſt, ſuppoſing the intereſt payable half yearly? Anſw. 64l. 1d.

7. What is the amount of 217l. forborn $2\frac{1}{4}$ years, at 5 per cent per annum, ſuppoſing the intereſt payable quarterly? Anſw 242l. 13s. $4\frac{1}{2}$d.

See more of Intereſt in Decimals.

REBATE or DISCOUNT.

REBATE, or DISCOUNT, is an allowance for paying money before the time appointed for payment, and is the difference between a ſum of money due

a certain time to come, and its present worth; which present worth, if put to interest for the time and rate for which the discount is to be made, would amount to the sum or debt then due.

RULE.

Multiply the time by the rate of interest; also multiply 100 by 1 year, taking the year in the same *denomination* as the time proposed: add the two *products* together, and say,

As the sum of the two products,
Is to the former of the two products;
So is the sum to be discounted, in what denomination so ever,
To the discount, in the same denomination.

The discount so found subtract from the sum discounted; and the remainder is the ready money to be paid down by the discounter.

This ready money may also be found, without finding the discount thus,

As the total of the two products,
Is to the latter of the two products;
So is the sum to be discounted, in what denomination so ever,
To the present worth, in the same denomination.

EXAMPLES.

1. What is the present worth of 463l. 10s. due 9 months hence, discount at 4 per cent. per annum?

9 months × 4 = 36 the 1st product,
100 × 12 months = 1200 the 2d product.

1236 the sum of the two products

Then as 1236 : 36 :: 463l. 10s. : 13l. 10s. the discount.
Then 463l. 10s.—13l. 10s. equal to 450l. the present worth
Or, as 1236 : 1200 :: 463l. 10s. : 450l. the present worth.

What

2. What is the present worth of 460 l. due 5 weeks hence, discount at 7 per cent. per annum?

5 weeks × 7 = 35 the 1ft. product.
52 × 100 = 5200 the 2d. product.

5235 the sum of the two products.

As 5235 : 5200 :: 460 l. : 456 l. 18s. 5d. 3 $\frac{2955}{5235}$ qr. the present worth required.

3. How much ready money may be given for a Note of 357 l. 12s. 6d. due 234 days hence, discount at 4 per cent. per annum?

234 × 4 = 936 the 1st. product.
365 × 100 = 36500 the 2d product.

37436 the sum of the two products.

357 l. 12s. 6d. = 14305 six-pences.
Then as 37436 : 36500 :: 14305 six pences : 348l. 13s. 8d $\frac{3104}{37436}$ qr. present worth.

Or, as 37436 : 936 :: 14305 : 8l. 18s. 9d. 3 $\frac{34332}{37436}$ discount.

4. What is the present worth of 767l. 17s. 6d. due $3\frac{1}{2}$ years hence, discount at 5 per cent. per annum?

7 half years × 5 = 35 the 1ft product.
2 × 100 = 200 the 2d. product.

235 the total of the two products.

Then as 235 : 200 :: 767l. 17s. 6d. : 653l. 10s. 2d. 2 $\frac{50}{235}$ the Answ.

A

5. A man has a Legacy of 500 l. left him, to be paid 20 years hence; but wanting money, he is deſirous to ſell the ſame; how much ready money muſt he receive allowing the purchaſer $5\frac{1}{2}$ per cent? Anſw. 238l. 1s. 10d. $3\frac{9}{2}$. qr.

6. What is the diſcount of 624l. for three months at 6 per cent. per annum? Anſw. 9l. 4s. 5d. +

Note, When the ſign more is annexed to the anſwer, it denotes the anſwer to be a ſmall matter more.

7. What is the rebate of 125l. 10s. payable 10 months hence, at $4\frac{1}{2}$d. per cent. per annum? Anſw. 4l 10s. $8\frac{1}{2}$d.

8. Sold goods to the amount of 166l. 12s. to be paid 6 months hence, what muſt I have at preſent, diſcount at 8 per cent. per annum? Anſw. 160l. 3s. 10d. +

9. Bought goods to the value of 17l. 14s. to be paid for 8 months hence, what muſt I pay in ready money, diſcount at 7 per cent. per annum? Anſw. 16l. 18s. $2\frac{1}{2}$d +

10. If a legacy of 1200l. is left me on the 12th of March 1765, to be paid on the Chriſtmas-day following, what muſt I receive when I allow 5 per cent. per annum diſcount? Anſw. 1154l. 9s. 1d. +

11. At 5 per cent. per. annum diſcount, what is the preſent worth of 60l. payable at two 3 months, i. e. half at 3 months, and the reſt at 6 months? Anſw. 58l. 17s. 11d. 2qr. +

12. Sold goods to the value of 420l. which is payable as follows, viz. 100l. at 3 months, 200l. at 6 months, and the reſt at 10 months; what muſt I have in preſent money, allowing diſcount at 6 per cent. per annum? Anſw. 406l. 19s. $7\frac{3}{4}$d +

Note. When goods are bought or ſold, and diſcount is to be made for preſent payment, at any rate, per cent. without regard to time, the intereſt of the ſum, as calculated for a year, is the diſcount.

Be

13. Bought goods to the value of 160l. 10s. 6d. having discount at 5 per cent. how much is the discount? Answ. 8l. $6\frac{1}{4}$d.+

14. Sold 46 dozen of knives at 10s per dozen, allowing discount at 1s. 6d. per L. what is the neat value?

First 46÷2=23l. the gross value;

	l.		six d.		l.		six d.	
Then as	1	:	3	::	23	:	69	=1l. 14s. 6d. the discount.

Hence 23l.—1l. 14s. 6d.=21l. 5s. 6d. the neat value.

Note. The method used among Bankers, &c. in discounting bills, is to find the interest of the sum drawn for, from the time the bill is discounted to the time when it becomes due, (including the days of grace) which interest they reckon as the discount, thereby making the discount more than it really is, the error being in proportion to the time.

EQUATION of PAYMENTS.

Equation of Payments is, when several sums of money are due at different times, to find the time, when the whole may be paid in at once, without loss to the debtor or creditor.

The common Rule is to multiply every sum of money by the time it is to continue in the hands of the debtor, and to divide the sum of the products by the sum of all the payments, the quote being the equated time.

Note. This rule is not exactly true, though very nearly so, and may serve very well in common business; only it makes the mean time a small matter too large.

E X A M P L E S.

1. A owes B 110l. whereof 50l. is to be paid at 2 years end, 40l at $3\frac{1}{2}$ years end, and 20l. at $4\frac{1}{2}$ years end; at what time may the whole debt be paid together, without prejudice to either party?

years

I.		years		
50	×	2	is equal to	100
40	×	$3\frac{1}{2}$	equal to	140
20	×	$4\frac{1}{2}$	equal to	90
110				

110)330(3 years. Anſwer.
330

0

2. A perſon has three ſeveral ſums of money due at different times, 50l. at the end of 5 months, 84l. at the end of 10 months, and 36l. a year and a half hence, but would receive the whole at once; in what time ſhall he receive the whole ſum? Anſw. $10\frac{38}{170}$ months.

3. A debt of 500l. is to be diſcharged thus, viz. 100l. at preſent, 300l. at 5 months, and the reſt at 15 months, what is the equated time for diſcharging the whole? Anſw. 6 months.

4. A debt is to be diſcharged by paying one half at three months, a third at 5 months, and the reſt at 17 months: What is the equated time for the whole?

Take any number at pleaſure that is diviſible into the propoſed parts without remainder, as 6.

			I.
Then half of 6	is	3.	3 × 3 equal to 9
a third of 6	is	2.	2 × 5 equal to 10
And 6—5	is	1.	1 × 17 equal to 17

6)36(6 months Anſw.

5. A debt is to be diſcharged thus, viz. one fourth at preſent, and one fourth every three months after, till the

whole

whole be discharged : What is the equated time for the whole ? Answ. $4\frac{1}{2}$ months.

The End of the First Chapter.

CHAP. II.

§ I. *Of Vulgar* FRACTIONS.

A FRACTION, simply and abstractedly considered, is some part or parts either of an unit, or any other number or quantity whatsoever.

A Vulgar Fraction consists of two numbers, called the Numerator and Denominator, placed one above the other with a line of separation between them, as $\frac{3}{4}$ Numerator / Denominator which is named three fourths : the Numerator being first named, then the Denominator.

The Numerator and Denominator are called Terms of the Fraction.

The Denominator shews how many equal parts the whole is divided into, and the Numerator expresses how many such parts the Fraction consists of. For instance : Imagine the line A B to represent one yard, one foot, one pound, one shilling, one acre, or any other integer whatsoever, and let it be divided into 4 equal parts ; then

A —— 1 —— 2 —— 3 —— B

One of those parts will be $\frac{1}{4}$, or one fourth.
Two of those parts will be $\frac{2}{4}$, or two fourths.
Three of those parts will be $\frac{3}{4}$, or three fourths.
Four of those parts will be $\frac{4}{4}$, or four fourths, equal

to

to the whole line. Consequently $\frac{4}{4}$ is improperly called a fraction.

Now it is easy to perceive, that 5 of the above parts, is one fourth part more than the whole line; that 6 is more than the whole by 2 such parts; that 7 exceeds the whole by 3 such parts; and that 8 is equal to 2 whole lines: for if one thing be divided into 4 equal parts, 8 such parts must be equal to 2 whole things, and consequently

$\frac{12}{4}$ equal to three whole things.

$\frac{16}{4}$ equal to four who things.

$\frac{20}{4}$ equal to five whole things, &c.

S C H O L I U M.

Hence, fractions whose Numerators are less than, equal to, or greater than their Denominators, are respectively less than, equal to, or greater than unity, or 1.

C O R R O L L A R Y I.

Whence it follows, that a fraction is but a quotient, signifying a part or parts of an Unit, exprest by a Numerator as a Dividend and a Denominator as a Divisor.

C O R R O L L A R Y II.

Therefore as the Numerator is to the Denominator, so is the Fraction to Unity.

Or as $3 : 4 :: \frac{3}{4} : 1$. i. e. $\frac{3\times4}{4\times3}=1$.

L E M M A I.

To multiply a fraction is to multiply the Numerator; but to divide a fraction is to multiply the Denominator, as in the example above.

C O R R O L L A R Y III.

Fractions having the ſame Denominators are one to another as their Numerators.

Thus $\frac{3}{8} : \frac{5}{8} :: 3 : 5$.

C O R R O L L A R Y IV.

As any fraction is to unity, ſo is unity to the inverſe, or reciprocal of the ſaid fraction.

Note. By the inverſe, or reciprocal of a fraction, is meant the fraction gotten by inverting its terms: thus the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$, and of 7 or $\frac{7}{1}$ is $\frac{1}{7}$.

Thus $\frac{2}{5} : 1 :: 1 : \frac{5}{2}$.

L E M M A II.

Multiplying or ÷ing both terms of a fraction by one and the ſame number alters not its value, but produces a new fraction equivalent to the given one.

Let both terms of the fraction $\frac{4}{8}$ be ×ed by 3.

$$\text{Then } \frac{4 \times 3}{8 \times 3} = \frac{12}{24} \text{ the new fraction.}$$

Now it is plain that theſe numbers produced are in proportion to each other as the numbers ×ed; becauſe as 4 is the half of 8, ſo 12 is the half of 24, and conſequently the two fractions are of equal value.

Or if we divide a pound into 8 equal parts, 4 of thoſe parts muſt be the half of it; and if a pound be ÷ed into 24 equal parts, 12 of thoſe parts muſt ſtill be the half.

Again, if both terms of the fraction $\frac{12}{24}$ be ÷ed by 3, and afterwards by 2.

$$\text{Thus, } \frac{12 \div 3}{24 \div 3} = \frac{4 \div 2}{8 \div 2} = \frac{2}{4} \text{; the new fraction.}$$

is ſtill equivalent to $\frac{12}{24}$.

L Let

Let both terms of the fraction $\frac{2}{3}$ be $\times$ed by 3, 4, and 5.

Then $\dfrac{2 \times 3 \times 4 \times 5 = 120}{3 \times 3 \times 4 \times 5 = 180}$ the new fraction. i. e. $\dfrac{2}{3} = \dfrac{120}{180}$

On the contrary both terms of the fraction $\frac{120}{180}$ divided by 60, give $\frac{2}{3}, = \frac{120}{180}$.

N O T A T I O N.

There are four sorts of Vulgar Fractions, distinguished by the names of Proper, Improper, Compound and Complex.

1. A proper Fraction (called also a pure, simple or single Fraction) is that whose Numerator is less than its Denominator, and represents a part or parts less than the Denominator, or a whole: as $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, &c which may represent $\frac{3}{4}$ of a foot, $\frac{2}{3}$ of an Acre, $\frac{1}{2}$ of ten thousand pound, &c.

2. An Improper Fraction is that whose Numerator is always greater, or equal to, its Denominator, and represents a number that is always greater, or equal to, an unit, or one whole thing, as $\frac{5}{3}$ or $\frac{3}{3}$.

3. A Compound Fraction, or Fraction of a Fraction, is that whose parts are Vulgar Fractions, and consists of more than one Numerator and Denominator connected with the word of; as $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{7}$; which read thus,—the half of two-thirds of three-fourths of four-sevenths.

4. A Complex Fraction is that whose Numerator, or Denominator, or both, is a Fraction or a mixt number; as $\dfrac{5\frac{3}{4}}{8}$, or $\dfrac{5}{8\frac{1}{2}}$ or $\dfrac{16\frac{3}{4}}{12\frac{1}{2}}$: and if any such Fraction occurs, it only denotes a division of the Numerator by the Denominator. Cor. 1.

If

If what has been already said, be perfectly understood, all operations relating to Fractions admit of very few or no difficulties. I now proceed toReduction of Vulgar Fractions.

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§ II. *Reduction of Vulgar Fractions.*

REDUCTION is the converting of Fractions out of one form or denomination into another, for the more ease in working, or in estimating of their value, and is chiefly a preparation of them for Addition or Subtraction.

PROBLEMS.

I. *To express a whole number Fractional wise.*

Rule. Place 1 under it for a Denominator.

Example. Suppose 3 the whole number, then $\frac{3}{1}$ is the Fraction required.

II. *To reduce a whole number to a Fraction of a given Denominator.*

Rule. Multiply the whole number by the given Denominator, and make the product the Numerator, under which write the given Denominator.

EXAMPLES.

1. Reduce 5 to a Fraction whose Denominator may be 7.

Thus $\frac{5\times7}{7} = \frac{35}{7}$ the Fraction required. i. e. $\frac{35}{7}=5$ by Lemma 2.

2. Reduce 14 to a Fraction whose Denominator may be 9.

Answ. $\frac{126}{9} = 14$

 Reduce

3. Reduce 24 to a Fraction whose Denominator may be 30.

$$\text{Anſw. } \frac{720}{30} = 24.$$

III. *To reduce a mixt number into an Improper Fraction of equal value.*

Rule. Multiply the whole number by the Denominator of the Fraction, and to the product add the Numerator, and the ſum is the new Numerator, under which write the Denominator.

E X A M P L E S.

1. Reduce $36\frac{3}{4}$ to an Improper Fraction.

$$\text{Thus } \frac{36\times4,+3}{4} = \frac{147}{4} \text{ the Fraction required.}$$

For ſuppoſe the whole number to repreſent 36 yards of cloth, and each yard divided into 4 equal parts, or quarters, the 36 yards ſo divided are of courſe equal to 144 quarters; i. e. $36\times4=144$; to which if the other 3 quarters be +ed, the whole is 147 quarters, or $\frac{147}{4}$ yards.

2 Reduce $2\frac{1}{7}$ to an improper Fraction: Anſw. $\frac{15}{7}$.

3 Reduce $13\frac{3}{10}$ to an improper Fraction, and it is $\frac{133}{10}$.

4. When $7\frac{1}{2}$, $11\frac{3}{4}$, and $14\frac{2}{3}$, are each reduced to an improper Fraction, they become $\frac{15}{2}$, $\frac{47}{4}$, $\frac{44}{3}$.

IV. *To reduce an improper Fraction into its equivalent whole or next number.*

Rule. Divide the Numerator by the Denominator, and the quotient is the whole or mixt number.

EXAM-

EXAMPLES.

1. Reduce $\frac{126}{9}$, $\frac{147}{4}$, $\frac{229}{7}$, and $\frac{728}{8}$, each to its whole or mixt number.

$126 \div 9 = 14$	$229 \div 7 = 32\frac{5}{7}$
$147 \div 4 = 36\frac{3}{4}$	$728 \div 8 = 91$

2. When $\frac{18}{3}$, $\frac{16}{3}$, $\frac{17}{3}$, $\frac{64}{5}$, $\frac{213}{7}$, and $\frac{403}{9}$, are each reduced to their whole or mixt numbers, they become 6, $5\frac{1}{3}$, $5\frac{2}{3}$, $12\frac{4}{5}$, $30\frac{3}{7}$, and $44\frac{7}{9}$.

V. *To find the greatest common measure (or ÷for) for any two numbers, or for the Numerator and Denominator of a Fraction.*

Rule. Divide the greater number by the less, and that Divisor by the Remainder, continuing to make the last ÷for a Dividend, and the last Remainder into a ÷for, till nothing remains, and the last ÷for is the greatest common measure.

Note. If the last Divisor be 1 the numbers are prime to each other.

EXAMPLES

1. Let the numbers proposed be 168 and 240: their greatest common measure is 24, as found by the Rule thus,

```
        168)240(1
            168
            ---
         72)168(2
            144
            ---
The greatest common measure is 24)72(3
                                  72
                                  --
```


2. What

2. What is the greatest common measure for $\frac{312}{1026}$?
Answ. 6.

3. What is the greatest common measure for $\frac{152}{384}$?
Answ. 8.

4. What is the greatest common measure of $\frac{91}{117}$?
Answ. 13.

VI. *To abbreviate or reduce a Fraction to its lowest terms.*

A GENERAL RULE.

Find the greatest common measure, by which ÷ both terms of the Fraction, and the quotients will be the terms of the Fraction required.

EXAMPLES.

1. Reduce the Fraction $\frac{168}{240}$ to its lowest terms. The greatest common measure being 24, as found above,

$$\frac{168 \div 24 = 7}{240 \div 24 = 10}, \text{ i. e. } \frac{7}{10} = \frac{168}{240}$$

2. Reduce $\frac{312}{1026}$ to its lowest terms. Facit $\frac{52}{171}$.

3. Reduce $\frac{75}{135}$ to its lowest terms. Facit $\frac{5}{9}$.

4. Reduce $\frac{152}{384}$ to its lowest terms. Facit $\frac{19}{48}$.

5. Reduce

5. Reduce $\frac{192}{576}$ to its lowest terms. Facit $\frac{1}{3}$

6. Reduce $\frac{5184}{6912}$ to its lowest terms. Facit $\frac{3}{4}$

PARTICULAR RULES.

1. Rule. Any Fraction may be abbreviated by a continual division by 2, if the terms end with an even number or a cypher.

7. Thus $\frac{256}{784} = \frac{128}{392} = \frac{64}{196} = \frac{32}{98} = \frac{16}{49}$ by a continual halving; therefore $\frac{16}{49} = \frac{256}{784}$.

8. $\frac{120}{2368}$ being continually halved is $\frac{15}{296}$.

2. Rule. When both terms end with 5, or one with 5 and the other with a cypher, divide, both by 5.

9. Let the Fraction be $\frac{465}{895}$; Then $\frac{465 \div 5 = 93}{895 \div 5 = 179}$

10. $\frac{325}{1300} = \frac{13}{52}$. For $\frac{325 \div 5 = 65 \div 5 = 13}{1300 \div 5 = 260 \div 5 = 52}$

3. Rule. If the sum of the digits of any number can ÷ed by 3, the number itself may also be ÷ed by 3.

As

As for example, the number 741 is divisible by 3, because 12, which is the sum of its digits 7, 4, and 1, is so.

EXAMPLES.

1. $\frac{63}{72} = \frac{7}{8}$, For $\frac{63 \div 3 = 21, \div 3 = 7}{72 \div 3 = 24, \div 3 = 8}$

2. $\frac{57}{135} = \frac{19}{45}$.

4. Rule. When both terms end with cyphers, cast off as many as are common to both: and reduce the significant figures to lower terms, by the foregoing Rules.

EXAMPLES.

1. $\frac{3500}{56000} = \frac{35}{560} = \frac{7}{112} = \frac{1}{16}$ in its lowest terms.

2. $\frac{15000}{27000} = \frac{15}{27} = \frac{5}{9}$

5 Rule. When any number which is expressed by several others, with the sign of addition or subtraction between them, is to be divided by any number, then must all the parts of it be ÷ed by this number.

Thus $\frac{6+9-12}{3} = 2+3-4 = 1.$

But if the given number is expressed by others with the sign of multiplication between them, only one of them must be ÷ed:

So $\frac{4\times9\times12}{3\times8} = \frac{4\times3\times12}{1\times8} = \frac{1\times3\times12}{1\times2} = \frac{1\times3\times6}{1\times1} = 18$

and

And in this caſe, when the ſame numbers are concern-ed in both Numerator and Denominator, they may be omitted, or caſt out of both.

EXAMPLES.

$$\frac{9\times5\times7\times3\times2\times8\times7}{5\times7\times2\times5\times3\times9\times6}=\frac{8\times7}{5\times6}=1\frac{26}{30}=1\frac{13}{15}.$$ by omitting the common terms, 9, 5, 7, 3, 2.

$$\frac{4\times3\times8\times7}{5\times4\times7\times3}=\frac{8}{5}=1\frac{3}{5}$$ by omitting, 4, 3, 7.

$$\frac{23\times14\times70\times8}{80\times2\times7\times230}=\frac{1\times7\times10\times1}{10\times1\times1\times10}=\frac{1\times7\times1\times1}{10\times1\times1\times1}=\frac{7}{10}$$

VII. *To reduce a Compound Fraction to an equivalent ſingle one.*

Rule. Multiply all the Numerators together for a new Numerator, and all the Denominators together for a new Denominator, of the ſingle Fraction.

Note. If part of the Compound Fraction be a whole or mixt number, reduce it to Fractions by the foregoing rules.

EXAMPLES.

1. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of a pound ſterling to a ſingle Fraction. Facit $\frac{1}{2}$ l.

Thus $\frac{2\times3}{3\times4}=\frac{6}{12}=\frac{1}{2}$ l. the Fraction required.

This is ſo clear that it carries its own evidence along with it: For the value of $\frac{3}{4}$ l. is 15 ſhillings, two thirds of which is 10 ſhillings or $\frac{1}{2}$ l.

2. Reduce

2. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of a pound troy or 12 oz. to a single Fraction. Facit $\frac{1}{4}$ lb.

Thus $\dfrac{1\times2\times3}{2\times3\times4}=\frac{1}{4}$, (by omitting the common terms 2 and 3) or 3 oz.

For $\frac{3}{4}$ of 12 is 9, $\frac{2}{3}$ of 9 is 6, and the $\frac{1}{2}$ of 6 is 3.

Or thus $\frac{1}{2}$ of 12 is 6, $\dfrac{2}{3}$ of 6 is 4, and $\frac{3}{4}$ of 4 is 3.

Or reckon the Fractions in any other order, the solution will be the same.

That a compound Fraction, when reduced to a single one by the Rule still retains the same value, is easily proved by the following example.

3. Reduce $\frac{1}{2}$ of $\frac{1}{2}$ of a yard of cloth to a single Fraction. Facit $\frac{1}{4}$ of a yard.

Now there is nothing more self evident than that the half of half a yard is one quarter, or $\frac{1}{4}$.

4 Reduce $6\frac{1}{2}$d to the Fraction of a pound sterling.

$6\frac{1}{2}=$d, $\dfrac{13}{2}$ then $\dfrac{13}{2}$ of $\dfrac{1}{12}$ of $\dfrac{1}{20}$ is $=\dfrac{13}{480}$ of a pound.

5. Reduce $3\dfrac{2}{5}$ of $1\frac{1}{2}$ of $\dfrac{1}{3}$ of 4 to a single Fraction.

Facit $\dfrac{204}{30}=6\dfrac{4}{5}$

6. Reduce $\frac{3}{8}$ of $\frac{1}{2}$ of $\frac{3}{4}$ to a single Fraction. Facit $\dfrac{9}{64}$.

7. $\dfrac{2}{3}$ of $\frac{3}{4}$ of $\dfrac{4}{5}$ of 6l. is $=2\dfrac{2}{5}=2$l. 8s.

VIII.

VIII. *To reduce a Complex Fraction to a single or simple one.*

1. RULE.

When the fractional part is annext to the Numerator.
1. Multiply the Numerator by the Denominator of the fractional part, and to that product + the Numerator of the fractional part, for a new Numerator.

2 Multiply the Denominator of the fraction, by the Denominator of the fractional part of the Numerator, for a Denominator.

EXAMPLES.

1. Reduce $\frac{3\frac{1}{2}}{6}$ to a single Fraction.

$\frac{3\times2+1}{6\times2}=\frac{7}{12}$ the single Fraction required.

For if we call the Denominator 6 pence, the Numerator must be 3 pence half-penny, because both terms of a Fraction are always made up of parts homogeneous, therefore $3\times2+1=7$ half-pennys, and $6\times2=12$ half pennys, consequently $\frac{3\frac{1}{2}}{6}=\frac{7}{12}$ which evidently demonstrates the truth of the operation.

2. Reduce $\frac{42\frac{3}{4}}{49}$ to a simple Fraction. Facit $\frac{171}{196}$

3. Reduce $\frac{34\frac{1}{2}}{46}$ to a simple Fraction. Facit $\frac{69}{92}$

4. Reduce

4. Reduce $\dfrac{7\frac{3}{4}}{8}$ to a ſimple Fraction. Facit $\dfrac{31}{32}$

5 Reduce $\dfrac{\frac{3}{4}}{6}$ to a ſingle Fraction. Facit $\dfrac{3}{24}=\frac{1}{8}$

2. RULE

When the Fractional part is annexed to the Denominator.

1. Multiply the Denominator of the Fraction by the Denominator of the fractional part, and to that product add the Numerator of the fractional part for a new Denominator.

2 Multiply the Numerator of the Fraction by the Denominator of the fractional part for a new Numerator.

EXAMPLES.

1. Reduce $\dfrac{4}{5\frac{3}{4}}$ to a ſingle Fraction.

Thus $\dfrac{4 \times 4 = 16}{5 \times 4 + 3 = 23}$ — the ſingle Fraction required.

The truth of this operation may be proved as the firſt Example in the foregoing Rule.

For 4d=16q. and $5\frac{3}{4}$d.=23q. therefore $\dfrac{4}{5\frac{3}{4}} = \dfrac{16}{23}$

2. Reduce $\dfrac{5\frac{3}{4}}{7\frac{1}{2}}$ to a ſingle Fraction.

Thus $\dfrac{5 \times 4, +3, \times 2}{7 \times 2 + 1 \times 4} = \dfrac{46}{60} = \dfrac{23}{30}$ the ſingle Fraction required.

Reduce

3. Reduce $\dfrac{7}{9\frac{1}{2}}$ to a single Fraction. Facit $\dfrac{14}{19}$

4. Reduce $\dfrac{50}{60\frac{3}{4}}$ to a single Fraction. Facit $\dfrac{200}{243}$.

IX. *To reduce Fractions of different Denominators, to those of equal value, having a common Denominator.*

Rule. Multiply each Numerator continually into all the Denominators, except its own, for each new Numerator; then × all the Denominators together for the common Denominator.

Note. When any of the proposed quantities are whole or mixt numbers, compound, or complex Fractions, reduce them by their proper rules to the form of simple Fractions.

E X A M P L E S.

1. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to a common Denominator.

$$\left.\begin{array}{l} 1\times4\times6\times8=192 \\ 3\times6\times8\times2=288 \\ 5\times8\times4\times2=320 \\ 7\times2\times4\times6=334 \end{array}\right\}\text{new Numerator for}\left\{\begin{array}{l} \frac{1}{2} \\ \frac{3}{4} \\ \frac{5}{6} \\ \frac{7}{8} \end{array}\right.$$

$2\times4\times6\times8=383$ the common Denominator: which put under every one of the Numerators last found, and you have a new set of Fractions,

viz. $\dfrac{192}{384}$, $\dfrac{288}{384}$, $\dfrac{320}{384}$, $\dfrac{336}{384}$, all of the same denomination as appears from the operation itself; and all of the same value with their respective original ones, as will appear by reducing each new Fraction to its lowest term.

term. Or thus, it evidently appears by the operation, that both terms of each Fraction are ×ed by the same numbers, and therefore (by Lemma 2) suffer nothing in their value.

2. Reduce $\frac{1}{3\frac{1}{2}}$, $\frac{42\frac{7}{8}}{49}$ $\frac{5\frac{3}{4}}{7\frac{1}{2}}$ $\frac{73}{131\frac{2}{5}}$, to a com. Denom.

These Fractions reduced to simple ones, and their lowest terms are $\frac{2}{7}$ $\frac{7}{8}$ $\frac{23}{30}$ $\frac{5}{9}$; reduced to a common Denominator they become $\frac{4320}{15120}$, $\frac{13230}{15120}$, $\frac{11592}{15120}$, $\frac{8400}{15120}$, the Fractions required.

3. $\frac{4}{7}$, $\frac{5}{8}$, $\frac{6}{9}$, and $\frac{7}{12}$, when reduced, become $\frac{3456}{6048}$ $\frac{3780}{6048}$, $\frac{4032}{6048}$, $\frac{3528}{6048}$.

2. Rule. If the Denominators have a common measure ÷them by it; then × both terms of each given Fraction by the quotes of all the other Denominators and by this method you will have new Fractions in lowest terms than by the 1st rule.

EXAMPLES.

1. Reduce $\frac{7}{12}$, $\frac{1}{6}$, and $\frac{8}{15}$, to a common Denominator.

Then

The common measure of all these Denominators being 3, the quotes are 4, 2, and 5, then $\frac{7}{12}=\frac{7\times2\times5}{12\times2\times5}=\frac{70}{120}$, $\frac{1}{6}=\frac{1\times4\times5}{6\times4\times5}=\frac{20}{120}$ and, $\frac{8}{15}=\frac{8\times2\times4}{15\times2\times4}=\frac{64}{120}$.

2. Reduce $\frac{3}{8}$, $\frac{5}{12}$, $\frac{3}{16}$, and $\frac{3}{20}$, to a common Denominator.

3. Reduce $\frac{3}{10}$, $\frac{7}{30}$, $\frac{2}{15}$, $\frac{3}{25}$, and $\frac{1}{20}$, to a common Denominator.

CORROLLARY, 1.

Whence in two Fractions, the terms of each must be multiplied by the Quote of the other's Denominator.

4. Reduce $\frac{5}{14}$ and $\frac{8}{21}$ to a common Denominator.

Here 7 being the greatest common ÷sor, the quotes are 2, 3. Then $\frac{5\times3}{14\times3}=\frac{15}{42}$, and $\frac{8\times2}{21\times2}=\frac{16}{42}$ the Fractions required.

3 Rule. When the greatest Denominator is divisible by all the Denominators, × the terms of all the other Fractions by the respective quotes.

 1. Reduce

EXAMPLES.

1 Reduce $\frac{2}{9}$, $\frac{1}{12}$, and $\frac{5}{36}$, to a common Denominator.

Here the greatest Denominator being ÷ed by the rest, the quotes are 4 and 3; then

$\frac{2\times4}{9\times4}=\frac{8}{36}$, $\frac{1\times3}{12\times3}=\frac{3}{36}$, and $\frac{5}{36}$ rests the same; therefore the required Fractions are $\frac{8}{36}$, $\frac{3}{36}$ and $\frac{5}{36}$.

CORROLLARY, 2.

Whence in two Fractions, × the terms of that which hath the less Denominator, by the quote arising from the division of their Denominators.

2. Reduce $\frac{2}{5}$ and $\frac{4}{15}$ to a common Denominator.

Here the quote is 3: then $\frac{2}{5}=\frac{2\times3}{5\times3}=\frac{6}{15}$ therefore the required Fractions are $\frac{6}{15}$ and $\frac{4}{15}$

SCHOLIUM.

By this Problem the greater of two or more Fractions may be discovered.

X *Several Fractions being given, to find as many whole numbers in the same proportion.*

RULE. Reduce the Fractions to a common Denominator,

nator, and the ſeveral Numerators will be to one another as the given Fractions.

EXAMPLES.

1. Suppoſe the Fractions were $\frac{5}{8}$ and $\frac{3}{4}$.

Theſe reduced to a common Denom. are $\frac{20}{32}$, $\frac{24}{32}$.

Then as 20 : 24 :: $\frac{5}{8}$: $\frac{3}{4}$.

2. The Fractions $\frac{1}{2}$, $\frac{3}{8}$, $\frac{3}{4}$, are as the numbers 32, 24, and 48.

XI. *To reduce Fractions of one denomination to their equivalents of another denomination.*

1. RULE. When the leſs denomination, or integer, *is an aliquot part of the greater*, conſider how many of each leſs denomination are contained in one of the greater; then if the reduction be from a greater denomination to a leſs, × the Numerator, but if from a leſs to the greater, × the Denominator, by all the denominations from the given one to that ſought.

EXAMPLES.

1. Reduce $\frac{3}{8}$l. to the Fraction of a ſhilling.

$\frac{3}{8}\text{l.} = \frac{3 \times 20}{8} = \frac{60}{8}\text{s.}$ the Fraction required.

2. Reduce $\frac{60}{8}$s. to the Fraction of a pound.

$\frac{60}{8}\text{s.} = \frac{60}{8 \times 20} = \frac{60}{160}\text{l.} = \frac{3}{8}\text{l.}$ in lower terms.

3. Reduce

3. Reduce $\frac{1}{336}$ l. to the Fraction of a farthing.

$\frac{1}{336}$ l. $= \frac{1\times20\times12\times4}{336} = \frac{960}{336}$ q. the required Fraction.

4. Reduce $\frac{20}{7}$ q. to the Fraction of a pound.

$\frac{20}{7}$ q. $= \frac{20}{7\times4\times12\times20} = \frac{20}{6720} = \frac{1}{336}$ l. the Answer.

5. Reduce $\frac{3}{8}$ of a lb. Troy to the Fraction of a grain.

$\frac{3}{8}$ lb. $= \frac{3\times12\times20\times24}{8} = 3\times12\times20\times3 = 2160$ grains.

6. Reduce $\frac{17280}{8}$ Grains to the Fraction of a lb. Troy.

$\frac{17280}{8}$ grs. $= \frac{17280}{8\times24\times20\times12}$ lb. $= \frac{17280}{46080} = \frac{3}{8}$ lb.

7. Reduce $\frac{3}{8}$ lb. Apothecaries weight to the Fraction of a grain.

$\frac{3}{8}$ lb $= \frac{3\times12\times8\times3\times20}{8} = \frac{17280}{8} = 2160$ gr.

8. Reduce $\frac{3}{4}$ of a mile to the Fraction of an inch.

$\frac{3}{4}$ m. $= \frac{3\times8\times220\times3\times12}{4}$ inch. $= \frac{190080}{4} = 47520$ in.

9. Reduce

9. Reduce $\frac{3}{4}$ of a pint of ale to the Fraction of a hhd.

Facit $\frac{3}{1632}$.

10. Reduce $\frac{3}{4}$ of a last to the Fraction of a pint.

Facit $\frac{15360}{4}$ pt. = to 3840 pts.

11. Reduce $\frac{3}{4}$ of an hour to the Fraction of a week.

Facit $\frac{3}{672}$ week.

Note. If a compound whole number be proposed, reduce it to the lowest denomination mentioned, and proceed as before.

EXAMPLES.

1. Reduce 6s. 8d. to the Fraction of a pound.

Facit $\frac{80}{240}$ l. = $\frac{1}{3}$ l.

2. Reduce $5\frac{3}{4}$d to the Fraction of a shilling. Facit $\frac{23}{48}$ s.

3. Reduce 3 oz. 17 dwts. 19 gr. Troy to the Fraction of a lb. Facit $\frac{1867}{5760}$ lb.

2. RULE If the less denomination be not an aliquot part of the greater, reduce the Fraction given to such a denomination, whereof a certain number are contained in

in the denomination to which the Fraction is to be brought; then reduce this Fraction to the denomination required.

E X A M P L E S.

1. Reduce $\frac{3}{8}$ of a pound to the Fraction of a guinea.

$$\frac{3}{8}\text{l.} = \frac{3\times 20}{8\times 21} = \frac{60}{168}\text{ gui.} = \frac{5}{14}\text{ guinea.}$$

2. Reduce $\frac{3}{4}$ of a Carolus to the Fraction of a Jacobus.

$$\text{Thus } \frac{3\times 25}{4\times 23}\text{ jacobus} = \frac{75}{92}\text{ jacobus.}$$

3 Reduce $\frac{3}{8}$ of a noble to the Fraction of a crown. Facit $\frac{1}{2}$.

4. Reduce $\frac{3}{4}$ of half a crown to the Fraction of a shilling.

$$\text{Facit } \frac{15}{8}\text{s.} = 1\tfrac{7}{8}\text{s.}$$

5. Reduce $\frac{3}{4}$ of a jacobus to the Fraction of a moidore.

$$\text{Facit } \frac{69}{108} = \frac{23}{36}\text{ moidore.}$$

XII. *To find the value of a Fraction, or to reduce it to the known parts of an Integer.*

Rule. Multiply the Numerator of the Fraction, by such a number of units of the next less denomination, as is equal to one of that name which the Fraction is of, dividing the Product by the Denominator, and the quote shews the parts sought.

If

If there be a remainder, × it by the Units of the next inferior Denomination, and ÷ by the Denominator as before, continuing this Work till it is brought to the lowest known part of an Integer.

EXAMPLES.

1. What is the value of $\frac{3}{7}$ a pound?

```
  3
 20
 ---
7)60(8 s.
  56
  ---
   4
  12
  ---
7)48(6 d.
  42
  ---
   6
   4
  ---
7)24(3
  21
  ---
   3 rem.
```

Answ. 8s. 6d. 3 $\frac{3}{7}$q.

2. What is the value of $\frac{5}{19}$ of a guinea?

```
     5
    21
19)105(5s.
    95
    ---
    10
    12
    ---
19)120(6 d.
   114
   ---
     6
     4
   ---
19)24(1 qr.
   19
   ---
    5 rem.
```

Answ. 5s. 6d. 1 $\frac{5}{19}$ q.

3. What is the value of $\frac{5}{7}$ of 7l. 11s. 7d. $\frac{3}{4}$?
Answ. 5l. 8s. 3d. 3 $\frac{2}{7}$ q.

4. What is the value of $\frac{67}{73}$ of a cwt?
Answ. 3 q. 18lb. 12 oz. 11 $\frac{29}{73}$ dr.

5. What is the value of $\frac{71}{97}$ of a lb. Troy?
Answ. 8 oz, 15 wts. 16 $\frac{8}{97}$ grs.

6. What

6. What is the value of $\frac{9}{13}$ of a cwt?
Answ. 2 qr. 21lb. 8 oz. 9 $\frac{11}{13}$ drs.

7. What is the value of $\frac{3}{4}$ of a lb. Avoirdupoise?
Answ. 12 oz.

8. What is the value of $\frac{7}{9}$ of a shilling?
Answ. 9d. 1 $\frac{1}{3}$ q.

9. What is the proper quantity of $\frac{4}{7}$ of a mile?
Answ. 4 fur. 125 yds. 2 feet, 1 inch $2\frac{1}{7}$ b. corns.

10. What is the proper value of $\frac{3}{5}$ of a tun of wine?
Answ. 2 hhds. 25 gal. $1\frac{3}{5}$ pt.

11. What is the proper value of $\frac{2}{15}$ of a hhd. of ale?
Answ. 6 gal $3\frac{1}{5}$ qrts.

XIII. *To reduce any given quantity to the Fraction of any greater denomination of the same kind.*

Rule. Reduce the quantity to the lowest denomination mentioned, annexing thereto the Fraction (if any) of that denomination, for a Numerator; then reduce the integral part to the same denomination for a Denominator; and lastly, reduce the Fraction to its lowest terms.

N. B. Problem XII. and XIII. prove each other.

E X A M P L E S.

1. Reduce 8s. 6d. $3\frac{3}{7}$q. to the Fraction of a pound. First 8s. 6d. $3\frac{3}{7}$q. $=411\frac{3}{7}$q. for a Numerator, 1l $=960$q. for a Denominator; therefore the required Fraction is $\frac{411\frac{3}{7}}{960}$l. which, reduced to a single Fraction, and then to its lowest terms, stands thus $\frac{411\times7+3}{960\times7}=\frac{2880}{6720}=\frac{3}{7}$l.

2. Reduce

2. Reduce 5s. 6d. $1\frac{5}{19}$q. to the Fraction of a guinea.

$$\frac{\text{5s. 6d. } 1\frac{5}{19}\text{q.} = 265\frac{5}{19}\text{q.}}{\text{1 guinea} = \text{1008 q.}} = \frac{5}{19} \text{ of a guinea.}$$

3. What part of 7l. 11s. $7\frac{3}{4}$d. is 5l. 8s. 3d. $3\frac{2}{7}$ q?

$$\frac{\text{5l. 8s. 3d. } 3\frac{2}{7}\text{q} = 5199\frac{2}{7}\text{q.}}{\text{7l. 11s. 7d. 3q.} = \text{7279 q.}} = \frac{36395}{50953} = \frac{5}{7} \text{ Answ.}$$

4. What part of a cwt. is 3qr. 18lb. 12 oz. 11 $\frac{29}{73}$ drs?

Answ. $\frac{67}{73}$

5. What part of a pound Troy is 8 oz. 15 dwts. $16\frac{8}{97}$ grs?

Answ. $\frac{71}{97}$

6. What part of a cwt. is 2 qr. 21 lb. 8 oz. 9 $\frac{11}{13}$ drs?

Answ. $\frac{9}{13}$

7. What part of a pound Avoirdupoise is 12 oz

Answ. $\frac{12}{16} = \frac{3}{4}$ lb. the Answ.

8. What part of a shilling is 9d. $1\frac{1}{3}$ q? Answ $\frac{7}{9}$.

XIV. *To reduce a Fraction to its equivalent that shall have any assigned Denominator.*

RULE. As the Denominator of the given Fraction is to its Numerator, so is the Denominator of the assigned Fraction to its Numerator.

EXAM-

EXAMPLES.

1. Reduce $\frac{3}{4}$ to a Fraction of the ſame value, whoſe Denominator ſhall be 12.

Thus as 4 : 3 :: 12 : 9 the new Numerator.

Therefore the new Fraction is $\frac{9}{12}$, $=\frac{3}{4}$

2. Reduce $\frac{7}{8}$ to a Fraction of the ſame value, whoſe Denominator ſhall be 49. Facit $\frac{42\frac{7}{8}}{49}$

For $7\times 49\div 8=42\frac{7}{8}$ for the Numerator.

3 Reduce $\frac{3}{4}$ to a Fraction of the ſame value, whoſe Denominator ſhall be 46. Facit $\frac{34\frac{1}{2}}{46}$

4. Reduce $\frac{5}{9}$ to a Fraction whoſe Denominator ſhall be $131\frac{2}{5}$ Thus $131\times 5+2\div 9=73$ the Numerator, hence $\frac{73}{131\frac{5}{9}}$ is the Fraction.

XV. *To reduce a given Fraction to its equivalent, that ſhall have any aſſigned Numerator.*

Rule. As the Numerator of the given Fraction is to its Denominator, ſo is the Numerator of the aſſigned Fraction, to its Denominator.

EXAM-

E X A M P L E S.

1. Reduce $\frac{3}{4}$ to a Fraction of the ſame value whoſe Numerator ſhall be 9. Facit $\frac{9}{12}$

For as $3 : 4 :: 9 : 12$ the new Denominator.

2. Reduce $\frac{7}{8}$ to a Fraction of the ſame value whoſe Numerator ſhall be $42\frac{7}{8}$.

Facit $\frac{42\frac{7}{8}}{49}$. For $42 \times 8 + 7 \div 7 = 49$ the new Denom.

3 Reduce $\frac{3}{4}$ to a Fraction of the ſame value whoſe Numerator ſhall be $34\frac{2}{4}$.

Faci: $\frac{34\frac{2}{4}}{46}$.

4. Reduce $\frac{5}{9}$ to a Fraction of the ſame value, whoſe Numerator ſhall be 73.

Facit $\frac{73}{131\frac{2}{5}}$

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§ III. *Addition of Vulgar Fractions.*

A GENERAL RULE.

REDUCE mixt numbers to improper Fractions; compound, and complex Fractions, to ſingle ones; and Fractions of different Denominators to a common Denominator Then add their Numerators into one ſum, under which ſubſcribe the common Denominator, and that Fraction will be the required ſum; then reduce it to its loweſt terms, and, if it be an improper Fraction, to its whole or mixt number.

EXAMPLES.

1. Add $\frac{7}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{1}{8}$ into one sum.

Here the Denominators being common, add the Numerators together; thus $7+3+5+1=16$ eighths, or $\frac{16}{8}, =2$.

2. What is the sum of $\frac{2}{3}$, $\frac{2}{7}$, and $\frac{4}{9}$? Answ. $1\frac{25}{63}$.

For when reduced to a common Denominator they are $\frac{126}{189}$, $\frac{54}{189}$, and $\frac{84}{189}$, Then $\frac{126+54+84}{189}=\frac{264}{189}$: the Denominator shewing that something is $\div$ed into 189 equal parts, and the Numerator denoting 264 such parts, $=1\frac{75}{189}=1\frac{25}{63}$.

3. Add $1\frac{1}{6}$, $\frac{1}{5}$, $\frac{1\frac{1}{2}}{2}$, and $\frac{2}{3}$ of $\frac{3}{4}$ together.

First $1\frac{1}{6}, =\frac{7}{6}$, $\frac{1\frac{1}{2}}{2}, =\frac{3}{4}$, and $\frac{2}{3}$ of $\frac{3}{4}=\frac{1}{2}$: then $\frac{7}{6}$, $\frac{1}{5}$, $\frac{3}{4}$, $\frac{1}{2}$, reduced to a common Denominator, and added together, $=2\frac{37}{60}$ the sum.

4. What is the sum of $\frac{7}{16}$, $\frac{11}{12}$, and $\frac{4}{9}$? Answ. $1\frac{115}{144}$

5. What is the sum of $9\frac{3}{20}$, $\frac{3}{5}$, and $\frac{4}{5}$ of $\frac{1}{3}$?

Answ. $10\frac{1}{60}$.

6. What

6. What is the ſum of $\frac{3}{5}$, $\frac{5}{9}$, and $2\frac{1}{6}$? Anſ. $3\frac{87}{270}$.

Note 1. If many Fractions are to be added, the beſt way is, firſt to add two of them together, and to the ſum add a third, and to that ſum a fourth, and ſo on.

EXAMPLES.

1. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{1}{6}$, together.

Firſt $\frac{1}{2}$ and $\frac{2}{3}$ are reduced to $\frac{3}{6}$ and $\frac{4}{6}$, whoſe ſum is $\frac{7}{6}$.

Then $\frac{7}{6}$ and $\frac{3}{4}$ are reduced to $\frac{28}{24}$ and $\frac{18}{24}$, whoſe ſum is $\frac{46}{24}=\frac{23}{12}$. Then $\frac{23}{12}$ and $\frac{4}{5}$ are reduced to $\frac{115}{60}$ and $\frac{48}{60}$ whoſe ſum is $\frac{163}{60}$. Then $\frac{163}{60}$ and $\frac{1}{6}$ are reduced to $\frac{978}{360}$ and $\frac{60}{360}$; whoſe ſum is $\frac{1038}{360}=2\frac{53}{60}$, the ſum of the five Fractions.

2. What is the ſum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$?

Anſw. $\frac{183}{60}=3\frac{1}{20}$.

Note 2. When mixt numbers are to be added, reduce their fractional parts to a common Denominator, and add their ſum to the total amount of the whole numbers.

E X A M P L E S.

1. Add $2\frac{1}{2}$, $3\frac{2}{5}$, $4\frac{1}{3}$ and $8\frac{1}{4}$ together.

First $\frac{1}{2}$, $\frac{2}{5}$, $\frac{1}{3}$, and $\frac{1}{4}$ are reduced to $\frac{30}{60}$, $\frac{24}{60}$, $\frac{20}{60}$, $\frac{15}{60}$;

Then $2\frac{1}{2} = 2\frac{30}{60}$

$3\frac{2}{5} = 3\frac{24}{60}$

$4\frac{1}{3} = 4\frac{20}{60}$

$8\frac{1}{4} = 8\frac{15}{60}$

$18\frac{29}{60}$ is the ſum required.

Here the ſum of the Fractions is $\frac{89}{60} = 1\frac{29}{60}$, and the ſum of the whole numbers 17;

therefore $17 + 1\frac{29}{60} = 18\frac{29}{60}$.

2. Add $476\frac{1}{3}$, $24\frac{3}{4}$, $74\frac{5}{6}$, $81\frac{3}{8}$, and $152\frac{5}{14}$ together.

Facit $809\frac{109}{168}$.

3. What is the ſum of 4, $3\frac{1}{2}$, 6, $4\frac{1}{3}$, 8, and $10\frac{3}{8}$?

Anſw. $36\frac{5}{24}$.

Note 3. If the Fractions given be of different denominations, reduce them to their proper value by Problem XII in Reduction, and add them as in Addition of different denominations.

EXAM-

EXAMPLES.

1. Add $\frac{1}{3}$ of a pound $\frac{3}{4}$ of a shilling and $\frac{3}{4}$d. into one sum.

	l.	s.	d.	
Thus $\frac{1}{3}$l. =	0	6	8	Reduced severally to their proper values
$\frac{3}{4}$ s. =	0	0	9	
$\frac{3}{4}$d. =	0	0	$0\frac{3}{4}$	
The sum	0	7	$5\frac{3}{4}$	

2. Add $\frac{2}{3}$ of a pound Troy, 1 fifth of an oz. $\frac{2}{3}$ dwt. and $17\frac{3}{4}$ grains together.

	oz.	dts	grs.
$\frac{2}{3}$ lb. =	8	0	0
$\frac{1}{5}$ oz. =	0	4	0
$\frac{2}{3}$ dwt. =	0	0	16
$17\frac{3}{4}$	0	0	$17\frac{3}{4}$
the sum	8	5	$9\frac{3}{4}$

3. Add $\frac{9}{13}$ cwt. and $\frac{5}{7}$ lb. & 3 fifths of an oz. together?

Facit 2 qr. 22lb. 4 oz. $1^{\circ}\frac{138}{455}$ drs.

4. What is the sum of $\frac{3}{7}$ of 7l. 11s. $7\frac{3}{4}$d, 5 nineteenths of a guinea, $\frac{1}{7}$ of a pound, and $\frac{7}{9}$ of a shilling?

Answ. 6l. 3s. 2d. $1\frac{124}{399}$ qr.

5. What

5. What is the sum $\frac{4}{7}$ of a mile $\frac{3}{5}$ of a furlong, $\frac{2}{3}$ of a yard, $\frac{3}{4}$ of a foot, and $\frac{1}{2}$ of an inch?

Answ. 5 fur. 38 yds. 1 ft. 11 inch. $0\frac{9}{14}$ br. c.

6. Add $\frac{5}{7}$ of a month, $\frac{7}{13}$ of a day, and $\frac{2}{3}$ of an hour, into one sum.

Facit 2 weeks, 6 days, 13 hrs. 35 m. $23\frac{1}{13}$ sec.

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§ IV. *Subtraction of Vulgar Fractions.*

THE Fraction being prepared here in all respects as in Addition, Subtract the Numerators, and under their difference, write the common Denominator.

EXAMPLES.

1. What is the difference between $\frac{3}{4}$ and $\frac{1}{4}$?

$$\frac{3}{4}-\frac{1}{4}=\frac{3-1}{4}=\frac{2}{4}=\frac{1}{2}, \text{ the Answ.}$$

2. What is the difference between $\frac{7}{8}$ and $\frac{3}{8}$? Answ. $\frac{4}{8}=\frac{1}{2}$.

3. From $\frac{5}{6}$ take $\frac{4}{9}$. These reduced to a common Denominator (by Cor. I. Problem IX. in Reduction) are

$\frac{15}{18}$, and $\frac{8}{18}$; then $\frac{15-8}{18}=\frac{7}{18}$ the Answ.

4. What

4. What is the difference between $\frac{3}{4}$ of $\frac{2}{5}$ and $\frac{3}{4}$?

$\frac{3}{4}$ of $\frac{2}{5} = \frac{3}{10}$ then $\frac{3}{10}$ and $\frac{3}{4}$, are reduced to $\frac{6}{20}$ and $\frac{15}{20}$;

therefore $\frac{15-6}{20} = \frac{9}{20}$ the Anſw.

5. From $5\frac{3}{8}$, take $\frac{2}{7}$ of $4\frac{1}{6}$.

Firſt $5\frac{3}{8} = \frac{43}{8}$, and $\frac{2}{7}$ of $4\frac{1}{6} = \frac{25}{21}$; then $\frac{43}{8}$ and $\frac{25}{21}$ are reduced to $\frac{903}{168}$ and $\frac{200}{168}$; therefore $\frac{903-200}{168} = \frac{703}{168} =$

$= 4\frac{31}{168}$ the Anſw.

6. What is the difference between $\frac{19}{20}$ and $\frac{1}{7}$ of $\frac{2}{3}$?

Anſw. $\frac{359}{420}$.

7. From $\frac{9}{13}$ take $\frac{2}{3}$ of $\frac{5}{8}$. Facit $\frac{43}{156}$.

8. From $\frac{7}{8}$ of 5 take $\frac{7}{13}$ of $\frac{19}{21}$. Facit $\frac{8491}{2184}$.

Note.

Note 1. In subtracting one mixt number from another, (or a proper Fraction from a whole number, when the Fraction in the subducend is greater than that in the minuend) Subtract the Numerator of the subducend from the Denominator, and to the difference add the Numerator of the minuend (if any;) and carry one to the integer (or cypher) in the subducend.

E X A M P L E S.

1. From $16\frac{1}{4}$ pence.
   Take $13\frac{3}{4}$ pence.
   Rem. $2\frac{1}{2}$d.

2. From 17
   Take $0\ \frac{1}{2}$
   Rem. $16\ \frac{1}{2}$

3. From $6\frac{1}{8}$
   Take $3\frac{3}{8}$
   Rem. $2\frac{6}{8}=2\frac{3}{4}$

4. From 1
   Take $0\frac{3}{4}$
   Rem. $0\frac{1}{4}$

Note 2. In mixt numbers, reduce the Fractions to a common Denominator; then take the Fractions from the Fractions, and the whole numbers from the whole numbers.

E X A M P L E S.

rom $16\frac{3}{4}=16\frac{21}{28}$
Take $9\frac{4}{7}=9\frac{16}{28}$
Rem. $7\ \frac{5}{28}$

2. From $86\frac{1}{3}=86\frac{5}{15}$
   Take $57\frac{4}{5}=57\frac{12}{15}$
   Rem. $28\frac{8}{15}$

3. From

3. From $34\frac{1}{4}$ take $\frac{2}{5}$ of 34. Facit $20\frac{13}{20}$.

4. From $\frac{1}{3}$ of 64 take $\frac{3}{4}$ of $\frac{1}{2}$ of 42. Facit $5\frac{7}{12}$.

5. From $1\frac{1\frac{1}{3}}{4}$ of $3\frac{2}{5\frac{1}{2}}$ of 6 take $\frac{4}{9}$ of $\frac{3}{4\frac{1}{5}}$.

Facit $26\frac{410}{693}$.

Note 3. In Subtracting Fractions of different denominations, reduce them to their proper value; and subtract as before.

E X A M P L E S.

		s.	d.	q.
1. From $\frac{3}{8}$ l.	=	7	6	0
Take $\frac{4}{13}$ s.	=	0	3	$2\frac{10}{13}$
Rem.		7	2	$1\frac{3}{13}$.

		s.	d.	
2. From $\frac{3\frac{1}{3}}{7}$ of a l.	=	9	6	$1\frac{5}{35}$
Take $\frac{3}{5}$ of $\frac{2}{3}$ of 1s.	=	0	4	$3\frac{7}{35}$
Rem.		9	1	$1\frac{33}{35}$

The Fractions being reduced to a common Denominator.

3. What

3. What is the difference between $\frac{3}{5}$ of a guinea, and $\frac{4}{5}$ of a mark? Answ. 1s. $11\frac{1}{5}$d.

4. From $\frac{3}{4}$ of a lb. Troy, take one sixth of an oz.
Facit 8 oz. 16 dwts. 16 grs.

5. From $\frac{4}{5}$ of a ton take $\frac{5}{6}$ of a lb.
Facit 15 c. 3 gr. 27lb. 2 oz. $10\frac{2}{3}$ dwts.

6. From $\frac{1}{6}$ of a yard take $\frac{2}{3}$ of an inch. Facit $5\frac{1}{3}$ in.

7. From $\frac{2}{3}$ of a qr. of corn, take $\frac{3}{4}$ of a bushel.
Facit 4 bushels $2\frac{1}{3}$ peck.

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§ V. *Multiplication of Vulgar Fractions.*

R U L E.

REDUCE mixt numbers to Fractions, compound and complex Fractions to single ones, also to their lowest terms; then × all the Numerators together for the Numerator, and all the Denominators together for the Denominator of the required product.

E X A M P L E S.

1. Multiply $\frac{2}{3}$ by $\frac{1}{2}$. Thus $\frac{3\times1}{3\times2} = \frac{2}{6} = \frac{1}{3}$ the product.

The

The product of any number ×ed by a proper Fraction is always less than that number. As is evident by the example above.

S C H O L I U M.

Whence the product of two proper fractions must of consequence, be less than either of them.

2. Multiply $\frac{3}{4}$ of a l. by $\frac{8}{9}$ of a l. Facit $\frac{2}{3}$ l.

For $\frac{3}{4}$ l. $\times \frac{8}{9} = \frac{24}{36}$ l. $= \frac{2}{3}$ l. $=$ 13s. 4d. but $\frac{3}{4}$ l. $=$ 15s and $\frac{8}{9}$ l. $=$ 17s. $9\frac{1}{3}$d. therefore their product is less than either quantity.

3. Multiply $7\frac{1}{2} = \frac{15}{2}$, by $\frac{2}{3}$. Thus $\frac{15 \times 2}{2 \times 3} = \frac{30}{6} = 5$ the product.

4. Multiply 41 $\frac{4}{5}$ l. by 12 l. Thus $\frac{209}{5}$ l. $\times \frac{51}{4} =$ $\frac{10659}{20}$ l. $= 532\frac{19}{20}$ l. $=$ 532l. 19s. the Answ.

5. What is the product when $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{2}{7}$ is multiplied by $\frac{2}{3}$ of $\frac{5}{8}$?

The compounds reduced are $\frac{6}{35}$ and $\frac{5}{12}$, in their lowest terms.

Then $\frac{6 \times 5 = 30}{35 \times 12 = 420} = \frac{1}{14}$ the Answ.

Or

Or thus, $\frac{4\times3\times2\times2\times5}{5\times4\times7\times3\times8}$ = (by omitting the common terms 4, 3, & 5) $\frac{2}{2}\times\frac{7}{8}=\frac{4}{56}=\frac{1}{14}$.

This example shews, that it is needless to reduce compound Fractions before multiplication be performed; because it evidently appears, that multiplication of Fractions, is only reducing a compound Fraction to a single one; for to $\times\ \frac{2}{7}$ by $\frac{1}{4}$, is no more than to take $\frac{1}{4}$ of $\frac{2}{7}$.

CONTRACTIONS.

I. When the Numerator of one and the Denominator of the other, can be $\div$ed by any number, take the quotes instead of them.

EXAMPLES.

1. Multiply $\frac{5}{12}$ by $\frac{6}{7}$. Divide by 6, and the Fractions become $\frac{5}{2}$ and $\frac{1}{7}$; then $\frac{5\times1}{2\times7}=\frac{5}{14}$ the Product.

2. Multiply $\frac{5}{8}$ by $\frac{4}{15}$. $\frac{5)5}{4)8}\times\frac{4}{15}=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$ the product.

3 Multiply $\frac{3}{8}$ by $\frac{4}{9}$. $\frac{3)3}{4)8}\times\frac{4}{9}=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$ the product.

4. Multiply $\frac{3}{8}$ by $\frac{4}{7}$. Facit $\frac{3}{14}$.

II. When

II. When a mixt number or Fraction is to be ×ed by a whole number, × the whole number by the whole number, then × the Numerator by the ſaid whole number, and ÷ by the Denominator, and add this qoute to the former product.

EXAMPLES.

1. Multiply three 5ths by 8. Thus $\frac{3\times8}{5}=\frac{24}{5}=4\frac{4}{5}$ the product.

2. Multiply $5\frac{2}{3}$ by 14. Thus $14\times2\div3=9\frac{1}{3}$; which when added to the product of 5 by 14, thus $14\times5+9\frac{1}{3}=79\frac{1}{3}$ the product required.

3. What is the product of $4\frac{3}{4}$ by 4? Anſw. 19.

4. What is the product of $3\frac{4}{7}$ by 13? Anſw. $46\frac{3}{7}$.

III. When a Fraction is to be ×ed by a number which happens to be the ſame with the Denominator, take the Numerator for the product.

EXAMPLES.

1. Multiply *four 7ths* by 7, the product is 4.
2. Multiply 4 by $\frac{3}{4}$, the product is 3.

IV. When ſeveral Fractions are to be ×ed, ſtrike out ſuch ×ers as are found both in the Numerators and Denominators, as before taught in Sect. II. Prob. VI. Rule V.

1. Multiply *four fifths, three eights, two thirds*, and *five ſixths* together.

Thus $\frac{4\times3\times2\times5}{5\times8\times3\times6}$ = (by omitting the common terms 3 and 5) $\frac{4\times1\times2\times1}{1\times8\times1\times6}$ = (by ÷ing by 4 and by 2) $\frac{1\times1\times1\times1}{1\times2\times1\times3} = \frac{1}{6}$ the product.

2. What is the product of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and 5 ?
Answ. $\frac{5}{4}$, or $1\frac{1}{4}$.

Note. A Fraction is best multiplied by an integer, by dividing the Denominator by it if possible; but if not, multiply the Numerator by it.

E X A M P L E S.

1. Multiply $\frac{3}{14}$ by 7. Thus $\frac{3}{14\div7} = \frac{3}{2} = 1\frac{1}{2}$.

2. Multiply $\frac{8}{27}$ by 9. Facit $2\frac{2}{3}$.

3. Multiply $\frac{4}{7}$ by 5. Thus $\frac{4\times5}{7} = \frac{20}{7} = 2\frac{6}{7}$.

4. Multiply *two 3ds* of 5 by 6. Facit 20.

5. What is the product of $\frac{1}{2}$, $\frac{2}{3}$, and 3 ? Answ. 1.

6. What is the product of 3l. 19s. $11\frac{3}{4}$d. by 3l. 19s. $11\frac{3}{4}$d ?

First as 3l. 19s. $11\frac{3}{4}$d.=3839 farthings for a Numerator, and 1l.=960 farthings for a Denominator, the Fraction becomes $\frac{3839}{960}$.

Then

Then $\frac{3839 \times 3839}{960 \times 960} = \frac{14737921}{921600}$ l. = 15l. 19s. 10$\frac{1}{3840}$d. the Anſwer required.

7. What is the product of 6l. 17s. 4$\frac{1}{4}$d. by 2l. 15s. 7$\frac{1}{4}$d?

Anſw. 19l. 2s. 1d. 3$\frac{623}{960}$qr.

8. What is the product of 4 lb. 9 oz. 4 dwts. 12 grs. by 2 lb. 5 oz. 10 dwts. 20 grs? Anſw. 11 lb. 8 oz. 17 dwts. 12$\frac{7}{8}$ grs.

9. What is the product of 3 yds. 2 feet, 6 inches, 2 br. corns; by 3 yds. 2 feet, 6 inches, 2 bar. corns?
Anſw. 14 yds. 2 feet, 6$\frac{1}{81}$ inches.

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§ VI. *Diviſion of Vulgar Fractions.*

FRACTIONS being prepared in all reſpects as for Multiplication, Diviſion is thus performed.

RULE.

When the Numerator and Denominator of the Dividend, can be exactly ÷ed by the Numerator and Denominator of the ÷for, take the quotients for the terms of the required Fraction; but if that cannot be done, multiply both terms of the Dividend by the Diviſor's reverſe Fraction, for the terms of the required quotient.

EXAMPLES.

1. Let $\frac{9}{20}$ be ÷ed by $\frac{3}{4}$. Thus $\frac{9 \div 3}{20 \div 4} = \frac{3}{5}$ the quotient. The truth of this evidently appears, and may be proved by Multiplication, thus $\frac{3 \times 3}{5 \times 4} = \frac{9}{20}$.

 2. Divide

2. Divide $\frac{6}{7}$ by $\frac{3}{5}$. The ÷for's reserve Fraction is $\frac{5}{3}$; then thus $\frac{6\times5}{7\times3}=\frac{30}{21}=1\frac{3}{7}$ the quote.

CONTRACTIONS.

I. If a Fraction is to be ÷ed by a whole number, ÷ the Numerator by it, if possible; but if not, × the Denominator by it.

II. When the two Numerators, or the two Denominators can be ÷ed by the same number, then ÷ them, and take the quotes instead thereof.

3. Divide $\frac{9}{11}$ by 3. Thus $\frac{9\div3}{11}=\frac{3}{11}$ quote.

4. Divide $\frac{9}{11}$ by 4. Thus $\frac{9}{11\times4}=\frac{9}{44}$ quote.

5. Divide $\frac{16}{17}$ by $\frac{12}{19}$. First $\frac{16\div4}{17}=\frac{4}{17}$, and $\frac{12\div4}{19}=\frac{3}{19}$. Then $\frac{4}{17}\div\frac{3}{19}=\frac{76}{51}=1\frac{25}{51}$ the quote.

6. Divide $\frac{8}{9}$ by $\frac{2}{45}$. First $\frac{8\div2}{9\div9}=\frac{4}{1}$, and $\frac{2\div2}{45\div9}=\frac{1}{5}$; then $\frac{4}{1}\div\frac{1}{5}=\frac{4\times5}{1\times1}=20$ the quote.

7. Divide

7. Divide $\frac{3}{4}$ l. by $\frac{4}{5}$ s. First $\frac{4}{5\times20}$ = $\frac{4}{100}$ l. = $\frac{1}{25}$ l.

Then $\frac{3}{4}$ l. $\div$ $\frac{1}{25}$ l = $\frac{3}{4}\frac{\times25}{\times1}$ = $\frac{75}{4}$ = $18\frac{3}{4}$ l.

8. What is the quote of $\frac{2}{5}$ by $\frac{2}{3}$? Answ. $1\frac{2}{5}$.

9. What is the quote of $3\frac{2}{3}$ by $5\frac{1}{4}$? Answ. $\frac{44}{63}$

10. What is the quote of $8\frac{1}{3}$ of $2\frac{1}{2}$ by $\frac{3}{4}$ of $\frac{2}{5}$? Ans $69\frac{4}{9}$

11. What is the quote of 15l. 19s. $10\frac{1}{3840}$ d. by 3l 19s $11\frac{3}{4}$d. Answ. 3l. 19s. $11\frac{3}{4}$d.

12. What is the quote of 19l. 2s. 1d. $3\frac{623}{960}$ q. by 2l. 15s. $7\frac{3}{4}$d? Answ. 6l. 17s. $4\frac{1}{4}$d.

13. What is the quote of 19l. 2s. 1d. $3\frac{623}{960}$ q. by 6l. 17s. $4\frac{1}{4}$d? Answ. 2l. 15s. $7\frac{3}{4}$d.

14. What is the quote of 11lb. 8 oz. 17 dwts. $12\frac{7}{8}$ grs. by 2lb 5 oz. 10 dwts. 20 grs?

Answ. 4lb 9 oz. 4 dwts. 12 grs.

15. What is the quote of 11lb. 8 oz. 17 dwts. $12\frac{7}{8}$ grs by 4lb. 9 oz. 4 dwts. 12 grs?

Answ. 2lb 5 oz. 10 dwts. 20 grs.

16. What is the quote of 14 yds. 2 feet, 6 [illegible] inches by 3 yards, 2 feet, 6 inches, 2 bar. corns?

Answ. 3 yds. 2 ft. 6 in. 2 bar. corns.

§ VII. *Of Decimal Fractions.*

NOTATION.

1. A *Decimal Fraction* is the Numerator of a Vulgar Fraction whose Denominator is an Unit with one or more Cyphers; as $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$ &c are Decimal Fractions.

2. In Decimals the Denominator is never expressed, but always understood to be an Unit with as many Cyphers as there are places in the Numerator, or given Decimal and thereby the Denominator, tho' not expressed, may be known.

3. Decimals are separated, and distinguished from Integers by a point or comma prefixed, and so wrote in one line like whole numbers.

Thus .3 is put for $\frac{3}{10}$, .03 for $\frac{3}{100}$, .003 for $\frac{3}{1000}$, &c.

Observe the same in mixt numbers, as 36.7 is put for $36\frac{7}{10}$, 3.67 for $3\frac{67}{100}$, 761.012 for $761\frac{12}{1000}$, 81.01261 for $81\frac{1261}{100000}$ &c.

4. The value of a Decimal is diminished by Cyphers on the left, in the same tenfold proportion as the value of an Integer is increased by having them on the right: thus $.5=\frac{5}{10}$, $.05=\frac{5}{100}$, $.005=\frac{5}{1000}$, $.0005=\frac{5}{10000}$, &c.

But

But Cyphers on the right hand of a Decimal alter not its value, for .5 and .50 and .500 &c. are all equal in value,

because $\frac{5}{10} = \frac{50}{100} = \frac{500}{1000}$ &c. $= \frac{1}{2}$, as is plain from Vulgar Fractions: and therefore Decimals are reduced to a common Denominator by annexing Cyphers. It may here be observed, that Cyphers on the right hand of a Decimal number (if nothing follow them) are as insignificant as Cyphers on the left hand of a whole number; and yet Cyphers are sometimes placed after Decimals, for the sake of regularity, or when we want to increase the number of decimal places.

5. In Decimals, the Integer or 1, is always supposed to be ÷ed into 10, 100, 1000, &c. equal parts; that is, 1 is supposed to be ÷ed into 10 equal parts, and each of these parts into 10 equal parts, and each of these parts into 10 parts more, and so on, by a continual Subdivision. For instance, suppose a foot rule (or any other measure) to be ÷ed into 10 equal parts; then each part is $\frac{1}{10}$; and if every one of those parts be ÷ed into 10 equal parts, the foot (or other measure) will be ÷ed into 100 equal parts; thus every part of the first division is $\frac{1}{10}$ or $\frac{10}{100}$; and every part of the second division, in respect of the whole, will be $\frac{1}{100}$: after the same manner we may conceive a foot, a yard, an acre, an hour, a bushel, a pound, a shilling, &c. to be ÷ed into 10, 100, 1000, 10000, &c. equal parts at pleasure.

6. If, as above, the Integer be ÷ed into 10, 100, &c. equal parts, the figures in the Decimal shews how many of such parts are taken as .1 denotes 1 tenth part, .3 three tenth parts, .5 five tenths, or half, .25 denotes 25 parts of 100, or $\frac{1}{4}$; and .75 denotes 75 parts of 100, or $\frac{3}{4}$; also .19896 denotes 19896 parts of 100000.

Now because $10 \times 10 = 100$, and $100 \times 10 = 1000$, it follows that the 10's are ten times greater than the 100's, the

100's, ten times greater than the 1000's, &c. if the figures were all equal; and in the Decimal .19896, if ever so many figures follow the first viz. .1, it is greater in value than them all; therefore unity or 1, is greater than .999999 &c. also $\frac{1}{10}$ or .1 is greater than .099999 &c. tho' infinitely continued.

7. The following Table will farther illustrate the Notation of Decimal Fractions.

The NOTATION TABLE.

Whole Numbers						Decimal parts.							
4	6	7	3	4	5	.6	7	8	9	3	2	4	&c.
100 thousands	10 thousands	thousands	hundreds	tens	units	tenth parts	hundredth parts	thousandth parts	10 thousandth parts	100 thousandth parts	millionth parts	10 millionth parts	

In whole numbers, the 1st place contains units, the 2d place to the left, tens; the 3d hundreds, &c. as before shewn; but in Decimals, the order of places is retrograde; for the 1st place in Decimals is tenths, the 2d place to the right is hundreth parts; the 3d, thousandth parts, &c. And as all figures on the left hand of the place of units, increase in their value, according to their distance from it, in a decuple proportion; so all figures on the right hand of the place of units, decrease in their value in a subdecuple proportion: as for instance, the number 345.6789 (where 5 stands in the place of units) is to be read thus; three hundred and forty five, six tenths, seven hundredth parts, eight thousandth parts, nine ten thousandth parts: or the Decimal parts may be read thus, six thousand seven hundred eighty nine ten thousand

ſand parts, by reducing them all to the Denominator of the greateſt, which is 10000, becauſe the laſt figure, 9 ſtands in the place of ten thouſandth parts. The reaſon of this latter way of reading is plain; for $\frac{6}{10}$ are $\frac{6000}{10000}$, and $\frac{7}{100}$ are $\frac{700}{10000}$, and $\frac{8}{1000}$, are $\frac{80}{10000}$, which being collected together with the remaining $\frac{9}{10000}$ make $\frac{6789}{10000}$. Therefore the numeration, or reading of Decimals, is the very ſame as that of whole numbers, with only adding the name of the parts ſignified by the Decimal.

§ VIII. *Addition and Subtraction of Decimals.*

RULE.

PLACE all the ſeparating points directly under each other, then tenths will ſtand under tenths, and hundredth parts under hundreths, &c. then add or ſubtract as in the whole numbers: and, laſtly, put a point under the other points, which will prick off the number of Decimal places in the ſum, or difference.

EXAMPLES in ADDITION.

	First	Second.	Third	Fourth
	,0200	,873	27.671	2,17
	,0603	,760	17,704	73.76
	,1700	,0731	1,612	,67
	,0120	,0129	0,160	1,61
	,0071	,9302	171,c61	,76
Sum	.2694	2.6492	218.208	78.97

5. What

5. What is the ſum of 736, 47.6737, 6.7361 360.721, .0126, and 24? Anſw. 1175.1434.

6. What is the ſum of 3.67, 761.2, 22.731, .72, 1.3612, .000276, and .062? Anſw. 789.744476

7. What is the ſum of .76, 7.6, 76, .076, .0076, .00076, and .760? Anſw. 844.44436.

EXAMPLES in SUBTRACTION.

	First.	Second.	Third.
From	.87	.1	1.
Take	.378	.09999	0.9999
Rem.	.492	.00001	.0001
Proof	.870	.10000	1 0000

4. What is the difference between 76.4 and 7.64? Anſw. 68.76

5. What is the difference of 1, and .1? Anſw. .9

6. What is the difference of 26.39 and 26.78? Anſw. .39

7. What number added to 739.712, will make it 801.31? Anſw. 61.598

8. What number +ed to .0679, will make it 20? Anſw. 19.9321

---

§. IX. *Multiplication of Decimals.*

RULE.

MULTIPLICATION in Decimals is alſo performed as in whole numbers, no regard being had to the Decimals as ſuch, till the Product is obtained; but then, ſo many Decimal places muſt be cut off from the right hand of the product, as there are in both factors. If there

there are not so many figures in the product as there ought to be Decimals, supply the defect with cyphers on the left.

EXAMPLES.

	First	Second	Third
Multiply	.3024	32.12	.0006
by	2.23	2.43	1.23
	9072	9636	18
	6048	12848	12
	6048	6424	6
Prod.	.674352	78.0516	.000738

4. What is the product of 741.983 by 3.48 ?
Answ. 2582.10084

5. Suppose there is a square garden, the length of whose side is 23.53 yards, how many square yards doth it contain ? Answ. 553.6609

6. In an oblong pavement, whose length is 78.451 feet, and breadth 27.8, how many square feet ?
Answ. 2180.9378

7. A cellar being dug whose length is 53.48, breadth 32.5 and depth 10.23 feet, how many solid feet of earth did it contain ?

length breadth depth
Answ. 53.48 × 32.5 × 10.23 = 17780.763 solid feet.

8. What is the product of .03217 by .0325 ?
Answ. .001045525

CONTRACTIONS.

I. When a Decimal, or mixt number, is to be ×ed by an Unit with Cyphers annext, as 10, 100, 1000, &c. you

you are only to remove the separating point so many places towards the right-hand in the multiplicand as there are cyphers annexed to the unit, subjoining cyphers if need be.

EXAMPLES.

1. The product of 7.374 by 10 is 73.74.

2. The product of 46.2 by 100 is 4620.

3. The product of .8460 by 10000 is 8460.

4. The product of .09642 by 1000 is 96.42

II. *It often happens in business, that the factors consist of many Decimal places; and to × them at large would be much trouble to little purpose, because a fewer number of places may do the business as well, if obtained true, which may be done as follows.*

1. Set the unit's place of the less factor, or ×er under such a number of the greater or multiplicand, as you would have kept in the product, and which stands immediately before the figures which are to be left out, either in whole numbers or Decimals.

2. Then set all the other figures of the ×er in a contrary, or inverse order, as in the example below.

3. Begin to × each figure in the multiplicand, by that figure in the ×er which stands directly under it, rejecting all the figures in the multiplicand which are on the right hand of your ×ing figure.

4. Set the products down so, that their right hand figures may fall strait below each other; and carry to such right-hand figures (for the increase that would arise from the foregoing figures of the multiplicand) 1 from 5 to 15, 2 from 15 to 25, 3 from 25 to 35, &c. and the sum of these simple products will exhibit so much of the product as was designed to be secur'd.

EXAMPLES.

1. Let 58,25435 be ×ed by 21.234, and have in the product (as sufficient) but two Decimal places.

Now

Now, if they are placed, and the operation wrought, as above directed, they'll stand thus,

```
 58,25435.
 432.12
 ---------
 116509
   5825
   1165
    175
     23
 ---------
 1236.97
 ---------
```

Or at large thus,

```
            58.25435
              21.234
      ----------------
      23 | 30:740
     174 | 76305
    1165 | 0870
    5825 | 435
  116508 | 70
  --------------------
  1236.97  286790
  --------------------
```

2. Let be required to × 1998612286 Integers by 5342945, and to have five places true in the product.— Also .23456 by itself and to have six Decimals true in the product.

```
        2d.                          3d.
  1998612286                      .234560
  5492435                         65432.0
  ----------                      --------
    99931                          46912
     5996                           7037
      799                            938
       40                            117
       18                             14
  ----------                      --------
   106784                         .055018
  ----------                      --------
```

Note 1. When whole numbers are multiplied by this contracted method, the whole multiplier must be inverted, as in Example 2.

2. You need not make use of any more figures in your ×er than what stands even with your multiplicand to the left, as in Example 2.

3. When it happens (as it does in Example 3d) that the number of figures produced by this contracted method of ×ing, are not so many as are assigned for the Decimal places in the product, the defect must be supplied by prefixing Cyphers, as was observed in the general Rule.

This contracted method is of excellent use in astronomical computations.

4. Multiply 3.141592 by 52.7438, retaining four Decimals in the product. Facit 165.6995.

5. Multiply .248264 by .725234, retaining five Decimals in the product. Facit .18004

6. Multiply 257,356 by 76,48, retaining no Decimals in the product. Facit 19682

7. Multiply 76.84375 by 8.21054, retaining five Decimals in the product. Facit 630.92868

8. Multiply .3570643 by .0210576, retaining nine Decimals in the product. Facit .007518917

---

§ X. *Division of Decimals.*

THE manner of operation is exactly the same in Division of decimals, as in whole numbers. The only difficulty (if any) is, to determine how many decimal places must be pricked off towards the right hand of the quote, which may be done by either of the following general Rules.

1. GENERAL RULE.

Consider the decimals as whole numbers, and ÷ them accordingly; then cut off from the right hand of the quote, as many decimal places as the dividend hath more than

than the ÷for: but when the quotient figures are too few, the want must be supplied with cyphers on the left: for there must always be just as many decimal figures in the ÷for and quotient together, as there are in the dividend alone; the reason whereof is manifest from IX Section.

For since the ÷for and quote ×ed together are to make the dividend, the ÷for and quote ought to have as many decimal places between them, as there are in the dividend.

Ex. 1. Let 69.9678 be ÷ed by 56.7,
Thus, 56,7)69,9678(1.234 the quote.

Here, by ÷ing as in whole numbers the qnote is 1234: but then considering that there were four Decimal places in the dividend, and but one in the ÷for, you must cut off three places from the right hand of the quote; and so make the true quote 1.234

For 1.234×56.7=69,9678 the proof.

Or the value of the quotient figures may be determined by the following Rule.

2. GENERAL RULE.

The first figure of the quotient, whether Integers or Decimals, is of the same degree, or value, as that figure of the dividend under which the units place of its product stands.

Ex. 2. Let .2274202 be÷ed by 5.326

```
5.326).2274202(.0427
        21304
        -----
        .14380
        10652
        ------
        .37282
         37282
         -----
        ......
```

Here since 1 (the units place of the product of the ÷for by the quotient figure 4) stands under the 2d place of Decimals in the dividend, therefore 4 must occupy the second place of Decimals in the quote.

From

From the above rules are deduced three cases, by which all operations in division of Decimals may be perfectly understood.

CASE I.

If the number of decimal places in the ÷for and dividend are equal, all the quote will be whole numbers. But note, if in this, or either of the following cases, there be a *remainder* after all the dividend figures are brought down, you may continue the quote to what number of decimals you please, by annexing a cypher continually to the last remainder, and ÷ing as before.

Ex. 3. Let 227,4202 be ÷ed by .5326 and the quote is 427 whole numbers.

Ex. 4. Divide 172,812 by 1.212.

```
1.212)172,812(142.584158 quote.
      1212
      ----
       5161
       4848
       ----
        3132
        2424
        ----
         7080
         6060
         ----
         10200
          9696
          ----
          5040
          4848
          ----
           1920
           1212
           ----
            7080
            6060
            ----
            10200
             9696
             ----
              504
```

Here, after all the dividend figures are brought down there is a remainder of 708, to which annex a cypher and ÷ as before, and you get one figure more, for a decimal, in the quote: and by annexing a cypher to each succeeding remainder, the quote is continued to six decimal places: thus by annexing cyphers you may continue the quotient at pleasure.

Ex.

Ex. 5. 412,992 ÷ .864=478.

Ex. 6. .0003264 ÷ .0000136=24.

Ex. 7. 69,9678 ÷ .0567=1234.

CASE II.

If the number of decimal places in the dividend exceed those in the ÷for, cut off their difference for decimals in the quote.

EXAMPLES.

69,9678 ÷ 56,7=1.234

.699678 ÷ 5.67=.1234

69,9678 ÷ .567=123.4

.00003264 ÷ ,136=.00024

CASE III.

When there are not so many decimal places in the dividend as there are in the ÷for, annex cyphers to the dividend to make them equal, then will the quote be all whole numbers, as in Case I.

EXAMPLES.

1. Divide 6727.5 by .345.
.345)6727.500(195 quote.

2 Divide 128.2 by 5.128
5.128)128.200(25 quote.

3. 11404.8÷3.456=3300 quote.

4. 13475÷.245=55000 quote.

5. ,75÷.0125=60 quote.

6. 1425÷.6252=2279 quote.

All the variety that can happen in divifion of decimals are contained in the following Examples, viz.

1. 234)809.64 (3.46

2. 2.34)809,64 (346

3. 2,34 80964.00 (34600

4. .0234)8.0964 (346

5. .0234).80964 (34,6

6. 2.34)8.0964 (3,46

7. 23,4).80964 (.0346

8. 234),80964 (.00346

9. .234)80964.000 (346000

CONTRACTIONS

I. When a decimal or mixt number is to be ÷ed by an integer with cyphers on the right, cut off the cyphers, and remove the feperating point in the dividend fo many places farther to the left as there were cyphers cut off from the right of the ÷for: then ÷ as before: prefixing cyphers if need be.

EXAMPLES.

1. Divide 1728.12 by 1200.

Thus 12)17.2812(1,4401 quote.

2. Divide

2. Divide 12.2416 by 40000.

4).00122416(.00030604 quote.

3d. 35.8845 $\div$ 470000 = .00007635 quote.

II. To $\div$ by an unit with cyphers, as 10, 100, 1000, &c. it is but removing the seperating point in the dividend so many places to the left, as there are cyphers annext to the unit.

EXAMPLES.

2873,67 $\div$ 100 = 28.7367

2873,67 $\div$ 10000 = .287367

5.6746 $\div$ 10000 = .00056746

III. In large divisions the work may be contracted as follows.

1. Find, by the 2d general Rule, what place of decimals or integers the first figure of the quote will possess.

2. Take as many of the left hand figures of the $\div$sor, as you judge necessary for your first $\div$sor, and just so many of the left hand figures of the dividend, as will yield you the first figure of the quote.

3. Having found the first figure of the quote; the following figures may be found thus; take each last remainder for a new dividend, and for a new $\div$sor prick off one figure from the right of each preceeding one; continue the operation till the $\div$sor is exhausted.

Note, In multiplying the quotient figure and divisor, leave out those figures prickt off, only particular regard must be had to the increase that would arise from the last figure so prickt off, and for such increase carry as in the second Contraction of Multiplication.

EXAM-

E X A M P L E S.

1. Divide 630,92878056 by 76,8437523

```
76,84375|23)630,92878|056(8,210541
 .......     61475000
             --------
              1617878
              1536875
              -------
                81003
                76843
                -----
                 4160
                 3842
                 ----
                  318
                  307
                  ---
                   11
                    7
                   --
                    4
```

This contraction is of excellent use in large divisions; especially in astronomical computations.

Explanation.

Here 8 is ×ed into 76 84375; then 2 is multiplied into 76,8437(carrying 1;) then 1 is multiplied into 76,843; the multiplication of 7684 by 0, is omitted; then 768 is multiplied by 5; then 76 by 4, lastly 7 by 1.

2. Divide 1236 9728679 by 21,234 so that the quote may contain three decimals. Facit 58,254

3. Divide 14169.206623951 by 384.672158 so that the quote may contain four decimals Answ. 36,8345

4. Divide 87,076326 by 9,365407 so as to have six places of decimals true in the quote. Facit 9,297655

---

§ XI. *Reduction of Decimals.*

1. TO reduce a vulgar fraction, to its equivalent decimal one.

RULE.

Add as many cyphers to the numerator as you would have decimal places; then ÷ that sum by the denomina- tor,

tor, and the quote (if nothing remains) will be a decimal equivalent to the given vulgar fraction. But when there is a remainder, you may by adding more cyphers, proceed so as to bring out the quote nearly equal.

EXAMPLES.

1. $\frac{1}{4}$=1.00÷4=.25 decimal, for 4 : 1 :: 1.00 : .25

2. $\frac{1}{2}$=1.0÷2=.5, for 2 : 1 :: 1.0 : .5

3. $\frac{3}{4}$=3.00÷4=.75, for 4 : 3 :: 1.00 : .75
By Problem XIV. Section II.

4. Reduce $\frac{2}{7}$ to a decimal.

Thus 2.000000 ÷ 7 = .285714 = $\frac{2}{7}$ nearly, not wanting $\frac{1}{1000000}$ part of an unit.

5. $\frac{5}{8}$ = .625 decimal.

6. $\frac{23}{64}$ = .359375 decimal.

7. $\frac{3}{32}$ = .09375 decimal.

Note 1. If the division ends at a certain number of places (as in the Examples above, except the 4th.) the decimal is called a finite; but that which no where ends, is called an infinite decimal: As in the following examples.

8. Reduce *one third* to a decimal.

Thus 1.0000 &c ÷3= ,3333 &c.

Note 2. When a figure continually repeats (as .3333 &c.) the decimal is called a single circulate, or recurring decimal. In all circulating numbers dash the first and last recurring figure.

9. Change $12\frac{17}{286}$ to decimal. First

286)17.0000000(.059440559440 &c.

Then $12\frac{17}{286}$=12.059440 nearly.

Note 3. When a certain number of figures continually repeat in the quote (as in the example above) the decimal is called

called a compound circulate; for by annexing another cypher to the remainder 170, the number is the ſame as when the diviſion firſt began: Conſequently the ſame figures viz. 59440 will repeat in the quote, ad infinitum.

10. Reduce $\frac{6}{7}$ to a decimal. Facit .857142857142.

11. $\frac{24}{13}$ = 1.8461538 &c.

12. $7\frac{7}{9}$ = 7,77777 &c.

13. $\frac{83}{9768}$ = .0084971334 &c.

14. $\dfrac{2}{17\frac{3}{4}}$ = .11267605 &c.

II. To reduce decimals or integers to equivalent decimals of a greater denomination.

CASE 1.

If the decimals or integers propoſed be ſimple, divide continually by all the denominations from the given one to that ſought: As in reductions of integers.

EXAMPLES.

1. Reduce .75 d. to the decimal of a l.

Thus, .75 ÷ 12 = .0625 s, ÷ 20 = .003125 l.

2. Reduce 1 grain to the decimal of an oz. Troy.

1.0000 ÷ 24 = .0416 dwts. ÷ 20 = .002083 oz.

3. Reduce 11 d. to the decimal of a l.

11.000 ÷ 12 = .916s &c. ÷ 20 = ,04583 l.

4. Reduce 3.216 pints of wine to the decimal of a hhd.

Facit 3.216 pts. = 0,42 gal. = ,00638 &c. hhd.

5. Reduce

5. Reduce 36 minutes to the decimal of a day.

36m. $=$,6 hr. $=$ .025 day.

6. Reduce 55 yards to the decimal of a mile.

55 yds. $=$,25 fur. $=$ .03125 mile,

7. Reduce .056 perches to to the decimal of an acre.
Facit .00035.

C A S E 2.

If the given part consists of several denominations, reduce them to that of the lowest for a numerator, and the integer to the same name for a denominator, then annex cyphers and divide as before.

Or thus, begin with the least and place down the several given denominations, for dividends, orderly under each other on the right hand of a perpendicular line, and on the left of it write against each dividend such a number, for a $\div$for, as will reduce it to the next greater name; then begin with the upper one, annex the quotient of each division, as decimal parts, on the right hand of the dividend next below it; and let this mixt number be $\div$ed by its $\div$for, Thus proced thro' all the denominations, and the last quote will be the decimal sought.

E X A M P L E S.

1. Reduce 17s. $6\frac{3}{4}$ d. to the decimal of a l.

First $\dfrac{\text{17s. } 9\frac{3}{4}\text{d.}}{\text{1l.}} \; \dfrac{= 843 \text{ q.}}{= 960 \text{ q.}}$ then 843,000000$\div$960

$=$ .878125 l. the required decimal.

Or thus,

4	3.00 q.
12	6.75 d.
20	17.5625 s.
	0.878125 l. the same as above.

2. Reduce

2. Reduce 11 oz. 16 dwts. 18 gr. to the decimal of a lb. Troy.

24	6	18
	4	(3
20		16.75
12		11,8375
		.9864583 &c. lb. the decimal reqd.

3. Reduce 3 qr. 16lb. 12 oz. Avoird. to the decimal of a cwt.

16	4	12
	4	(3
28	4	16.75
	7	4.1875
	4	3.59821428571 qr.
		.8995535714 cwt.

Reduce 2 ft. 9 in. 2 b. c. to the decimal of a yard.

3	2.00 b. c.
12	9.6666666 &c. in.
3	2.8055555 &c. feet.
	.935185185 &c. yds.

5. Reduce 59 yds. 2 ft. 9.5 in. to the decimal of a mile.

12	9.5 in.
3	2.791666 &c. ft.
220	59.930555 &c.
8	0.272411616 fur.
	.034051452 m.

6. Reduce 48 m. 37 seconds 54 thirds to decimals; 1 degree being integer.

60	54,0 thirds.
60	37.9 seconds.
60	48.6316666 m.
	.810527777 deg.

8. Let 16 ac. 3 ro. 16 pls. be all express'd in acres.

40	1.6
4	3.4
	16,85 acres.

Hence

Hence the following decimal TABLE is made:

Money.	Avoirdupoise Weight.
1 l. the integer.	1 cwt. the integer.
1 s. = ,05	1 qr. = ,25
1 d. = ,0041666	1 lb. = ,00892857
1 q. = ,0010416	1 oz. = ,00055803
Troy Weight.	**Long Measure,**
1 lb. the integer.	A yard the integer
1 oz. = ,0833333	1 ft. = ,3333333
1 dwt. = ,0041666	1 in. = ,0277777
1 gr. = ,0001736	1 b.c. = ,0092592
Apothecaries Weight.	**Square and Solid Measure:**
1 oz. the integer.	1 in. = ,006945, the decimal of a square foot.
1 dr. = ,125	1 in. = ,0005787 the decimal of a cubic foot.
1 scr. = ,0416666	
1 gr. = ,0020833	
Avoirdupoise Weight.	**Time.**
1 lb. the integer.	1 day the integer.
1 oz. = ,0625	1 hr. = ,0416666
1 dr. = ,00390625	1 min. = ,0006944
	1 sec. = ,0000115

III *To reduce a decimal of a superior denomination to its value in the inferior ones.*

RULE.

Multiply the given decimal by the number of parts contained in the next inferior denomination, cutting off the decimals from the product; (as taught in Multiplication) then × the decimals so cut off by the next lower denomination, and you'll have the parts of that denomination; thus proceed to the lowest name required, or till the decimals pointed off are all cyphers: then the number on the left of the points will express the value of the decimal.

EXAMPLES.

1. What is the value of ,878125 l?

```
 ,878125 l.
      20
 ----------
17,562500 s.
      12
 ----------
 6,750000 d.
       4
 ----------
 3,00000 q.
```

Answ. 17s. $6\frac{3}{4}$d.

2. What is the value of ,9864583 lb. Troy?

```
 ,9864583 lb.
       12
 -----------
11,8374996 oz.
       20
 -----------
16,7499920 dwt.
       24
 -----------
 29999680
 14999840
 -----------
17,9998080 grs.
```

Answ. 11 oz. 16 dwts 18 grs. nearly.

3. What is the value of ,8995535 cwt.?

```
 ,8995535 cwt.
       4
 -----------
 3,5982140 qrs.
       28
 -----------
16,7499920 lb.
       16
 -----------
11,9998720 oz.
```

Answ. 3 qr. 16 lb. 12 oz. nearly.

4. What is the value of ,935185 yd?

```
 ,935185 yd.
      3
 ----------
2,805555 ft.
      12
 ----------
9,666660 in.
      3
 ----------
1,999980 b. c.
```

5. What is the value of ,034051452 mile? Answ. 59 yds. 2 ft. 9,49 in.

6. What

6. What is the value of 16,85 acres? Anſw. 16 acres 3 roods, 16 perches.

7. What is the value of ,6875 yard of cloth?
Anſw. 2 qr. 3 nls.

8. What is the value of ,071428 of a hhd. of wine?
Anſw. 4 gal. 1 qt. 1,999712 pt.

9. What is the value of ,3673 of a year?
Anſw. 134 days 1 hr. 32 m. 52 ſeconds, 48 thirds.

10. What is the value of ,548 degree of a circle?

Anſw. 32 min. 52 ſeconds 48 thirds.

XII. *The Rule of Three Direct, in Vulgar and Decimal Fractions.*

The RULE for *Vulgar Fractions.*

HAVING prepared the numbers by the rules in Reduction, ſo that the firſt and third terms be of one name, multiply continually together the 2d and 3d terms and the reciprocal of the 1ſt, for the anſwer.

The Rule of Three in decimals, is ſo much the ſame with that in whole numbers, that nothing need be ſaid of it, except to illuſtrate it by a few examples.

My chief reaſon for treating the Rule of Three in Vulgar and Decimal Fractions together, is, becauſe I have often found, by experience, that a learner will gain a clearer idea of both rules when wrought together, than when treated ſeparately.

Note. For the ſake of brevity, after the vulgar fractions propoſed in the queſtion, I have placed their equivalent decimals; eſpecially in thoſe that I have ſolved by decimals.

E X A M P L E S.

1. If $\frac{3}{5}$ = ,6 of a yard of cloth coſt $\frac{3}{8}$ l. = ,375 l. what will $\frac{3}{4}$ = ,75 of a yard coſt? Anſw. 9s. 4½d.

Stating. If $\frac{3}{5}$ yd. : $\frac{3}{8}$ l. :: $\frac{3}{4}$ yd:

$$\text{Operation } \frac{5\times3\times3}{3\times8\times4} = \frac{15}{32}\text{ l.} = 9\text{s. } 4\tfrac{1}{2}\text{d.}$$

By Decimals.

If ,6 yd. : ,375 l. :: ,75 yd. : ,46875 l. = 9s. 4½d. Anſw.

2. A father dying left his ſon a certain portion, of which he ſpent $\frac{1}{4}$th = ,25 part, and $\frac{1}{2}$ = ,5 of what remained, and had then 600l. left: how much was his portion at firſt?

Let $\frac{4}{4}$ = 1, repreſent his portion at firſt; from which take $\frac{1}{4}$, and there remains $\frac{3}{4}$ = ,75 part; $\frac{1}{2}$ of which is $\frac{3}{8}$ = ,375 part, which ſtill remains: then ſay,
If $\frac{3}{8}$ part : 600l. :: 1 whole : 600×8÷3=1600 l. Anſw.

By Decimals.

If ,375 part : 600 l. :: 1 whole : 1600 l. his portion at firſt.

3. After paying away *one fourth*, = ,25 and *one fifth*, = ,2 of my money, I found 66 guineas left in my bag: what was in it at firſt? Anſw. 120 guineas.

For $\frac{1}{4}+\frac{1}{5}=\frac{9}{20}$=,45 part paid out: then let $\frac{20}{20}$ = 1 be the quantity at firſt, from which take $\frac{9}{20}$ or ,45 and there remains $\frac{11}{20}$ or ,55 part in = 66 guineas: therefore ſay, as 11 : 66 :: 20 : 120 guineas: the denominators being rejected becauſe common.

By Decimals.

As ,55 : 66 :: 1 : 120 guineas, the Anſwer as above.

4. Out

4. Out of a caſk of wine, which had leaked away *one third*, 21 gallons were drawn; and then being gauged, it appeared to be half full: how much did it hold?

Anſw. 126 gallons.

For $\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$ part leaked out, and left in, which taken from 1, leaves $\frac{1}{6}$ part drawn out, = 21 gallons.

Then as $\frac{1}{6} : 21 :: \frac{6}{6}$: or, $1 : 21 :: 6 : 126$ gallons Anſw.

5. A ſum of money is to be ſhared between two perſons, A and B; ſo that as often as A takes 9 l. B is to take 4 l. Now it happened, that A received 15 l. more than B: their reſpective ſhares are required.

Firſt $4 + 9 = 13$; then it is evident A had $\frac{9}{13}$ part and B $\frac{4}{13}$ part, their difference being $\frac{5}{13}$ part = 15 l.

Then as 5 : 15 l. :: 9, or as 1 : 3 :: 9 : 27 l. A's ſhare,

And as 5 : 15 l. :: 4, or as 1 : 3 :: 4 : 12 l. B's ſhare.

6. It is found by experiment, that heavy bodies near the ſurface of the earth, in their fall deſcend $16\frac{1}{12}$ (equal $\frac{193}{12}$) Engliſh feet in the firſt ſecond of time; how many feet will they deſcend in 4 ſeconds?

Note 1. The ſpaces deſcribed, and the diſtances run thro' by falling weights, will always be as the ſquares of their times ſpent in falling.

2. To ſquare a number is to multiply it by itſelf. The ſquare of 1 is 1; and the ſquare of 4 is 16.

Then as $1 : \frac{193}{12} :: 16 : 257\frac{1}{3}$ feet the Anſw.

7. If a body falls 20 feet in 2 ſeconds of time, how far will it fall in 10,5 ſeconds? Anſw. 551 feet.

8. If one bricklayer, A, can build a wall in 40 days, B, in 30, C, in 20, D, in 10, and E, in 5; in how many

days can they finish the same, if they all work together?

SOLUTION.

It is evident that $\frac{1}{40}$ part of the work is to be done by A; $\frac{1}{30}$ by B; $\frac{1}{20}$ by C; $\frac{1}{70}$ by D; and $\frac{1}{5}$ by E. Now these fractions reduced to a common denominator (by the 3d Rule in Prob. IX. of Reduction,) and added together make $\frac{49}{720}$. Then say, as 49 : 1 :: 120 : $2\frac{22}{49}$ days the Answer. That is, as the sum of the numerators, is to unity; so is the common denominator to the mean time.

9. Some boys A, B, C, D, E, F, G, H, and I, robbed an orchard; A had for his share $\frac{1}{3}$, B $\frac{1}{12}$, C $\frac{1}{8}$, D $\frac{1}{20}$, E $\frac{1}{7}$, and F $\frac{1}{4}$ part of the whole; G had 310, H 425, and I 140 apples; how many apples had they in all?

SOLUTION.

The above fractions being added together the sum is $\frac{143}{168}$ (in the lowest terms) part, taken by A, B, C, D, E, and F (all together;) and $310+425+140=875$ apples, the residue, taken by G, H, and I, together; now let $\frac{168}{168}=1$, represent the whole taken by them all together; from which take $\frac{143}{168}$, and there remains $\frac{25}{168}$ part, $=875$ apples: therefore, by proportion, say,
as 25 : 875 ap. :: 168 : 5880 apples the Answer.

10. A person dying left his wife with child, and making his will, ordered, that if she went with a son, *two thirds* of his estate should belong to him, and the remainder to his mother; and if she went with a daughter, he appointed the mother *two thirds* and the daughter *one third*: but it so happened that she was delivered both of a son and a daughter; by which she lost in equity 2000l. more than if it had only been a girl: what would have been her dowry had she only had a son?

SOLUTION.

Here if the son have $\frac{2}{3}$, and his mother $\frac{1}{3}$, the son has twice as much as his mother, and by the same way of reasoning

reasoning, the mother will have twice as much as the daughter, therefore take three numbers to represent each person's part, that are in proportion to one another, as 2 to 1, such are the numbers 4, 2, 1. Then $4+2+1=7$; and as $7:4::1:\frac{4}{7}$ the son's part,

$7:2::1:\frac{2}{7}$ the mother's part,

$7:1::1:\frac{1}{7}$ the daughter's part.

according to the present conditions of the question: but had she been delivered of a girl only, the mother's part would have been $\frac{2}{3}$; which is now but $\frac{2}{7}$ of the estate; then $\frac{2}{7}$ taken from $\frac{2}{3}$ leaves $\frac{8}{21}$ their difference.

Then as, $\frac{8}{21} : 2000 :: 1 : 5250$l. the whole estate, $\frac{1}{3}$ of which is 1750l. the mother's share if she had only had a son.

11. A person was possessed of $\frac{3}{5}$ share of a silk mill, and sold $\frac{3}{4}$ of his interest therein for 1710 l what was the value of the whole mill? Answ. 3280 l.

12. If $\frac{7}{8}$ of $\frac{4}{5}$ of $\frac{1}{2}$ of a ship be worth $\frac{4}{7}$ of $\frac{5}{9}$ of $\frac{11}{13}$ of the cargo, valued at 12000l: what did both the and cargo stand the owners in?

Answ. 15223l. 8s. 10d. $1\frac{45}{91}$q.

13. A younger brother received 2200l. which was *five twelfths* of his elder brother's fortune, and $3\frac{1}{8}$ times the elder's money was as much as the father was worth: now when the two sons had received their fortunes, the father gave the residue of his money to his six daughters, according to their minority as follows, *namely*, as often as A had $6\frac{1}{2}$l. he gave B 5l. C 4l. D 3l. E 2l. and F $1\frac{1}{2}$l. how much was each daughter's fortune?

Answ.

Answ. A's 2665l. B's 2050l. C's 1640l. D's 1230l E's 820l. and F's 615 l.

14. B. and C. together can build a boat in 18 days; with the assistance of A, they can do it in 11 days: in what time would A do it by himself? Answ. $27\frac{2}{7}$ days.

15. Sound is said to move 1142 English feet in one second of time, how far will it move in 1,5 minute?
Answ. 102780 feet.

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§ XIII. *The Inverse Rule of Three in Vulgar Fractions.*

RULE.

MULTIPLY continually together the 1st. and 2d. terms and the reciprocal of the 3d. for the answer.

EXAMPLES.

1. If the penny white loaf weighs 9 oz. Troy, when wheat is at $4\frac{2}{3}$s. $=\frac{14}{3}$s. a bushel; what ought it to weigh, when wheat is at $6\frac{3}{4}$ s. $=\frac{27}{4}$s. a bushel?

$$\text{If } \frac{14}{3}\text{s.} : \frac{9}{1}\text{ oz.} :: \frac{27}{4}\text{s.} : \frac{4\times14\times9}{27\times1\times3} = \frac{4\times14}{3\times3} = 6\frac{2}{9}\text{ oz.}$$

the Answ.

2. It is found by experiment that a pendulum must be 39,2 inches long to swing seconds; that is, to make 60 vibrations in a minute; how long then must a pendulum be, to make 120 vibrations in a minute?

Note. The length of pendalums are as the squares of their vibrations reciprocally. And contrarywise.

Therefore $60\times60=3600$, and $120\times120=14400$:
Then as 3600 : 39.2 :: 14400: 9,8 inches, the Answ.

§ XIV.

§ XIV. *The Double Rule of Three in Vulgar Fractions.*

RULE.

HAVING placed down and marked the terms, (according to the rule in page 71) take the continual product of the multiplying terms, and the reciprocals of the dividing terms for the answer.

EXAMPLES.

1. If the carriage of $3\frac{1}{3}$ cwt. $= \frac{10}{3}$ cwt. $150\frac{1}{4} = \frac{601}{4}$ miles costs $2\frac{1}{10}$ l. $= \frac{21}{10}$ l. what will the carriage of $7\frac{2}{5}$ cwt. $= \frac{37}{5}$ cwt. for $60\frac{3}{5} = \frac{303}{5}$ miles cost at the same rate?

$$\div \frac{10}{3}\text{ c.} : \frac{21}{10}\text{ l.} :: \frac{37}{5}\text{ c.} \times$$

$$\div \frac{601}{4}\text{ m.} : \text{———} :: \frac{303}{5}\text{ m.} \times$$

$$\frac{21 \times 37 \times 303 \times 4 \times 3}{10 \times 5 \times 5 \times 601 \times 10} = 1\text{l. } 17\text{s. } 7\tfrac{1}{4}\text{q.} +$$

2. Suppose the weight of a beam, whose length is $40\frac{2}{3}$ feet, breadth $3\frac{1}{5}$ feet, thickness $2\frac{2}{3}$ feet, be 467 stones, what will the weight of another beam be whose length is 60 feet, breadth 5 feet, and thickness $3\frac{1}{2}$ feet?

Answ. $1418\frac{241}{288}$ stones.

3. If $36\frac{1}{2}$ lb. of bread be sufficient for 3 soldiers $3\frac{1}{2}$ days, how many days will 216lb. suffice 12 soldiers?

Answ. $5\frac{13}{73}$ days.

4. If I pay 16s. 4d. for the carriage of $5\frac{1}{4}$ cwt. 20 miles, what must be paid for the carriage of $17\frac{3}{4}$ cwt. $7\frac{1}{2}$ miles? Answ. 1l. $8\frac{1}{2}$d.

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§ XV. *Single Fellowship.*

SINGLE FELLOWSHIP (or Partnership) is that which determines how much gain or loss is due to every Partner concerned; by having the whole gain or loss, and their particular stocks, given.

1. A GENERAL RULE.

Say by the Rule of Three, as the whole stock : is to the whole gain or loss : : so is every man's particular stock : to his particular part of the gain or loss.

EXAMPLES.

1. Two Partners A, and B, make a stock of 120 l.; A puts in 40l.; and B 80l.; they gain 50l. by Trade. What is the gain of each?

$$\begin{array}{ll} \text{stock} & \text{gain} \\ \text{As } 120\text{ l.} : & 50\text{l.} \; : : \end{array} \left\{ \begin{array}{l} 40\text{ l.} : 16\text{ l. } 13\text{s. } 4\text{d.} = \text{A's} \\ \\ 80\text{l.} : 33\text{ l. } 6\text{s. } 8\text{d.} = \text{B's} \end{array} \right\} \text{gain.}$$

50l. 0s. 0 d. proof.

The proof is made, by adding together all their shares, which must be equal to the whole gain or loss. Or thus, as the whole gain or loss : is to the whole stock : : so is each man's share of the gain or loss, : to his share in the Stock.

2. Three

2. Three men A, B, C, freighted a ſhip with Wine; A had 284 tuns; B 140, and C 64. By a ſtorm at ſea they were obliged to caſt 100 tuns over board. What loſs doth each ſuſtain?

Firſt 284 + 140 + 64 = 488 the whole ſtock.

$$\begin{array}{llll} & \text{ſtock} & \text{loſs} & \\ \text{As} & 488 & : 100 :: & \left\{\begin{array}{r} 284 \\ 140 \\ 64 \end{array}\right\} \begin{array}{l} : 58\frac{96}{488} = \text{A's} \\ . \ 28\frac{336}{488} = \text{B's} \\ : 13\frac{56}{488} = \text{C's} \end{array} \left.\vphantom{\begin{array}{l}a\\a\\a\end{array}}\right\} \text{loſs.} \end{array}$$

100 proof.

3. Four men bought a barrel of ale for 20s. of which A was to pay one third part, B one fourth, C one fifth, and D one ſixth; what muſt each pay?

Firſt, one third, one fourth, one fifth, and one ſixth of 20s, when added together make juſt 19s. = 228d. Then ſay,

$$\begin{array}{llll} \text{As } 228\text{ d.} : 20 :: & \left\{\begin{array}{r} 80\text{ d} \\ 60\text{ d} \\ 48\text{ d.} \\ 40\text{ d.} \end{array}\right\} & \begin{array}{l} : 7\text{s. 0d. } 0\frac{48}{57}\text{q. A's} \\ : 5\text{s. 3d. } 0\frac{36}{57}\text{q. B's} \\ : 4\text{s. 2d. } 2\frac{6}{57}\text{q. C's} \\ : 3\text{s. 6d. } 0\frac{24}{57}\text{q. D's} \end{array} & \left.\vphantom{\begin{array}{l}a\\a\\a\\a\end{array}}\right\} \text{ſhare;} \\ \quad\text{contracted} & & & \\ \text{Or } \ 57 : 5 :: & & & \end{array}$$

20s. proof.

2. RULE.

Where many Partners are concerned; find the ſhare of 1 integer, by dividing the whole gain or loſs by the whole ſtock, and the quotient will be a common multiplier; by that multiply every man's part of the ſtock, and it will give his ſhare of the gain or loſs.

4. Eight Men trade together, and compoſe a ſtock of 2400 l. A put in 96l. 10s. = 96,5 l. B 108,25 l. C 162,3 l. D 196.3 l. E 260.4 l. F 300 75 l. G 346.25 l. H 929.25 l; and they gained 288 l. What is the ſhare of each?

2400

2400)288.00(.12 a common ×er, or the gain of 1l.

$$\left.\begin{array}{r} \text{l.}\\ 96.5\\ 108.25\\ 162.\ 3\\ 196.\ 3\\ 260.\ 4\\ 300.75\\ 346.25\\ 929.25 \end{array}\right\} \times .12 = \left\{\begin{array}{r} \text{l.}\\ 11.580\\ 12.990\\ 19.476\\ 23.556\\ 31.248\\ 36.090\\ 41.550\\ 111.510 \end{array}\right\} = \left\{\begin{array}{l} \\ \text{A's share.}\\ \text{B's}\\ \text{C's}\\ \text{D's}\\ \text{E's}\\ \text{F's}\\ \text{G's}\\ \text{H's} \end{array}\right.$$

1800l. proof.

5. Eight captains plundered the enemy of 1800l. the first had 10 men, the 2d 15, the 3d 20, the 4th 28, the 5th 60, the 6th 76, the 7th 112, the 8th 129. What must each captain have in proportion to his number of soldiers?

The number of soldiers is 450; then 1800÷450=4 a common ×er, or the share of one man. Now each captain's men ×ed by 4 will give his share of the money: so the 1st captain's share is 40l. the 2d 60l. the 3d 80l. the 4th 112l. the 5th 240l. the 6th 304l. the 7th 448l. the 8th 516l.

6. Three men composed a stock thus, A put in 40l. B and C together 170l. they gained 126l. of which B took 42l. for his share. B and C's stock, and A and C's gain are required? Answ. A's gain is 24l. and C's 60l. B's stock is 70l. and C's 100l.

7. Two men entered into partnership thus, A put in 170l and took 4 sevenths of the gain; what did B put in? Answ. 127l. 10s.

8. A, B, C, and D, joined their stocks in trade and gained a certain sum of money, of which A, B, and C took 60l. B, C, and D 90l. C, D, and A 80l. D, A and B 70l. what is each person's distinct gain? Answ. A's 10l. B's 20l. C's 30l. and D's 40l.

§ XVI.

§ XVI. *Double Fellowship.*

THE *Double Rule of Fellowship* is that which determines how much gain or loss is due to every partner concerned; by having the whole gain or loss, and the particular stocks, and their time of continuance given.

1. RULE.

Multiply every man's stock by the time it is employed; then by the Rule of Three, say, as the sum of these products : is to the whole gain or loss :: so is each of these products : to each man's gain or loss.

EXAMPLES.

1. Three merchants A, B, C, enter into partnership. A put in 60l. for 4 months; B, 50l for 3 months; C 30l. for 2 months. They gain 30l. What is each man's share of the gain?

$$\left.\begin{matrix}60\times4\\50\times3\\30\times2\end{matrix}\right\}=\overset{\text{products}}{\left\{\begin{matrix}240\\150\\60\end{matrix}\right\}}=\left\{\begin{matrix}\text{A's}\\\text{B's}\\\text{C's}\end{matrix}\right\}\text{stock}\times\text{ed into his time.}$$

450 the sum of the products.

1.

$$\begin{matrix}\text{As } 450 : 30 :: \\ \text{Or } 15 : 1 ::\end{matrix}\left\}\begin{matrix}240 : 16 = \text{A's}\\150 : 10 = \text{B's}\\60 : 4 = \text{C's}\end{matrix}\right\}\text{gain.}$$

contracted by ÷ing by 30

30l. proof.

2. Four men A, B, C, D, hold a pasture in common, for which they pay 120l. A had 48 oxen 32 days; B 24 oxen 48 days; C 32 oxen for 24 days; and D had 20 oxen for 30 days. What must each pay?

oxen		days		products	
48	×	32	=	1536	the product of each man's oxen ×ed by the days they were feeding.
24	×	48		1152	
32	×	24		768	
20	×	30		600	
				4056	sum of the products.

l.		l.	s.	d.	q.	
As 4056 : 120 ::	1536 :	45	8	10	$2\frac{6}{169}$	A's share.
Or 169 : 5 ::	1152 :	34	1	7	$3\frac{89}{169}$	B's
contracted by dividing by 24.	768 :	22	14	5	$1\frac{3}{169}$	C's
	600 :	17	15	0	$1\frac{71}{169}$	D's
		120 proof.				

2. RULE.

When many partners are concerned, divide the whole gain or loss by the first term or sum of the products, and the quotient will be a common multiplier; by which multiplying the several products, you'll have the several shares.

3. Six merchants trade after this manner.

A puts in 50l. for 6 months, and then puts in 60l. more for 4 months longer.

B puts in 90l. for 8 months.

C puts in 160l. for 5 months, and then takes out 60l. for 4 months more.

D puts in 200l. for 7 months, and then puts in 100l. more for 5 months.

E puts in 300l. for 10 months, and 100l. more for months.

F

F puts in 400l. for 3 months, and 200l. for 4 months more, and then takes out 400l. for 5 months more.

They gained 686,4l. what is the gain of each merchant ?

The ſeveral products of the ſtock and time will be as follows.

$$\left.\begin{array}{l} 50\times6 = 300 \\ 60\times4 = 240 \end{array}\right\}\text{add} \qquad 540 \text{ for A.}$$

$$90\times8 = 720 \text{ for B.}$$

$$\left.\begin{array}{l} 160\times5 = 800 \\ 100\times4 = 400 \end{array}\right\}\text{add} \qquad 1200 \text{ for C.}$$

$$\left.\begin{array}{l} 200\times7 = 1400 \\ 100\times5 = 500 \end{array}\right\}\text{add} \qquad 1900 \text{ for D.}$$

$$\left.\begin{array}{l} 300\times10 = 3000 \\ 100\times2 = 200 \end{array}\right\}\text{add} \qquad 3200 \text{ for E.}$$

$$\left.\begin{array}{l} 400\times3 = 1200 \\ 200\times4 = 800 \\ 200\times5 = 1000 \end{array}\right\}\text{add} \qquad 3000 \text{ for F.}$$

For A. B. C. D. E. F.

$540+720+1200+1900+3200+3000=10560$ the ſum of the products.

10560)686,400(,065 the ſhare of 1l. being a common multiplier.

$$\text{Therefore}\ \begin{array}{c}\text{products}\\ \left.\begin{array}{r} 540 \\ 720 \\ 1200 \\ 1900 \\ 3200 \\ 3000 \end{array}\right\}\end{array} \times,065 = \begin{array}{c}\text{l.}\\ \left\{\begin{array}{r} 35,1 \\ 46,8 \\ 78,0 \\ 123,5 \\ 208,0 \\ 195,0 \end{array}\right\}\end{array} = \left\{\begin{array}{l} \text{A's share of} \\ \text{B's the gain} \\ \text{C's} \\ \text{D's} \\ \text{E's} \\ \text{F's} \end{array}\right.$$

686,4l. proof.

 4. Two

4. Two merchants traded together: A put into ftock 400l. for 12 months, B put into ftock a certain fum which continued in trade 8 months, and they gained 300l. of which B had 160l. what was B's ftock?

Firft, $400 \times 12 = 4800$ the product of A's ftock and time, $300l. - 160l = 140l. =$ A's gain: Then

As $140 : 4800 :: 160 : 5485,714 =$ B's ftock and time, then $8)5485,714(685,714251 =$ B's ftock.

5. Three perfons A, B, and C, entered into partnerfhip, their common ftock being 3822l. A's money was in 3 months, B's 5 months, and C's 7 months; they gained 234l. which was fo divided, as the half of A's gain was equal to *one third* of B's gain, and equal to $\frac{1}{4}$ of C's gain what did each gain and put in?

Suppofe A's gain was 4l then B muft have 6l. and C 8 according to the conditions of the queftion; then $4+6+8=18$, and by the Rule of Three, fay,

$$\left.\begin{array}{l} \text{As } 18 : 234 :: \\ \text{Or } \ 1 : 13 \ :: \end{array}\right\} \begin{array}{l} 4 : \ \ 52 \text{ A's gain.} \\ 6 : \ \ 78 \text{ B's gain.} \\ 8 : 104 \text{ C's gain.} \end{array}$$

Having the whole ftock and each man's gain (or lofs) and time given, each man's ftock may be obtained by the following general rule.

*RULE.

Multiply each man's gain (or lofs) by all the other time, except his own, then fay, as the fum of thefe products: is to the whole ftock :: fo is each of thefe particular products: to each man's particular ftock. Now in the

* I have never as yet met with a true rule, in any Author for finding each man's ftock, by having the whole ftock each man's gain (or lofs) and time given: notwithftanding it is

the present question: $52\times5\times7=1820$; likewise $78\times3\times7=1638$; and $104\times3\times5=1560$; then $1820+1638+1560=5018$, the sum of the products. Now say,

	products	l.	s.	d.	
As 5018 : 3822 ::	1820 :	1386	4	4 $\frac{572}{2509}$	A's stock;
contracted.	1638 :	1247	11	11 $\frac{13}{2509}$	B's stock.
Or 2509 : 1911 ::	1560 :	1188	3	8 $\frac{1924}{2509}$	C's stock.
		3822 l. proof.			

The above question is the same with the 6th in HILL's *Arithmetic*, page 284, but there calculated by a false rule, by which he finds A's stock=468l. B's=1170l. and C's =2184l.

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infallible a method of proving the double rule of Fellowship: and for that reason, I have here inserted my method of investigating the rule, which will be easily understood by any one who hath a small smattering of Algebra: and altho' this step is a little out of the way, yet I hope the candid reader will pardon it.

Put $a=52$l. $b=78$l. $c=104$l. $r=3$, $n=5$ $m=7$ months, $s=3822$l. stock, and let x be put for A's stock: now, as in question 4th, $rx=$ the product of A's stock and time;

then as $a : rx :: b : \frac{brx}{a}$, the product of B's stock and time, which $\div$ed by B's time viz. n, gives $\frac{brx}{na}=$ B's stock: Again as $a : rx :: c : \frac{rxc}{a}$, the product of C's stock and time, which $\div$ed by m, gives $\frac{rxc}{ma}=$ C's stock: then $x+\frac{brx}{na}+\frac{rxc}{ma}=s$; which equa-

6. A, B, and C, company: A puts into ſtock 1386l. 4s. $4\frac{572}{2509}$d. for 3 months; B puts in 1247l. 11s. $11\frac{13}{2509}$d. for 5 months; C puts in 1188l. 3s. $8\frac{1224}{2509}$d. for 7 months; and they gained 234l. Required each perſon's ſhare thereof? Anſw. A's ſhare is 52l. B's 78l. and C's 104l.

7. A, B, and C company: A put in his ſhare of the ſtock for 5 months, and took one fifth of the gain; B, put in his for 8 months; C, advanced 400l. for 7 months, and required on the balance two thirds of the gain: what is the ſtock of A, and B? Anſw. A's 168l. and B's 70l.

8. Four merchants trade after this manner. A puts in 100l. for 8 months. B puts in 80l. for 5 months, and then puts in 40l. more for 3 months longer. C puts in 176l. for 4 months, and then takes out 50l. for 4 months more. D puts in 230l. for 6 months, and then takes out the whole. They gained 212l. 10s. what is the gain of each merchant?

		l.	s.	d.
Anſw.	A's ſhare is	40	19	8
	B's	38	18	$8\frac{1}{4}$
	C's	61	17	$8\frac{1}{2}$
	D's	70	13	$11\frac{1}{4}$.

§ XVII.

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tion reduced gives $x = \dfrac{sanm}{anm + brm + crn}$, which gives the following analogy,

As $anm + brm + crn : s :: anm$: to A's ſtock: and in like manner, if x be put to repreſent B and C's ſtock ſeverally, by proceeding as above, we ſhall have $x = \dfrac{sbrm}{anm + brm + crn}$ = B's ſtock, and $x = \dfrac{scrn}{anm + brm + crn}$ = C's ſtock. That is,

As, $anm + brm + crn : s :: \begin{cases} brm : \text{to B's ſtock,} \\ crn : \text{to C's ſtock.} \end{cases}$

Which is the ſame as the rule above given, in words at length.

§ XVII. BARTER.

BARTERING is the exchanging of commodities, so as neither party is supposed to sustain any loss.

RULE.

1. Find the value of that commodity whose quantity is given, then find what quantity of the other commodity, at its proposed rate, will amount to the same money, and so much of the latter commodity, must be given for the former.

2. When one commodity is advanced above the ready money price, find the advanced price of the other in the same proportion, thus: by the Rule of Three, say, as the ready money price of the one : is to its advanced price :: so is that of the other : to its advanced price.

3. When the commodities to be exchanged are of unequal value, the defect is supplied by money, or some commodity equivalent in value.

EXAMPLES.

1. How much coffee at 5s. per lb. must be given for 367lb. of tea at 8s. per lb? Answ $587\frac{1}{5}$lb.

First as 1lb. : 8s. :: 367lb. : 2936 s. the value of the tea. Now there is no more to do but to find how much coffee at 5s. per lb. may be bought for 2936 s. Thus

As 5s. : 1lb :: 2936s. : $587\frac{1}{5}$lb, the Answ.

2. How much sugar at 8d. per lb. may be bought for 12 cwt. of tobacco at 4l per cwt? Answ. 1440lb.

First $12 \times 4 = 48$ l. the value of the tobacco, $=$ 2880 groats: then if 2 : 1lb. :: 2880 : 1440lb. the Answ.

3. A and B barter thus: A hath 100 yards of cloth at 6s. 8d. per yard ready money, but in barter he will

have

8s. per yard. B hath hops at 1l. 14s. per cwt. ready money: it is required to find how many cwt. of hops B must give A for his cloth, to make his gain in the barter equal to A's.

First 6s. 8d.=20 groats, and 34s.=102 groats,

Then as 20. : 8s. : : 102 : 40,8 s. the advanced price of one cwt. of B's hops. Now proceed as in the two examples above.

Thus 100×8=800 s: the advanceed value of the cloth.

Then as 40,8 s. : 1 cwt : : 800 s. : 19 cwt. $3\frac{3}{51}$ qr. the Answ. required.

4. Two merchants A and B barter with each other thus, A has 460 yards of broad cloth, worth 9 s. 2d. per yard, but in barter he will have 11 s. per yard. B has tobacco worth 2s. per lb. which he charges at 2s. 6d. How much tobacco must A receive for his cloth; and what does he gain or lose by the bargain?

In this question, first find what the cloth comes to at the advanced price; then how much tobacco at its advanced price, may be bought for that money, and lastly the true value of both.

1 yd. : 11s. : : 460 y. : 5060s. the advanced value of the cloth: then as $2\frac{1}{2}$s. : 1lb. : : 5060s. : 2024lb. of to bacco received. Next find the true value of both at their prime cost. Thus,

1 y. : $9\frac{1}{6}$s. : : 460 y. : $4216\frac{2}{3}$ s. the value of the cloth.

and 1lb. : 2s. : : 2024lb : 4048 s. the value of the tobacco.

Difference = $168\frac{2}{3}$ s. which A lost by the bargain.

5. A and B barter: A has 140 l. 11 oz. of plate, at 6s. 4d. per oz. which in truck he rates at 7s. 2d an ounce and allows a discount on his part, to have *one seventh* of that in ready money. B has tea worth 9s. 6d. the pound, which he rates at 11s. 2d. when they struck the balance

lance, A received but 7 c. 2 qr. 18 lb. of tea: what discount did A allow B; which of them had the advantage, and how much, in an article of trade thus circumstanced?

First 140 lb. 11 oz. = 1691 oz. 7s. 2d. = 86 d.

then 1 oz : 86d. :: 1691 oz. : 145426 d. = 605 l. 18s. 10 d the value of the plate at the advanced price; *one seventh* of which is 86 l. 11 s. $3\frac{1}{7}$ d the ready money to be paid, which taken from 605 l. 18 s. 10 d. leaves 519 l. 7 s. $6\frac{6}{7}$ d. = $124650\frac{6}{7}$ d. to be laid out in tea at 11 s. 2d. = 134 d. per lb.

Then as 134 d. : 1 lb. :: $124650\frac{6}{7}$ d. : 8 c. 1 qr. $6\frac{108}{469}$ lb. from which take the quantity received, viz. 7 c. 2 qr. 18 lb and there remains 2 qr. $6\frac{108}{469}$ lb. which at 11 s. 2d. per lb. amounts to 40 l. 6 s. $6\frac{6}{7}$d. the discount.

	l.	s.	d.
B's gain at 20 d. per lb. upon the tea he delivered, viz. 7 c. 2 qr. 18 lb. amount to	71	10	0
A's gain at 10 d. per oz. upon his plate delivered, viz. 140 lb. 11 oz. amounts to	70	9	2
The difference of their gain in barter is	1	0	10
But A allows B discount to the amount of	40	6	$6\frac{6}{7}$
Therefore B has the advantage by	41	7	$4\frac{6}{7}$

6. How much tea at 9s. per lb. can I have in barter for $4\frac{1}{2}$ cwt. of chocolate, at 4s. per lb? Answ. 2 cwt.

7. How many knives at 5s. 2d. per doz. must be delivered in barter for 3 cwt. 2 qr. 16 lb. of steel, at 37 s. 4d. per cwt.

Answ. $26\frac{10}{31}$ doz.

8. A and B barter thus: A has $53\frac{5}{8}$ quarters of corn, at 1 l. 10 s. per quarter; for which B would give $13\frac{1}{7}$ cwt. of

ſugar at 4 l. 12 s. per cwt. and the balance in raiſins, at $6\frac{1}{2}$ d. per lb. how many pounds of raſins muſt be given ?

Anſw. $737\frac{67}{91}$ lb.

9. A and B barter: A gives to B 90 doz. of penknives at 7s. 8d. per doz. for which B gives to A $10\frac{1}{2}$l. in money and 500lb. of tobacco; what was it valued at per lb? Anſw 11 d. 2,08 qr.

10. A and B barter: A hath 20 cwt. of cheeſe at 21,5s, per cwt. B hath 8 doz. of handkerchiefs at $3\frac{7}{10}$ l. per doz. I deſire to know who muſt receive the difference, and how much ? Anſw. B muſt receive of A 8 l. 2 s.

11. A and B barter: B hath cheeſe at 30 s. per cwt. which is vendable but for 26 s. he would have *one third* ready money, and gain 10 per cent A hath handkerchiefs at 4 s. per piece ready money : how muſt the handkerchiefs be valued per piece, to equal the barter ?

Anſw. $5\frac{13}{15}$ s.

12. A has tea at 6 s. per lb. which he barters with B at 7 s. 6 d. per lb. and is willing to loſe 10 l. per cent. to have *one third* ready money : what is the juſt price of a yard of velvet delivered at 22 s. to equal the barter ?

Anſw. 18,7 s.

13. If 56 yards of cloth be given for $6\frac{1}{2}$ cwt. of raiſins at 2,4 l. per cwt. what is the cloth valued at per yard?

Anſw. 5s. 6d. $3\frac{3}{7}$q.

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§ XVIII. LOSS and GAIN.

BY this rule we are taught how to raiſe or ſettle the price of any commodity, ſo as to gain or loſe ſo much per cent. or by the whole, and is of excellent uſe to traders. A great variety of queſtions may be ſtarted in.

in this rule, which may be eaſily ſolved, if you conſider that the gains and loſſes are always in proportion as the given quantities, and the contrary. Therefore this rule is only direct proportion.

EXAMPLES.

1. Bought 5 cwt. 3 qr. 21 lb. of ſugar, at 2 l. 16 s. per cwt. and ſold it again for 8 d. per *pound*: how much was gained or loſt by the whole, and how much per cent?

Firſt 5 cwt. 3 qr. 21 lb. = 665 lb. and 2 l. 16 s. = 2.8 l. then 112 lb. : 2,8 l. :: 665 lb. : 16,625 l. = 16 l. 12 s. 6 d. = 3990 d. the prime coſt:

Again, 1 lb. : 8 d. :: 665 lb. : 5320 d. = 22 l. 3 s. 4 d. ſold for: therefore the gain upon the whole is 5 l. 10 s. 10 d. Next to find the gain per cent. ſay,
As 3990 d. : 5320 d. :: 100 l. : 133 l. 6 s. 8 d. therefore the gain per 100 l. is 33 l. 6 s. 8 d.

2. A Mancheſter chapman going to a fair, ſold fuſtians for 11 s. 6 d. the end, wherein was gained 15 l. per cent. and ſeeing no other chapman had ſo good, raiſed them at the latter end of the fair to 12 s. I demand what he gained per cent. by this latter ſale? Anſw. 20 l.

For 15 l. + 100 = 115 l. what 100 l. is advanced to by the firſt ſale; then as 11,5 s. : 115 l. :: 12 s. : 120 l. the amount of 100 l. by the latter ſale; then 120 l. minus 100 l. = 20 l. the Anſwer.

N. B. *This queſtion is the ſame with the eighth queſtion in loſs and gain in* HILL's Arithmetic *page* 289, *but there calculated by a falſe method; for the anſwer there brought out is* 15,652*l.* *which is* 4,348*l.* *too little; and in like manner ſeveral Authors have run into the ſame error in queſtions of this ſort, by making the gain or loſs of* 100*l* *the ſecond term in the ſtating, inſtead of its amount (in caſe of gain) or depreſſion (in caſe of loſs).*

3. A grocer bought coffee at 7 s. per lb. which, being damaged, he is willing to ſell out ſo as to loſe 8 l. per cent.

cent. at what rate per pound must it be sold? Answ. 6 s. 5 d. 1,12 q.

For 100—8=92 the depression of 100 l. then say, As 100 l. : 92 l. :: 7 s. : 6,44 s. the Answ.

4. If 5 cwt. of chocolate cost 252 l. at what rate per lb. must it be sold, that the gain upon the whole may be 18 l. 13 s. 4 d? Answ. at 9 s. 8 d. per lb.

5. If tea be bought for 7 d. per ounce, and sold for $8\frac{3}{4}$ d. what is the gain per cent? Answ. 25 l.

6. If needles be bought for 2 s. 6 d. a gross, how many may be sold for 1 d. to gain 20 l. per cent? Answ. 4.

7. A had 15 pipes of wine which he parted with to B, at 4 l. 6 s. 8 d. per cent. profit, who sold them to C, for 38 l. 11 s. 6 d. advantage; C made them over to D for 500 l. 16 s. 8 d. and cleared thereby, $6\frac{1}{2}$ l. per cent. what did this wine cost A per gallon? Answ. 4 s. $4\frac{3}{4}$ d.

8. If when I sell cloth at $8\frac{1}{2}$ s. per yard, I lose 5 *per cent*. what do I gain or lose per cent. when I sell it for 9 s. per yard? Answ. 11 s. $9\frac{3}{17}$ d. gain.

9. Paid 69 l. for 1 ton of steel, which is retailed at 6 d. per lb. what is the profit or loss by the sale of 14 tons? Answ. 182 l. loss.

10. Having sold 12 lb. of tea for 5 l. 14 s. and thereby gained 8 l. per cent. what was the prime cost of a lb? Answ. 8 s. 9 d. $2\frac{2}{9}$ q.

§ XIX. ALLIGATION.

ALLIGATION teaches how to mix several sorts of simples or ingredients together, so that the composition may be of a middle quality. Alligation is commonly distingished into two principal cases, called *alligation medial* and *alligation alternate*.

CASE

Case I. ALLIGATION MEDIAL.

Alligation Medial teaches how to find the mean rate of a mixture or compound, when the particular quantities are mixt, and their ſeveral rates are given.

Note. That by the rates are meant the numbers which determine, or define the proportion of the qualities of the ſimples and compound; ſuch as the given prices of their integers, or numbers expreſſing their degrees of fineneſs, or any other numbers proportional to them. And if any one of the ſimples be of little or no value with reſpect to the reſt, its rate is ſuppoſed to be 0; as water mixt with wine, or alloy with gold or ſilver.

R U L E.

Multiply each quantity by its rate; then divide the ſum of the products, by the ſum of the quantities, (or the whole compoſition) and the quote will be the rate of the mixture (or compound) required.

E X A M P L E S.

1. If 20 buſhels of wheat at 5s. 6d. per buſhel, be mix'd with 30 buſhels of rye at 3s. 6d. a buſhel. At what price per buſhel muſt the mixture be ſold?

buſh.		d.		d.	
20	×	66	=	1320	the value of the wheat.
30	×	42	=	1260	the value of the rye.
—				——	
50				50)2580(51,6d.	=4s. 3d. 2,4 qr. the mean

rate, or value per buſhel of the miſlegin.

The proof is made, by finding the value of the whole mixture at the mean price; which muſt be equal to the total value of the ſeveral ingredients.

Thus, 51,6 d. × 50 = 2580d. as above.

2. A vintner would mix 40 gallons of malaga, at 8 s. the gallon; with 25 gallons of ſherry at 5s. 6d. 16 gallons of canary at 7s. and $14\frac{1}{2}$ gallons of white wine at 4s. 6d. how muſt the mixture be ſold?

Anſw. 6s. 7d. $3\frac{7}{191}$ q.

3. A maltster mixed 24 quarters of high dried malt, at 25s. a quarter, with 30 quarters of brown malt, at 28s. and 46 quarters of pale malt at 30s. What is the value of 1 quarter of this mixture? Answ. 28,2s.

4. A mixture being made of 20 gallons of ale at 10d. a gallon, 18 at 8d. 14 at $6\frac{1}{2}$d. 10 at 5d. and $7\frac{1}{2}$ at 4d. what will it be worth per gallon? Answ. 7d. $1\frac{89}{139}$ q.

5. Having melted together 10 oz. of gold of 22 carracts fine, $30\frac{1}{2}$ oz. of 20 carracts fine, and 16 oz. of 18 carracts fine: I demand the fineness of the composition? Answ. $19\frac{89}{113}$ carracts fine.

6. Suppose I mix 4lb. of silver of 10 oz. fine, with 6lb. 8 oz of 8 oz. fine, and 2lb. 6 oz of alloy: of what fineness is that composition? Answ $7\frac{7}{79}$ oz. fine.

Case II. ALLIGATION ALTERNATE.

Alligation alternate shews how to find the particular quantities concerned in any mixture, when the particular rates of each sort, and also the mean rate, are given. *Alligation alternate* is the *reverse* of *alligation medial*, and may be proved by it.

PREPARATION.

Set down the several rates in order from the greatest to the least, as the letters *a*, *b*, *c*, *d*; and the mean rate (*m*) to the left in its due order.

$$m \begin{array}{l|l} a & p \\ b & q \\ c & r \\ d & s \end{array}$$

Connect or link every two rates together by an arched line or brace, so as one rate may be greater and another less than the mean, till all be coupled.

Where *note*, that one rate may be coupled with several others one by one, as oft as you please.

Take the difference between each rate and the mean rate, and place it down *alternately*, that is, against all its

yoke-

yoke fellows. Do thus with all the rates; and the differences will stand as p, q, r, s. When several differences happen to stand against one rate, add them all together. Then,

1. RULE.

When no quantity is given of any of these sorts, the numbers (or differences) standing against the several rates, are the quantities required.

EXAMPLES.

1. A vintner would mix canary at 2s. 9d a quart, with sherry at 1s. 6d. per quart, to sell the whole at 2s. 3d. per quart. How much of each must he take?

	d.			
m. 27	33	9	quarts of canary	the Answer.
	18	6	sherry	

Which may be proved by alligation medial: Thus,

9 qrts. canary at 33d. amounts to 297d.
6 sherry at 18d. amounts to 108d.

15 total quantity. Total amount. 405 d.

Then $405 \div 15 = 27$d. the mean rate.

Note. The questions of this case are called indeterminate, because each admit of various answers: from an Algebraic process it appears that they will have infinite varieties of answers; nay, if the expression may be allowed, that they will admit of infinite varieties of infinite varieties of answers: In the question above, altho' 9 and 6 do truly answer the conditions, yet any two numbers that bear the same proportion to each other, as 9 to 6, will as truly answer it. Thus,

$$9 : 6 :: \left\{\begin{array}{l} 3 : 2 \\ 6 : 4 \\ 12 : 8 \\ 15 : 10 \end{array}\right\} \text{\&c. ad infinitum.}$$

Here 3 and 2, or 12 and 8, or 15 and 10, are numbers that will as truly anſwer the conditions of the queſtion (as thoſe found by the rule) 9 and 6.

2. How much ſherry at 2s 3d. canary at 2s. 9d. and white port at 1s. 2d. per quart, muſt a vintner mix together to be ſold for 18d. per quart?

18d.	33	4	4 qrts. canary	Anſwer.
	27	4	4 ſherry	
	14	9.15	24 w. port	

Here the difference between 18 and 14 is 4, which I ſet againſt 33, and alſo againſt 27. The difference between 18 and 27 is 9, which I place againſt 14.

The difference between 18 and 33 is 15, which I alſo ſet againſt 14. Then 9 added to 15 is 24. So the differences, to work by, will be 4, 4, 24.

3. How much malaga, at 2s. 11d. per quart, canary at 2s. 6d. ſherry at 1s. 8d. and white port, at 1s. 2d. will compoſe a mixture to be ſold for 2s. 8d. per quart?

	d.			
32d.	35	2 . 12 . 18	32 qrts malaga	Anſwer.
	30	3	3 canary	
	20	3	3 ſherry	
	14	3	3 w. port	

4. How much malaga, at 2s. 11d. per quart, canary at 2s. 6d. ſherry at 1s. 11d. and white port at 1s. 2d muſt be mixed, to be ſold at the mean rate of 20d. per quart?

Anſw. malaga, canary, ſherry, 6 qrts. each, and 28 qrts. of w. port.

5. Suppoſe it be required to make a mixture of ſix ſorts of wine, viz. malaga at 2s. 11d canary at 2s. 6d: gallicia at 2s. ſherry at 1s. 8d. rhenish at 1s. 6d. and white port at 1s. 2d. per quart, how much of each muſt be uſed, that the compoſition may be ſold for 1s. 11d. per quart?

Anſw.

Answ. malaga 9, canary 5, gallicia 3, sherry 1, rhenish 7, white port 12 quarts. And by various coupling of the quantities, various answers may be obtained; or by the rule of proportion.

2. RULE.

ALLIGATION PARTIAL.

In alligation partial, where one of the quantities (to be mixed) is given; say, by the Rule of Three,

As the difference standing against the price of the given quantity:
Is to the given quantity ::
So are the several other differences:
To the respective quantities required.

EXAMPLES.

1. A farmer would mix 15 bushels of *wheat* at 5s. with *rye* at 3s. *barley* at 2s. 9d. and *oats* at 1s. 10d. to be sold at 2s. 10d. per bushel. How much rye, barley, and oats must he take?

			B	
m. 34	wheat	60	12	The differences found by the first rule.
	rye	36	1	
	barley	33	2	
	oats	22	26	

Note. These differences are not the very quantities that the limits of the question require, but corresponding proportionals to them; from which the quantities themselves may be found (according to the above rule) as follows.

As	12 : 15 ::	1 : $1\frac{1}{4}$	bushels of rye		to be mixed with 15 bushels of wheat.
Contr	4 : 5 ::	2 : $2\frac{1}{2}$	barley		
		26 : $32\frac{1}{2}$	oats		

This may be proved by alligation medial.

2. A tobacconist would mix three sorts of tobacco, at 20d. at 14d. and at 10d. per lb. with 8lb. at 24d. per lb.

How much of each fort muft he take, fo as the whole compofition may be fold for 18d. per lb ?

1ft. m. 18	24	8 lb.	Or thus	2d m. 18	24	4 lb.
	20	4			20	8
	14	2			14	6
	10	6			10	2

Now the differences gained by the firft method, viz. 4lb. at 20d. 2lb. at 14d. and 6lb. at 10d. per lb. are fuch as will fufficiently anfwer the propofition, becaufe 8lb found by alligation, is the fame as the quantity given in the queftion ; but the differences gain'd by the fecond method will require the rule of proportion to bring out the true quantities.

			d.		
As 4 : 8 ::	8 : 16lb.	at	20	amount to	320
i. e. 1 : 2 ::	6 : 12		14		168
	2 : 4		10		40
And the given quantity (8lb) at 24d.					192

The total quantity 40)720(18d mean rate.

3. How much filver bullion of 10 dwts. and another fort of 12 dwts. fine, muft be melted down with 20lb. that is 6 dwts. fine, fo that the whole mixture may bear 9 dwts fine ? Anfw. 5lb of each.

4. How much gold of 22, and 24 carracts fine, alfo how much alloy, muft be mixed with 20 ounces of 20 carracts fine, that the whole mafs may be 18 carracts fine ? Anfw. 20 oz. of 22 carracts fine, alfo 20 oz. of 24 carracts fine, and $13\frac{1}{3}$ oz. of alloy.

3. RULE.

ALLIGATION TOTAL.

In alligation total, where the total fum of the quantities (to be mixed) is given; add up all the differences together, then fay by the Rule of Three,

As

As the fum of the differences :
Is to the total quantity given ::
So is every particular difference :
To its refpective quantity.

EXAMPLES.

1. A goldfmith would mix gold of 24 carracts, with fome of 23 carracts, fome of 22 carracts, and fome other of 21 carracts fine, and with a due quantity of alloy; fo that 200 oz. might bear 20 carracts fine: how much of each fort muft he take?

Here the mean rate muft be fubtracted from the feveral given rates, and the feveral differences placed againft 0: and 0 from the mean rate, to be placed againft the feveral given rates; as below.

m.	24	20	20	Here alloy is
20	23	20	20	reckoned 0 car-
	22	20	20	racts.
	21	20	20	
	0	4 · 3 · 2 · 1	10	

The fum of the difference 90

Then 90 : 200 :: 20 : $44\frac{4}{9}$ oz. of the 4 forts of gold.
or 9 : 20 :: 10 : $22\frac{2}{9}$ oz. of alloy.

2. A refiner has fix ingots of filver of different finenefs, viz. of 4, 5, 6, 8, 11 and 12 ounces fine, of which he would mix 500lb. weight, fo as to make it 9 oz. fine: how much of each fort muft he take?

In the folution of this queftion, I fhall fet down four ways of obtaining the differences between the mean rate and the feveral fimple rates, but fhall leave the linking of the feveral fimple rates for the learner's practice.

First way.

m. 9			
	4	2	2
	5	2	2
	6	3	3
	8	3	3
	11	4 . 5	9
	12	1 . 3	4

Second way.

m. 9			
	4	3	3
	5	3	3
	6	2 . 3	5
	8	2 . 3	5
	11	1 . 3	4
	12	3 . 4 . 5 . 1	13

Third way.

m. 9			
	4	2	2
	5	2 . 3	5
	6	2 . 3	5
	8	3	3
	11	3 . 4 . 5	12
	12	1 . 3 . 4	8

Fourth way.

m. 9			
	4	3	3
	5	2 . 3	5
	6	2 . 3	5
	8	2 . 3	5
	11	1 . 3 . 4	8
	12	1 . 3 . 4 . 5	13
			39

The operation by the last way, is thus

$$39 : 500 :: \begin{cases} 3 : 38\frac{18}{39} \text{ oz. of 4 oz. fine.} \\ 5 : 64\frac{4}{39} \text{ of 5, 6, and 8 oz. fine.} \\ 8 : 102\frac{22}{39} \text{ of 11 oz.} \\ 13 : 166\frac{26}{39} \text{ of 12 oz.} \end{cases}$$

S C H O L I U M.

Although the several ways of combining and coupling the rates, as before directed, afford so many different solutions to the question; yet they do not give all the answers the question is capable of. To remedy which, and to make the method more general, you may repeat any two alternate (or corresponding) differences as often as you will; and the like for any other two, &c. This will give a great variety of solutions, from which the easiest and most suitable may be selected. Or rather proceed by the following rule.

4. RULE, Universally.

Having coupled the rates as before directed, instead of any couple of the differences, take any equimultiples thereof;

thereof; that is, multiply them both by any number you will, do the like for any other couple, &c. By this means you'll have a new set of differences to work with.

3. A grocer would mix three sorts of sugar, at 6d. 9d. and 12d. per lb. to have a quantity of 50 lb. to be sold at 8d. per lb. How much of each sort must he take?

Common way.

$$\begin{array}{l} \text{m.} \\ 8 \end{array} \left\{ \begin{array}{r} 12 \\ 9 \\ 6 \end{array} \right. \left| \begin{array}{l} 2 \\ 2 \\ 4.1 \end{array} \right| \begin{array}{l} 2 \\ 2 \\ 5 \end{array} \qquad \begin{array}{l} \text{m.} \\ 8 \end{array} \left\{ \begin{array}{r} 12 \\ 9 \\ 6 \end{array} \right. \left| \begin{array}{l} 2\times3 \\ 2\times4 \\ 4\times3.1\times4 \end{array} \right| \begin{array}{r} 6 \\ 8 \\ 16 \\ \hline 30 \end{array}$$

Here the couple of differences against 12 and 6 being 2 and 4, × them both by 3, and they become 6 and 12. Again the couple of differences against 9 and 6 being 2 and 1, × them both by 4 and they become 8 and 4. Then you'll have 6, 8, and 16 for a new set of differences; whose sum is 30.

$$30 : 50 :: \left\{ \begin{array}{lll} 6 & : 10 \text{ lb.} & \text{at 12d.} \\ 8 & : 13\frac{1}{3} & \text{at 9d.} \\ 16 & : 26\frac{2}{3} & \text{at 6d.} \end{array} \right.$$

The proof 50

4. A wholesale brewer having four sorts of ale, at 12d. 10d. 5d. and 4d. per gallon, desires to have a mixture of 100 gallons made out of them, so as that it may be sold for 7d. per gallon: how much of each sort must he take?

$$\begin{array}{l} \text{m.} \\ 7\text{d} \end{array} \left\{ \begin{array}{r} \text{d.} \\ 12 \\ 10 \\ 4 \\ 5 \end{array} \right. \left| \begin{array}{l} \\ 2\times2=4 \\ 3\times3=9 \\ 3\times3=9 \\ 5\times2=10 \\ \hline \end{array} \right\} \text{A new set of differences.}$$

Sum of the new differences 32

$$\begin{array}{l} \text{As } 32 : 100 :: \\ \text{Or } \ \ 8 : 25 :: \end{array} \left\{ \begin{array}{l} \quad\ \ \text{gal.} \\ 4 : 12\frac{1}{2} \text{ at 12d} \\ 9 : 28\frac{1}{8} \text{ at 10 d, and also at 4d:} \\ 10 : 31\frac{1}{4} \text{ at 5d.} \end{array} \right.$$

5. A

5. A Goldſmith would make a maſs containing 80 oz, and bearing 18 carracts fine, of the following ſorts of gold, viz. that of 23, 22, and 21 carracts fine, and of alloy, of each a ſufficient quantity: how much of each ſort muſt he make uſe of?
Anſwer $21\frac{9}{11}$ oz. of each ſort of gold, and $14\frac{6}{11}$ oz. of alloy.

§ XX. EXCHANGE.

EXCHANGE properly conſiſts in giving monies in one *place* to receive the value at a certain price agreed on in another, and is much like the bartering of commodities.

There are five things to be defined before we come to the practice of Exchange; and theſe are, 1ſt. The real money of every *country*. 2d. The imaginary money of it. 3d. The *par* of *exchange*. 4th. The *courſe* or current rate of exchange. and 5th. The *ageo*.

1ſt. The real money of any *country*, is a quantity of ſome metal coined by the authority of that ſtate and current at a certain rate; as a *guinea*, a *crown*, a *ſhilling*, &c. in *England*. 2d. The imaginary money is a *denomination* which has no real *ſpecies* or *coin* to anſwer it; as a *pound*, a *mark*, an *angel*, a *penny*, &c. in *England*.

3d. The par of exchange is, when any two *coins* of different countries do contain the ſame quantity of pure gold or ſilver, although of a different weight. Hence it has been found that the ecu or French crown is, to the Engliſh crown, as 9 to 10, and therefore the *par* of the French crown is 4s. 6d. in ſterling.

4th. The courſe of exchange does not always run at *par*. but often riſes above and ſettles below it, according to the plenty or ſcarcity of money or bills, and the preſent rate at any time, is called the *courſe* of *exchange*.

5th.

5th. The ageo is, the difference of the value of current *money*, and *bank money*: for the bank money in foreign places is finer than the current money in them.

Note. It is by comparing the bank money with sterling that the par is ascertained. Also the exchange is always supposed to be made in bank money; and if there is a necessity for taking currency in case of a defect of the bank to answer the bills, the more current money must be received, and that in proportion to the ageo.

It will not be consistent with my intended brevity, to treat of the exchanges of every country, for that would make a volume of itself, neither is it any wise necessary: I shall therefore confine myself to the exchanges made between England and the chief countries of commerce in Europe. The course of exchange of any country being given, the questions appertaining thereto, may be solved by the Rule *of* Three, *or* Practice; *and since these rules have already been sufficiently treated of, I think it unnecessary to express any operations at large; but only to shew the proportion by the stating of the question in each case, and subjoin the answer.*

1. *With* F R A N C E.

In France accompts are kept in livres, sols, and deniers; but they exchange with other kingdoms, &c. upon the crown (or ecu) of 60 sols, or 3 livres tournois.

Note 1. The livres are imaginary money, but the crowns, sols, deniers are real: and the term tournois is only a note of distinction of the French money, as sterling is of the English.

2. In the French exchange you must distinguish between exchange money and book money. The exchange money is called money d'or, and is expressed in crowns, sols, and deniers d'or, and these are summed up like l. s. d.; for 12 deniers make a sol of the crown, and 20 sols of the crown, make a crown.

The book money is called tournois, and is expressed in livres, sols, and deniers tournois; and these are summed up like

like l. s d. ; for 12 deniers make a sol, of the livre, and 20 sols of the livre make a livre. So that crowns, sols, and deniers d'or (or exchange money) multiplied by 3 through out, give livres, sols and deniers, turnois, or book money. On the contray, book money divided by three throughout, gives exchange money.

E X A M P L E S.

1. How many livres &c. will 345l. 6s 7d. sterling, amount to, exchange at $31\frac{3}{4}$ d. per ecu ?
As $31\frac{3}{4}$: 1 ecu :: 345 l 6s. 7d. : 2610 ecus 7 sols 3 den' d'or, near. Which ×d by 3, gives 7831 liv. 1s. 9 den. tournois.

2. To how much sterling will 7831 livres, 1 sol 9 den amount to at $31\frac{3}{4}$d. per ecu ?
As 3 liv. : $31\frac{3}{4}$d, :: 7831 livres 1 sol 9 den : 345l. 6s. 7d. sterling. the Answer.

3. How much sterling must be paid in London, to receive in Paris 14875 livres tournois, at 23 livres per pound sterling ?

As 23 liv. : 1l. :: 14875 liv. : 646 6l. 14s. 9 $\frac{9}{23}$d. sterling. the Answer.

4. London remits to Paris 646 l. 14s. 3 $\frac{9}{23}$d. what is the value in French livres, at 23 livres per pound sterling ?

As 1l. : 23 liv. :: 646l. 14s. $9\frac{9}{23}$d. : 14875 liv. the Answ.

5 Remitted to Paris 168l. 18s. sterling at $23\frac{7}{8}$d per crown ; how many crowns, sols, and deniers d'or does it come to ?

Answer 1697 cr. 16 sols. $10\frac{18}{191}$ den.

6. How much sterling for 2064 crowns d'or, exchange at 56d. each ?

Answer 467 l 12s.

7: How

7 How many ecus, for 345l. 6s. 7d. sterling, at 32d. per ecu? Answ. 2589 ecus. 19 sols $4\frac{1}{2}$ den.

8. Marseilles remits to London 47329 livres, 10 sols. 6d. exchange at 27 liv. 10s. per pound sterling; what sterling money will the said remittance amount to?

Answ. 1721l. 1s. $5\frac{3}{4}$d.

2. *With* S P A I N.

In Spain they keep their accompts in piastres, rials, and maravedies; reckoning 34 maravedies to a rial, and 8 rials to a piastre, *peso*, or piece of *eight*, old money, by which they exchange, whose par is 4s. 6d. The piastre is imaginary, but the rials and maravedies are real. London exchanges with Spain in pesos (or piece of eight) old money, of 8 rials, or 272 maravedies; and in new money, of 10 rials.

E X A M P L E S.

1. If London draws or remits on Spain 555l. 5s. 5d. at $43\frac{3}{4}$d. sterling per peso, how much must be paid, or received there?

	d.		peso		l.	s.	d.		pesos	marav.	
As	$43\frac{3}{4}$	:	1	::	555	5	5	:	3046	16	the Answ.

2. Cadiz remits to London 7075 pesos, 3 rials, 10 marav. at $52\frac{1}{4}$d. per peso; what will this remittance amount to in London?

	peso		d.		pesos	r.	m.		l.	s.	d.	
As	1	:	$52\frac{1}{4}$	::	7075	3	10	:	1540	7	$6\frac{1}{4}$	the Answ.

3. How many piastres, &c. shall I receive at Bilboa for 612l. 10s. 4d. sterling exchange at $52\frac{7}{8}$d. sterling per piastre? Answ. 2780 piastres, 1 rial, $25\frac{67}{423}$ marav.

4. A merchant at Cadiz remits to London 2547 pesos, at 56d. sterling each; how much sterling is the sum?

Answ. 594l. 6s.

5. How many pieces of eight, &c. will 2500 l. amount to at $57\frac{1}{2}$d. sterl. per piece of eight? Answ. 10434 pesos, 6 rials, $8\frac{20}{23}$ marav.

3. *With* I T A L Y.

At Leghorn and Genoa they keep their accompts in livres, sols, and deniers, as in France, but they exchange by the piastre or dollar, as in Spain, which at Genoa is accounted 5 livres, and at Leghorn 6. Also at *Venice*, accounts are kept by some in the same manner. and by others in ducats and gross. Also at Florence by ducatoons. A ducatoon is equal to 4s. 4d. at *par*.

Note, 6 soldi = 1 gross, and 24 gross = 1 ducat.

E X A M P L E S.

1. Suppose there be owing to me by a correspondent at Genoa 3675 livres, 15 sols, 10 deniers; how much sterling does it amount to, exchange at 50d. sterling per piastre *dollar* or piece of eight?

First 3675 liv. 15 sols, 10 den, ÷ 5 = 735 dollars, 3 sols, 2 den.

	dol.	d.		dol.	sol.	den.		l.	s.	d.	
As	1 :	50	::	735	3	2	:	153	3	$1\frac{1}{2}$	the Answ.

2. A merchant remitted 153 l. 3s. $1\frac{1}{2}$d. sterling to Genoa; how many dollars must he receive there, the exchange being 50d sterling per dollar?

First 153l. 3s. $1\frac{1}{2}$d. = $36757\frac{11}{12}$d. and 36757 × 12 + 11 = 441095 twelfths of pence;

Also 50 × 12 = 600 twelfths of pence; then

			dol.	sols.	den.	
As 600 :	1 ::	441095 :	735	3	2	the Answ.

3. Genoa draws upon London for 8504 liv. 17 sols 1 den: how much sterling will satisfy the draught, exchange at $53\frac{3}{8}$d. sterl. per dollar?

Answ. 378l, 5s, 9d. $1\frac{4}{15}$q.

5. How

4. How many livres, &c. muſt be received at Leghorn for 705l 16s. 4d. ſterling, exchange at $51\frac{1}{2}$d. ſterling per piaſtre?

Anſw. 19735 liv. 9 ſols 1 $\frac{53}{103}$ den.

5 A merchant ſold goods to Florence, for 250 ducatoons, at 54d. ſterl each; what is the value in pounds ſterling? Anſw. 56 l. 5s.

At Venice ſome keep their accompts in ducats of the bank, and others in ducats current, but the Republic keeps them in ducats groſs. The ducat of the bank (which is imaginary) is worth 24 groſs; and 100 ducats banco are worth 120 ducats current; ſo that 5 ducats banco are equal to 6 ducats current; and the ageo conſtantly 20 per cent.

To reduce bank money into current.

RULE.

Divide the given ſum by 5, the quote is the ageo which added to the bank money gives the current.

EXAMPLE.

Reduce 13650 ducats banco into current.

$13650 \div 5 = 2730$ ageo, and $13650 + 2730 = 16380$ ducats current.

To reduce current money into bank money.

RULE.

Divide the current money by 6 and the quote will be the ageo, which ſubtract from the current money, and the remainder will be the bank money.

EXAMPLE.

Reduce 16380 ducats current into ducats banco.

$16380 \div 6 = 2730$ ageo, and $16380 - 2730 = 13650$ ducats banco.

E X A M P L E S.

1. London remits to Venice 764l. fterl. the exchange 52d. fterl. per ducat banco, how many ducats banco will this remittance come to? First 764l = 183360d.

	d.		duc.		d.		ducats	grofs
As	53	:	1	::	183360	:	3526	$3\frac{2}{13}$d. banco, and 4231

duc. $9\frac{3}{13}$ grofs current.

2. Suppofe my correfpondent at Venice owes me 3526 duc. $3\frac{2}{13}$ grofs banco, the exchange at 52d. fterl. per ducat, what fterling money will this remittance produce at London?

	duc.		d.		ducats	grofs		l.
As	1	:	52	::	3526	$3\frac{2}{13}$	:	764 fterling.

3. How many Venice ducats at 4s. 5d. fterl. each, for 60l. 14s. 7d. fterling? Anfw. 275.

4. How many pounds fterling, for 275 Venice ducats, at 4s. 5d. fterl. per ducat? Anfw. 60l. 14s. 7d.

4. *With* P O R T U G A L.

The Portugefe keep their books and accompts in milrees, which are real, feparating the thoufands from the inferior fums called rees: therefore 1000 rees = 1 milree, whofe *par* is about 6s. $8\frac{1}{2}$d. or 6s. 9d. fterling.

E X A M P L E S.

1. To how many milrees will 199l. 15s. amount, exchange at 5s. 8d. per milree?

As 5s. 8d. : 1 mil. :: 199l. 15s. : 705 milrees.

2. To how much fterling will 705 milrees amount, exchange at 5s. 8d. per milree?

As 1 mil. : 5s. 8d. :: 705 mil. : 199l. 15s. fterling.

3 Lifbon

3. Liſbon remits to London 4617 milr, 216 rees, exchange at $65\frac{3}{4}$d. ſterling per milree: what muſt be received at London?

	rees	d	milr.	rees	l.	s.	d.
As	1000 :	$65\frac{3}{4}$ ::	4617	,216 :	1264	18	5 $3\frac{101}{125}$q.

ſterling.

4. To how many milrees at $65\frac{3}{4}$d. each, will 1264l. 18s. 6d. amount? Anſw. 4617 milr. $216\frac{192}{263}$ rees.

5. *With* HOLLAND, FLANDERS, *and* GERMANY.

I take theſe countries together, becauſe they keep their accompts, and make exchange with London in the ſeme manner.

In *Amſterdam*, *Antwerp*, Bruſſels, *Rotterdam*, and *Hamburgh*, ſome keep their accompts in *pounds ſhillings* and pence, (or groats) as in *England*; others in guilders, ſtivers and pennings;

16 pennings = 1 ſtiver, or 6 groats
20 ſtivers = 1 guilder, or florin.
and 50 ſtivers = 1 rixdollar.

The money of *Holland* and *Flanders* is diſtinguiſhed by the name *flemiſh*, and they exchange with London by the pound ſterl = 33s. 4d. flemiſh at *par*.

1 penny *flemiſh*, is $\frac{1}{2}$ ſtiver, or 8 pennings.
1 ſhilling *flemiſh*, is 12 pence flemiſh, or 6 ſtivers.
1 pound *flemiſh*, is 20 ſhillings flemiſh or 6 guilders.

Holland. The bank of *Amſterdam* was eſtabliſhed by an ordinance of the States in 1609, in which it was decreed, that all bills of exchange ſhould be paid for in bank money, and for merchandize in current or groſs money, unleſs the ſum ſhould be under 360 florins.

Note. The ageo, or difference between the bank money and current money, is 3, 4, 5 or 6 per cent. ſometimes more, and ſometimes leſs.

To reduce current money into bank money.

RULE.

As 100 current money with the ageo added to it, is to 100 bank money, so is the current money given, to the bank money sought.

EXAMPLE.

Change 8162 guilders 12 stivers current money into bank money, the ageo being $3\frac{1}{4}$ per cent?

	guil. cur.		guil. b.		guil.	stiv.		guil.	stiv.	penn.
As	$103\frac{1}{4}$	:	100	::	8162	12	:	7905	13	5 +

banco.

Answ. 7905 guil. 13 stiv. 5 pennings banco, with a small remainder.

To change bank money into current money.

RULE.

As 100 guilders bank, is to 100 with the ageo added; so is the bank given, to the current required.

EXAMPLE.

Suppose the ageo $5\frac{1}{8}$ per cent. how much current money will 4954 guil. 9 st. 8 penn. bank amount to?

	bank		current		g. bank	st.	p.		guil.	st.	p.
As	100	:	$105\frac{1}{8}$	::	4954	9	8	:	5208	7	$13\frac{12}{100}$

current.

To change sterling money into flemish.

RULE.

As 1l. sterl. is to the given rate; so is the sterl. given to the flemish sought.

To change flemish into sterling.

RULE.

As the given rate, is to 1l. sterling, so is the flemish sum given, to the sterling required.

EXAMPLES.

1. A merchant at *London* remits to Rotterdam 3 bills of exchange, 200l. each, one at 33s. 6d. another at 33s. 5d. and the other at 33s. $5\frac{1}{2}$d. flemish per pound sterling. How

How many guilders, &c. bank money does each of these bills come to? And how much current money in them all, ageo at $4\frac{7}{8}$ per cent?

	l.		s. flem.		l.		l. fl.	s.	d.		guild.	st.	p.
As	1	:	$33\frac{1}{2}$	::	200	:	335	0	0	=	2010	0	0
	1	:	$33\frac{5}{12}$	::	200	:	334	3	4	=	2005	0	0
	1	:	$33\frac{11}{24}$	::	200	:	334	11	8	=	2007	10	0
										Banco	6022	10	0
at $4\frac{7}{8}$ per cent. the ageo of the whole is											293	11	15
										current money	6316	1	15

2. Suppose there be due to me at Rotterdam 6316 guil. 1 st. 15 pennings, current money, and the ageo be $4\frac{7}{8}$ per cent. how much sterling money does it come to, exchange at 33s. $5\frac{1}{2}$d. flemish per pound sterling?

First reduce the current money into banco, thus

As $104\frac{7}{8}$ cur. : 100 bank :: 6316 cur. 1 st. 15 p. : 6022 guil. 10 stiv. banco.

Next reduce the banco money into flemish pounds, thus 6022 guil. 10 st. ÷ 6 = 1003l. 15s. flemish; which reduce into sterling, (exchange being 33s. $5\frac{1}{2}$ per l. sterl.)

thus as 33s. $5\frac{1}{2}$d. : 1l. :: 1003l. 15s. : 600l. sterling, the Answ.

3 To how much flemish will 754l. 10s. sterling amount, exchange at 33s. 6d. flem. per l. sterling? Answ. 1263l. 15s. 9d. flem.

4. To how much sterling will 1263l. 15s. 9d. flemish amount, exchange at 33s. 6d. per l. sterling? Answ. 754l. 10s.

5. How many guilders, &c. may I have for 173l. 14s. 2d. sterling, exchange at 35s. $3\frac{1}{2}$d. flem. per l. sterling? Answ. 1839 guil. 2 stiv. $11\frac{5}{6}$ pennings.

6. How

6. How much flemiſh currency will 290l. 11s. 10d. ſterling amount to, exchange at 33s. 10d. flem. per l. ſterling, and the ageo $4\frac{1}{2}$ per cent ?

Anſw. 513l. 14s. 1d. $1\frac{917}{3000}$q.

7. How much ſterling will 8000 guilders amount to, at 11 guil. 3 ſtiv. 10 pen per l. ſterling ?

Anſw. 715l. 9s. $8\frac{76}{1789}$d.

8. To how much ſterling will 6712 guil. 10 ſtiv. current money amount, at 34s. 10d per l. ſterl. and ageo at $4\frac{3}{4}$ per cent ? Anſw. 596l. 2s 0d. 3q. +

6. *With* IRELAND, AMERICA, *and the* WEST-INDIES.

Accompts are kept in Ireland, America, and the Weſt-Indies, in pounds, ſhillings, and pence, as in England: and exchange per cent. ſterl. 100l. ſterling = $108\frac{1}{3}$ l. Iriſh at par, or 1l. ſterl. = 1l. 1s. 8d. Iriſh: alſo 5l. ſterl. is accounted worth 7l of the currency of the Weſt-Indies, becauſe of the great plenty of the foreign coins there; but on the continent of America there is very little coin of any ſort circulating.

E X A M P L E S.

1. How much in Dublin for 123l. 4s. 5d. in London, exchange at 107 per cent?

ſterl.	Iriſh	ſterl.	
As 100 :	107 l. ::	123l. 4s. 5d. :	131l. 16s. $11\frac{11}{100}$d.

the Anſw.

2. Dublin remits to London 345l. at 108 per cent, how much muſt be received at London ?

Iriſh	l. En.	Iriſh	l. s. d.	
As 108 :	100 ::	345 :	319 8 10 Eng.	Anſw.

3. Lon-

3. London remits to Dublin 123l. 4s. 5d. at $107\frac{1}{8}$ per cent. what must be received there? Answ. 132l. Irish.

4. London remits to Barbadoes 789l. 10s. 11d. what currency must be received for it, exchange at $107\frac{1}{2}$ per cent?

Answ. 848l. 15s. 2d. $3\frac{3}{10}$q. Barbado. cur.

5. How much English for 436l. 8s. $10\frac{1}{4}$d. Barbadoes currency, at 131 per cent?

Answ. 333l. 3s. 2d. $3\frac{115}{131}$q.

6. London remits to Jamaica for 678l. 9s. 10d. sterling: what must be received for it, exchange at 135l. per cent?

Answ. 9?5l. 19s. 3d. $1\frac{1}{5}$q.

7. Jamaica remits to London for 135l. 7s. $9\frac{1}{4}$d. currency: what must be received for it, exchange at $110\frac{1}{2}$ per cent?

Answ. 122l. 10s. 5d. $2\frac{138}{221}$q. Eng.

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§ XXI. CONJOINED PROPORTION.

THIS chiefly consists in comparing several different sorts of things together, as to their value; and the following rule teaches to find, how many of one sort is equal to a given number of another sort.

RULE.

Place the terms in two perpendicular columns, so that there may not be found in either column, two terms of one kind. Then the numbers in the less column must be xed for a divisor; and the numbers in the greater column, where the odd term i, for a dividend. The quotient is the Answer.

Note: To abridge the work, throw out any numbers that you can find in both columns.

EXAM-

EXAMPLES.

1. If 100lb. at London be equal to 92lb. at Paris, and 46lb. at Paris be equal to 44lb. at Rouen, and 88lb. at Rouen be equal to 91 at Rochelle, how many pounds at Rochelle, for 50lb. at London?

100lb. Lond.	=	92lb. Paris
46lb. Paris	=	44lb. Rouen
88lb. Rouen	=	91lb. Rochelle
How many lbs Rochelle	=	50lb. London

thus.

$$\frac{92\times44\times91\times50}{100\times46\times88}=\frac{18418400}{404800}=45\tfrac{1}{2}\text{ lb. the Answ.}$$

2. If 3 pair of gloves be worth 2 yards of lace, 3 yards of lace worth 7 dozen of buttons, 6 dozen of buttons equal to 2 penknives, and 21 penknives to 18 pair of buckles; how many pair of gloves is equal to 28 pair of buckles?

3 gloves = 2 lace,
3 lace = 7 buttons,
6 buttons = 2 knives,
21 knives = 18 buckles,
28 buckles ——
—— 504
31752

504)31752(63.

Or thus.

$$\frac{3\times3\times6\times21\times28}{2\times7\times2\times18}=\frac{3\times21\times28}{2\times7\times2}=3\times21=63.$$

3. Exchange between Dublin and London being 107l. per cent. between London and Amsterdam 36 sch. 1 den. gros per l. sterl.; what is the amount at Amsterdam of 234 Irish remitted from Dublin, by the way of London, allowing London $\frac{1}{2}$ per cent. or a 200th part, for commission and charges?

Note 1. When commission is allowed in one or several places, it is best to make calculations for these commissions, on the dividend, before the last division is made. If the affair

under

under confideration be a circulation or chain of remittances, fubtract the commiffion found : again on the remainder calcu-late the fecond commiffion, which fubtract from the faid re-mainder ; and fo continue to calculate for as many commiffions as are to be allowed.

But in a circulation of draughts, the commiffion found muft be added, and the new commiffions calculated on the totals laft found.

2. The rate of commiffion is different in different places, in fome it is $\frac{1}{4}$ per cent. that is to fay, the 400th part, in fome one third per cent or the 300th part ; in others $\frac{1}{2}$ per cent, or the 200th part.

If 107l. Irifh	=	100 l. Eng.
and 1l. Eng.	=	433 den gros.
how much at Amft.	=	234 l. Irifh ?

Divifor 107. From 10132200 dividend = x.
deduct a 200th part of x. = 50661 Lond. commiffion.

107)10081539(94220 den. gros. near.

4. Exchange between London and Antwerp being 36 fch 8d. gros per l. fterl. ; betwixt Antwerp and Paris 57 den gros per ecu of 3 liv. tourn ; betwixt Paris and Oporto 450 rees per ecu tourn ; how much in Oporto for 456 l. fterl. remitted from London thro' Antwerp and Paris, allowing Antwerp and Paris each $\frac{1}{2}$ per cent. for commiffion and charge ?

1l. fterl.	=	440 den. gros.
57 den. gros	=	1 ecu tourn.
1 ecu tourn	=	450 rees.
How many rees	=	456l. fterl.

$$\frac{440 \times 450 \times 456}{57} = \frac{440 \times 450 \times 8}{1} =$$

1584000 rees.
a 200th part is 7920 for Antw. commiffion.

1576080
a 200th part is 7880 for Paris commiffion.

1568,200 amount in Oporto.
mil. rees.

5. A remittance circulating entirely; that is to say, ending where it began.

London remits 500l. to Cadiz, at 40d. sterling per piece of 8, to be remitted thence to Leghorn, at 125 pieces of 8 for 100 Leghorn dollars; thence to Venice at 90 Leghorn dollars for 100 ducats banco; thence to Lyons at 66 duc. banco for 100 ecus tourn; thence to London at 33d. sterl. per ecu tourn; how much will this remittance produce in London, allowing to each of the other four place, ½ per cent. for commission?

40d. sterl	=	1 pce of 8.
125 pieces of 8	=	100 Leghorn d.
90 Leg. dol.	=	100 duc. bo.
66 duc. bo	=	100 ecus tourn,
1 ecu tourn	=	33d sterl.
how much is	=	500l. sterling?

From these two columns, the divisor will be found to be 9, and the dividend 5000, when each are properly contracted.

From the dividend 5000 = x
Subtr. the 200th part of x = 25 Cadiz Comm.

From 4975 = a
Subtr. the 200th part of a = 24,875 Leg. Comm.

From 4950,125 = b
Subtr. the 200th part of b = 24,750 Ven. Comm.

From 4925,375 = c.
Subtr. the 200th part of c. = 24,627 Lyons Comm.

4900,748 = d.

d ÷ 9 = 544.527 l. in London; or 544 l. 10s. 6½d. sterl. the neat produce.

§XXII.

§ XXII. INVOLUTION.

INVOLUTION is the raising of powers from any proposed number, called the root or first power, and is nothing but a continual multiplication of the given quantity or root into itself a certain number of times.

Thus: 4 is the root, or first power of 4.
$4 \times 4 = 16$ is the square, or second power of 4.
$4 \times 4 \times 4 = 64$ is the cube, or third power of 4.
$4 \times 4 \times 4 \times 4 = 256$ is the biquadrate, or 4th power of 4.

SCHOLIUM.

Some of these powers have borrowed their denomination from local extension For a line having but one dimension, viz. length, ×ed into itself, produces a square-plane. And that a square having two dimensions, viz. length and breadth, ×ed into itself, produces a cubed-solid. This cube has three dimensions, viz. length, breadth, and thickness; but the nature and property of space admits of no other extension.

Whence it follows, that the root or first power being taken as a side, the second power will be a square, the third a cube.

A TABLE *of the first three powers of the nine digits.*

Roots	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	[illegible]
Cubes	1	8	27	64	[illegible]	216	[illegible]	[illegible]	[illegible]

Note 1. The number which exceeds the multiplications by one is called the index, or exponent of the power: so the index of the first power is 1, that of the second power is 2, that of the third is 3, &c.

2. Powers are commonly denoted by writing their indices above the first power: so the second power of 4 may be denoted thus 4^2, the third power thus 4^3 the fourth power thus 4^4, &c. and if the given number consists

consists of several figures, a crooked line is sometimes drawn between them and the index: thus $\overline{408}\rceil^5$, denotes the fifth power of 408.

EXAMPLES.

1. What is the 5th power of 8? Answ. 32768.

For $8 \times 8 = 64$ the 2d power, and $64 \times 8 = 512$ the 3d power, and $512 \times 8 = 4096$ the 4th power, and $4096 \times 8 = 32768$ the 5th power.

2. What is the biquadrate of 27? Answ. 531441.

3. What is the sixth power of 503?
Answ. 16196005304479729.

Note, 1. To involve a fraction, raise each of its terms to the power required.—And if a mixt number be proposed, either reduce it to an improper fraction, or reduce the vulgar fraction to a decimal, and proceed as above.

2. The sum of any two or more indices, or exponents of powers, will be the index of a power resulting from the multiplication of the several powers into each other.

As $5^2 \times 5^3 = 5^5 = 3125$. Also $3^1 \times 3^2 \times 3^4 \times 3^5 = 3^{12} = 531441$. and $3^{12} \times 3^{12} = 3^{24}$: by this method the raising of powers may be a little shortened, for if any power be multiplied into itself, a power is produced whose index is double to that which was multiplied: for the 2d power multiplied into itself gives the fourth power, and the 4th multiplied into itself gives the 8th power, and this into itself will give the 16th power, &c.

EXAMPLES.

1. What is the third power of $\frac{7}{8}$? Answ. $\frac{343}{512}$

For $\frac{7 \times 7 \times 7}{8 \times 8 \times 8} = \frac{343}{512}$ the Answ.

2. What is the 5th power of $\frac{5}{7}$? Answ. $\frac{3125}{16807}$

3. What

3. What is the 2d power of $4\frac{4}{25}$? Answ. 17,3056.

4. What is the sixth power of ,09?

First ,09×,09×,09=,000729 the third power, or cube, and ,000729,×,000729=,000000531441 the Answ.

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§ XXIII. EVOLUTION.

ENVOLUTION, or the extraction of roots, being directly contrary to Involution, or raising of powers, is performed by converse operations, in order to find the roots from the given powers, either accurately or in decimals till the error be insignificant.

NOTATION *of* ROOTS.

Roots are mostly represented by writing $\sqrt{\ }$ before the power with the index of the root against it; so the third root of 60 is $^{3}\sqrt{60}$, and the second root of it is $\sqrt{60}$, the index 2, being omitted; which index is always understood when a root is named or wrote without one. But if the power is expressed by several numbers with the sign +, or —, &c. between them, then a line is drawn from the top of the radical sign, over all the parts of it: so the third root of 60+14 is $^{3}\sqrt{\overline{60+14}}$, or of 60—14 is $\sqrt{\overline{60-14}}$. And sometimes roots are expressed like powers by a crooked line drawn over the number, with the reciprocal of the index of the root above it: so the square root of 60 is $\overline{60}\rceil^{\frac{1}{2}}$, the cube or third root of it is $\overline{60}\rceil^{\frac{1}{3}}$: Also the third root of 60+14—6 is $\overline{60+14-6}\rceil^{\frac{1}{3}}$; the fourth root of it is $\overline{60\times14-6}\rceil^{\frac{1}{4}}$, and the *m* root of $\overline{60\times14-6\times2}\rceil^{3}$ is $\overline{60\times14-6\times2}\rceil^{\frac{3}{m}}$.

RULE *for extracting the square root of* integers *or* decimals, or both mixt together.

1. Divide the given number into periods of two figures each: Thus begin at the units place, and put a point over

over it, alſo put a point over the top of every other figure, to the left hand in whole numbers, and to the right in decimals: ſo will the given number be pointed into ſo many periods, as there will be figures in the required root.

2. Find the greateſt ſquare that is contained in the firſt period, towards the left hand. Set the root in the quotient and ſubtract the ſquare from the figures of that period.

3. To the remainder bring down the two figures under the next point, for a reſolvend. This is always to be repeated.

4. Double the quotient for a diviſor, and ſee how oft it is contained in the reſolvend (except the laſt figure); and ſet the anſwer in the quotient, and alſo after the diviſor. This muſt always be repeated; for a new diviſor muſt be found for every figure.

5. Then multiply the whole diviſor by that quotient figure, and ſubtract the product from the whole reſolvend; but if that product be greater, a leſs figure muſt be placed in the quotient. Thus go on till all the figures or periods are brought down, but if there be a remainder bring down cyphers two at a time, and ſo carry the root into decimals at pleaſure.

Note, Inſtead of doubling the quotient every time for a diviſor, you may always add the laſt quotient figure to the laſt diviſor, for a new diviſor, and proceed as before.

E X A M P L E S.

1. Extract the ſquare root of 32176552,863844 and the operation will be as follows.

root

```
 .  .  .  .  .  .  .  .      root
32176552,863844(5672,438
25
--------------
    106 | 717
     +6 | 636
-------------------
   1127 | 8165
     +7 | 7889
-------------------
  11342 | 27652
     +2 | 22684
-------------------
 113444 | 496886
     +4 | 453776
-------------------
1134483 | 4311038
     +3 | 3403449
-------------------
11344868 | 90758944
         | 90758944
------------------------------
```

The proof is, to multiply the root by itself, and add the remainder; which must be equal to the number to be extracted, if the work be right.

Thus 5672,438 × 5672,438 = 32176552,863844 the given number.

Note, In large operations the work may be much abbreviated thus, when one more than half the figures of the root are found, by the common method, all the rest may be found as truly, by dividing the last dividend by the last divisor, annexing a cypher to every dividual, as in division of decimals; or rather, without annexing cyphers, by omitting continually the right hand figure of the divisor, after the same manner of the third contraction in division of decimals, in page 170.

Ex. 2. Extract the ſquare root of 0,002468.

```
·   · · ·
0 002468(0.049678969394 the root near.
     16
  ---------
  89) 868
  +9  801
  ---------
 986) 6700
  +6  5916
  -----------
9927)  78400
  +7   69489
  -----------
99348) 891100
   +8  794784
  --------------
993569)  9631600
         8942121
         --------
          689479
          596141
          -------
           93338
           89420
           -----
            3918
            2981
            ----
             937
             894
             ---
              43
              40
              --
               3
```

Here having by the rule found the firſt ſix figures, I find the five laſt by plain diviſion : only it may here be obſerved, that the laſt figure found by diviſion is moſtly a ſmall matter too large; as 4, the laſt figure in this Example by the rule wou'd but come out 3.

3. What is the ſquare root of 3272869681?

Anſw. 57209.

4. What

4. What is the ſquare root of 787833.76 ?
Anſw. 7,6

5. What is the ſquare root of ,001234 ?
Anſw .0351283361 nearly

6. Extract the ſquare root of 3, to thirteen places ?
Facit. 1.732050807568

To extract the ſquare root of a Vulgar Fraction.

RULE.

1. Reduce the Fraction to its loweſt terms.

2d. When the terms of the given Fraction are both of them perfect ſquare numbers, extract the root out of the Numerator and Denominator, for the reſpective terms of the root required.

3. When either of the terms of a propoſed Fraction hath not a perfect ſquare root, then reduce it to decimals, and proceed as before.

4. Mixt numbers may be either reduced to improper Fractions, and extracted as above, or you may reduce the Vulgar Fraction to a Decimal, and annex it to the whole number, then extract the root of the whole.

EXAMPLES.

1. What is the ſquare root of $\frac{36}{49}$? Anſw, $\frac{6}{7}$.

2. What is the ſquare root of $\frac{384}{864}$? Anſw. $\frac{2}{3}$

For $\frac{384}{804}$ in its loweſt terms is $\frac{4}{9}$ whoſe ſquare root is $\frac{2}{3}$

3. What is the ſquare root of $6225\frac{21}{100}$? Anſw. 78.9

4. What is the ſquare root of $85\frac{3}{4}$? Anſw. 9.26 &c.

Ap-

Application of the Square root.

To find a mean proportional between any two given numbers.

RULE. Multiply the two numbers together, and extract the ſquare-root of the product.

Ex 1. What is the mean proportional between 4 and 9?
$\sqrt{9 \times 4} = 6$ the Anſw.

2. What is the mean proportional between 49 and 64? Anſw. 56

3. What is the diameter of a circle equal in area to an ellipſis whoſe longeſt diameter is 768 and ſhorteſt 48? Anſw. 192.

To find a ſide of the ſquare equal in area to any given ſuperficies.

RULE. Extract the ſquare-root of the given area, for the ſide of the ſquare ſought.

Ex. 1. If the area of a circle be 1324,96 what is the ſide of an equal ſquare. Anſw. 36 4.

2 An army of 32400 men is to be formed into a ſquare battle; how many men muſt be in rank, and how many in file? Anſw. 180.

3. Suppoſe 12544 ſoldiers be diſpoſed into the form of an oblong, ſo that the number of men in rank, may be 4 times as many as thoſe in file; how many muſt be placed in rank, and how many in file?

Firſt $12544 \div 4 = 3136$, whoſe ſquare root is 56 in file; and $12544 \div 56 = 224$ in rank.

4. A certain ſociety collect among themſelves a ſum amounting to 15l. 5s. $0\frac{1}{4}$d, every one contributing as many farthings as there were members in the whole Society: I demand the number of members. Anſw. 121 members.

Two

Two ſides of a right angled triangle being given to find the other ſide.

CASE 1. The two perpendicular ſides, or legs being given to find the other ſide, or hypothenuſe.

RULE. Square each ſide, add the two ſquares together, and the ſquare root of this ſum gives the hypothenuſe required.

Ex. Suppoſe a garriſon's wall the height of which is 18 feet, with a moat ſurrounding it, whoſe breadth is 24 feet, what muſt be the length of a ſcaling ladder, to reach from the outſide of the moat, to the top of the wall?

Note. The moat and wall are the two perpendicular legs of a right angled triangle, and the ladder the hypothenuſe therefore the ſquare of 18 added to the ſquare of 24 equal 900, whoſe ſquare root is 30 feet, the ladder's length.

CASE 2. The hypothenuſe and either leg being given to find the other leg.

RULE. From the ſquare of the hypothenuſe ſubtract the ſquare of the given leg, and the ſquare root of the remainder will be the leg required.

Ex. A ladder whoſe length is 30 feet, being fixed on the brink of a moat, will reach the top of a wall on the oppoſite ſide whoſe height is 18 feet: from hence is required the breath of the moat.

From the ſquare of 30 take the ſquare of 18, and the ſquare root of the remainder is 24, the breadth of the moat.

If the length of the ladder and breadth of the moat be given to find the height of the wall, then from the ſquare of 30 take the ſquare of 24, and the ſquare root of the remained is 18.

A general rule for extracting the roots of all powers.

1. Prepare the given number for extraction, by pointing it into periods of two, three, four, five or ſix, &c. figures

gures each, according as the index of the power directs: beginning at the units place and from thence proceeding to the left hand in whole numbers, and to the right in Decimals. By this means the number will be divided into so many periods as there are figures in the required root.

2. Enquire which is the greatest cube, biquadrate, or 5th power, &c. in the first period, and the root of that power will give the first figure of the required root. Subtract the greatest cube, biquadrate, or 5th power, &c from the first period, and to the remainder annex the first figure of your second period, which will give your dividend.

3. Involve the root, or quotient figure, already found to a power less by unity than that whose root is sought, × it by the index of the given power for a divisor, by which divide the dividend, and the quotient will be the second figure of the root required.

4. Involve the part of the root already found, to the power whose root is required for a subducend, and if that subducend be found equal to, or less than the two first periods of the given number, the second figure of the root is right. But if it be found greater, you must diminish the second figure of the root till the subducend be found equal to, or less, than those periods of the given number.

5. Take the subducend from the figures of the two first periods, and to the remainder annex the first figure of the next period for a new dividend.

6. Find a new divisor, and proceed thro' the whole given number in all respects as before, finding the third figure by means of the two first, as you found the second by the first, and afterwards find the fourth figure (if there be a 4th period) after the same manner from the three first.

Note 1. After you have gone thro' the number proposed, if there is a remainder, you may continue the operation by adding periods of cyphers to that remainder, and carry the root into decimals, to any degree of exactness.

2. In extracting the cube root, after you have got two or three figures in the quotient, (or root) you may bring down a whole

whole period, a figure at a time, and so get three places of figures more by division, as in Example 2d: The same may be observed in other roots.

3. The extraction of roots is greatly expedited by observing what integer numbers multiplied together produce the index of the required root, and making such extractions as are denominated by these numbers.

Thus, instead of the biquadrate root, extract the square root twice, i. e. extract the square root, and then the square root of that root; instead of the sixth, extract first the square root, and then the cube root of that; instead of the eight, extract thrice the square root; and instead of the ninth, extract twice the cube root.

EXAMPLES.

1. What is the cube root of 146363183?

```
            .  .  .
            146363183(527 the root of
            125 = cube of 5.
            ---------
5²×3=75)    213 = dividend
            ---------
            146363 = two first periods.
Subducend   140608 = cube of 52.
            ---------
52|²×3=8112)57551 = new dividend.
            ---------
            146363183 the three periods.
Subducend   146363183 = cube of 527.
            ---------
                0 remains.
```

EXPLANATION.

Here the greatest root in the first period is 5, whose cube is 125, which taken from the 1st period leaves 21, to which annex 3 the first figure of the second period and you have 213 for the dividend, and 3 times the square of 5, =75 for the divisor, which goes twice in 213, so set 2 after 5 in the root and you have 52, whose cube is 140608; which taken from the two first periods leaves 5755, to which bring down 1, the first figure of the third period, and

and you have 57551 for a new dividend, and 3 times the square of $52 = 8112$ for a divisor, which goes 7 times in 57551, so set 7 after 2 in the root and it makes it 527 whose cube is 146363183 equal the given number: therefore 527 is the true cube root sought.

2. What is the cube root of ,067507824239?

,067507824239(,407178 root.
,064 = cube of 4.

$4^2 \times 3 = 48$) 35 dividend

,067507 the two first periods
,064000 = cube of ,40

$\overline{40}|^2 \times 3 = 4800$)35078 dividend.
33600

,067507824 = 3 first periods
,067419143 = cube of ,407

$\overline{407}|^2 \times 3 = 496947$)886812 dividend
496947

3898653
3478629

4200249
3975576

224673

Here because the cube of .407 is so nearly equal to the three first periods, I take in three of the last figures by division, but if this method in any case seems not to hold true, you may proceed according to the rule, viz. by cubing the root for every new figure annexed, &c.

3. What

3. What is the cube root of 2? Answ. 1.2599, &c.

$$\dot{2}.\dot{0}0\dot{0}00\dot{0}00\dot{0}00\dot{0}00\dot{0} \;(1.2599$$

$$1 = \text{cube of } 1$$

$$1^2 \times 3 = 3)10 \text{ dividend}$$

$$2\,000 = \text{two first periods}$$

$$\text{subduct } 1,728 = \text{cube of } 12.$$

$$\overline{12}|^2 \times 3 = 432)2720 \text{ dividend}$$

$$2.000000 = \text{the three first periods.}$$

$$\text{subducend} = 1.953125 = \text{the cube of } 1.25$$

$$\overline{125}|^2 \times 3 = 46875)468750$$

$$2.000000000 = \text{the 4 first periods.}$$

$$\text{subducend} = 1.995616979 = \text{the cube of } 1,259$$

$$\overline{1259}|^2 \times 3 = 4755243)43830210 \text{ dividend.}$$

4. What is the sursolid, or 5th root of 12309502009375

$$12309\dot{5}020\dot{0}9375\dot{(}415 \text{ root.}$$

$$1024 = 4|^5$$

$$\overline{4}|^4 \times 5 = 1280)2069 = \text{the dividend}$$

$$123095020 = \text{the two first periods.}$$

$$\text{subducend} = 115856201 = \text{the 5th power of } 41$$

$$\overline{41}|^4 \times 5 = 14128805)\ 72388190 = \text{the dividend.}$$

$$12309502009375 = \text{the 3 periods.}$$

$$\text{subducend} = 12309502009375 = 415|^5$$

5. What is the 7th root of 3829.86553955078125 ?

$$3829{,}86553955078125(3{,}25$$

$$\overline{3}\rceil^{7}=2187$$

$$\overline{3}\rceil^{6}\times 7=5103)16428 \text{ dividend}$$

$$38298655395=\text{two first periods.}$$

$$\overline{32}\rceil^{7}=34359738368=\text{the subducend.}$$

$$\overline{32}\rceil^{6}\times 7=7516192768)39389170275 \text{ dividend}$$

$$\overline{325}\rceil^{7}=382986553955078125 \text{ Subducend.}$$

o remains

6. What is the 9th root of

40452954761505126953125 ?

$$40452954761505126953125(325 \text{ root.}$$

$$\overline{3}\rceil^{9}=19683$$

$$\overline{3}\rceil^{8}\times 9)207699 \text{ dividend}$$

$$40452954761505=\text{two first periods.}$$

$$32\rceil^{9}=35184372088832=\text{subducend.}$$

$$\overline{32}\rceil^{8}\times 9)\ 5268582672673 \text{ dividend.}$$

$$325\rceil^{9}=40452954761505126953125 \text{ Subducend.}$$

o remains

In dividing the dividend by its respective divisor, the quotient figure will sometimes come out too much, but in

in ſuch caſe, the ſubducend will be greater than the number of periods from which it is to be taken: therefore you may any time know how to diminiſh the quotient figure if it happens to be too large. In example 3d, the 1ſt diviſor 3, may be had 3 times in the dividend, 10, but then the ſubducend ariſing therefrom, viz. the cube of 13, would be greater than 2,000 the two firſt periods. And in example 5th the diviſor 5103 wou'd go three times in the dividend 16428; but then the 7th power of 33, would be greater than the two firſt periods.

7. What is the biquadrate root of 1035330554199747·5180024563336?

Firſt extract the ſquare root and it makes 32176552,863844. Again extract the ſquare root of 32176552,863844 and you'll have 5672,438 for the root required. See Note 3. page 234.

8. What is the biquadrate root of 1680362,620425216256? Anſw. 36,004.

9 What is the cube root of 46671553.728064? Anſw. 360,04

10. What is the cube root of 182519501173,87753167.2? Anſw. 5672,438.

11. What is the fifth root of 2? Anſw. 1,148699 &c.

12. What is the fifth root of 8349416423424? Anſw. 384,3 &c.

13. What is the ſixth root of 21035896,12735? Anſw. 16,61474 &c.

14. What is the ſeventh root of 1231171548132409344? Anſw. 384,42 &c.

To extract any root of a vulgar fraction.

RULE.

1. If the given fraction be a complete power, i. e. have a finite root of the kind required, extract the root out of the *numerator* and *denominator*, for the terms of the root required.

2. But if the fraction is not a complete power, reduce it into a decimal, and then extract the required root.

3. Mixt numbers may either be reduced to improper fractions or decimals, and then extracted.

EXAMPLES.

1. What is the cube root of $\frac{27}{343}$? Answ. $\frac{3}{7}$.

2. What is the cube root of $8\frac{4}{7}$? Answ. 2,08

3. What is the cube root of $\frac{512}{729}$? Answ. $\frac{8}{9}$.

4. What is the cube root of $\frac{6}{7}$? Answ. .949 &c.

5. What is the cube root of $42\frac{21}{64}$? Answ. $3\frac{1}{2}$.

The APPLICATION *of the Cube Root.*

PROBLEMS.

I. *To find the side of a cube that shall be equal in solidity to any given solid, as a cone, prism, globe, cylinder, &c.*

Rule. Extract the cube root of the solidity for the side of the required *cube*.

Ex. If the solidity of a *cone prism, globe,* or *cylinder* be 15625 inches, what is the side of a cube of equal solidity? The cube root of 15625 = 25 inches the Answ.

¶ All similar solids are in proportion to one another as the cube of their sides, and all spheres as the cube of their diameters.

II. *The side of one cube being given, to find the side of another cube, that shall be 2, 3, 4 times, &c. greater or less.*

Rule 1. To make a cube any number of times greater, × the cube of the side by that number, and extract the cube root of the product.

2. To

2. To make a cube any number of times leſs, ÷ the cube of the ſide by that number, and the cube root of the quotient will be the ſide required.

Ex. 1. If the ſide of a cube be 12 inches, and its ſolidity 1728: required the ſides of ſix other cubes, whoſe ſolidity of the firſt three ſhall be 2, 3, and 6 times greater; of the latter three 2, 3, and 6 times leſs than the given cubes.

$\sqrt[3]{1728 \times 2} = 15{,}1$ Alſo $\sqrt[3]{1728 \times 3} = 17{,}3$ and $\sqrt[3]{1728 \times 6} = 21{,}8$ the ſides of the greater cubes.

$\sqrt[3]{1728 \div 2} = 9{,}5$ Alſo $\sqrt[3]{1728 \div 3} = 8{,}3$ and $\sqrt[3]{1728 \div 6} = 6{,}6$ inches, the ſides of the three leſs cubes.

Ex. 2. There is a ſphere, or globe whoſe diameter is 6 inches and ſolidity 113,097; what is the diameter of another ſphere whoſe ſolidity is 8 times greater than the former?

Multiply 113,097 by 8, extract the cube root of the product, and you'll have 9.6 the diameter ſought.

III. *To find two mean proportionals between two given numbers.*

Rule. Divide the greater number by the leſs, take the cube root of the quotient, which, ×ed by the leſs number, gives the leſs mean, × the ſaid cube root by the leſs mean, and the product will be the greater mean proportional.

Ex. 1. What are the two mean proportionals between 9 and 576?

Firſt $576 \div 9 = 64$, whoſe cube root is 4. And $9 \times 4 = 36$ the leſs mean. And $36 \times 4 = 144$ the greater mean.
Proof 9 : 36 :: 144 : 576.

2. What are the two mean proportionals between 8 and 512? Anſw. 32 and 128.

¶ Like ſolids are in a triple proportion to their homologous, or like, ſides, diameters, lines, &c. Therefore the weight of globes are as the cube of their diameters, and the burthen of ſhips, &c. as the cube of their like dimenſions.

QUESTIONS.

1. If a bullet, whoſe diameter is 4 inches, weigh 9lb. what will a bullet of the ſame metal weigh, whoſe diameter is 8 inches?

Firſt $4\times4\times4=64$, and $8\times8\times8=512$.
Then as 64 : 9lb. :: 512 : 72lb. Anſw.

2. If a bullet whoſe diameter is 4 inches weighs 9lb. what is the diameter of another bullet of the ſame metal, whoſe weight is 72lb.? Here $4\times4\times4=64$, the cube of the diameter. Then

As 9lb. : 64 :: 72lb. : 512 whoſe cube root is 8, the Anſw.

3. If the diameter of a globe be 1 inch, and the ſolidity thereof ,5236 of an inch; what is the ſolidity of another globe, whoſe diameter is 10 inches?

	cube D.		ſolid.		cube D.		ſolid.
As	1	:	,5236	::	1000	:	523,6 inches Anſw.

4. If the length of a ſhip's keel piece be 44 feet, depth of the hold 9 feet, and midſhip beam 20 feet, what muſt theſe dimenſions be in another ſhip of the ſame mould to carry a double burthen?

The cube of 44 ×ed by 2 = 170368, whoſe cube root is 55,44 feet for the keel's length ſought.

$$\text{Then as } 44 : 55{,}44 :: \begin{cases} 20 : 25{,}22 \text{ midſhip beam.} \\ 9 : 11{,}34 \text{ depth of the hold.} \end{cases}$$

Thus you ſee, having found any one of the dimenſions, the reſt may be had by the Rule of Three.

§ XXIII.

§ XXIV. *Arithmetical Progression.*

ARithmetical Progression, or Proportion, is when numbers do proceed by equal differences, either increasing or decreasing. Such a rank of numbers is sometimes termed a series. If the succeeding terms of a progression exceed each other, it is called an ascending progression or series; if the contrary, a descending series.

As, 1. 2. 3. 4. 5. 6. 7. 8. &c. is an ascending series, increasing by the continual addition of (the common difference) 1. And 8. 7. 6. 5. 4. 3. 2. 1. &c. is a descending series, decreasing by the continual subtraction of 1.

Note. The first and last terms of a progression are called the extreams; and the other terms the means.

THEOREMS.

In any number of continued arithmetical proportionals;

1. *If the number of terms be even*; the sum of the extreams, and that of every two terms equally distant from them, are equal.

2. *If the number of terms be odd*; then those sums are equal to the double of the middle term.

As suppose 2. 4. 6. 8. 10. 12. be an even number of terms, then $2+12=4+10=6+8$; and if 2. 4. 6. 8. 10. be an odd number, then $2+10=6\times 2$.

In Arithmetical Progression, these five things are to be observed, viz.

$$\text{The}\left\{\begin{array}{l}\text{Least Term}\\ \text{Greatest Term}\\ \text{Common Difference}\\ \text{Number of Terms}\\ \text{Sum of the Series}\end{array}\right\}\text{ which put }\left\{\begin{array}{c}\\ \\ =\\ \\ \\ \end{array}\right\}\left\{\begin{array}{l}l.\\ g.\\ d.\\ n.\\ s.\end{array}\right\}$$

Any three of these terms being given, the other two are easily found. But not having room here to treat the whole at large, I shall therefore be obliged to express the rules (or theorems) algebraically, only some of the most material in words.

The

The rules algebraically expressed (which in my opinion is far the best way of expressing them) may be easily understood, and applied, by a common Arithmetician; and turned into words if required.

PROBLEMS.

I. *Given one of the extreams, the common difference* (d), *and the number of terms* (n) *of an arithmetical series; to find* 1. *The other extream.* 2. *The sum of the series* (s).

1. RULE. $\left\{\begin{array}{l} g=l+\overline{d\times n-1} \\ l=g-\overline{d\times n-1} \end{array}\right\}$ That is, to or from the given term, according as it is the least or greatest, add or subtract the product of the common difference ×ed into 1 less than the number of terms, and the sum or difference will be the term required.

2. RULE. $s=\overline{g+l}\times\frac{1}{2}n$: That is, multiply the sum of the extreams by half the number of terms, and the product will be the sum of the series.

EXAMPLES.

1. What is the greatest term and sum of an arithmetical series, whose least term (l) is 5, common difference (d) 3, and number of terms (n) 52?

1. RULE. $g=5+\overline{3\times51}=158$, the greatest term.

2. RULE, $s=\overline{158+5}\times\frac{1}{2}52=4238$ the sum of the series

2. Given the greatest term (g) 128, the common difference (d) 4, and the number of terms (n) 31, to find (l) the least term, and (s) the sum of the series.

1 RULE. $l=128-\overline{30\times4}=8$ the least term.

2. RULE. $s=\overline{128+8}\times31\div2=2108$ the sum of the series.

3. If a debt can be discharged in 2 years, by paying 2s. the first week 5s. the second, and so on, encreasing every

week's payment by 3s. what is the last payment, and how much the debt? Answ. The last payment will be 15l. 11s and debt 813l. 16s.

4. What is the greatest term, and sum of the series $0+1+2+3+4$ &c. continued to 1000 places? Answ. the greatest is 999, and sum of the series 24975.

II. *Given the extreams* (l *and* g) *and number of terms* (n,) *to find*, 1. *the common difference* (d), 2. *the sum of the series* ( ).

1. Rule. $d=g-l\div n-1$: That is, divide the difference of the extreams by 1 less than the number of terms, the quote will be the common difference.

2. Rule. $s=l+g\times\frac{1}{2}n$: As the 2d. Rule Prob. I.

EXAMPLES.

1. Given the extreams 5 and 158. and the number of terms 52; what is the common difference, and sum of the series?

1. Rule. $d=g-l\div n-1=158-5\div 52-1=3$, the common difference.

2. Rule. $s=g+l\times\frac{1}{2}n=4238$ the sum of the series.

2. Find three arithmetical means between 2 and 12.

Here the number of terms is 5; therefore $12-2\div 4=2\frac{1}{2}$ the common difference.

Then $2+2\frac{1}{2}=4\frac{1}{2}$, the 2d term; $4\frac{1}{2}+2\frac{1}{2}=7$, the 3d term; and $7+2\frac{1}{2}=9\frac{1}{2}$, the 4th term: Therefore the required means are $4\frac{1}{2}$, 7, $9\frac{1}{2}$. And the series, 2, $\frac{1}{2}$, 7, $9\frac{1}{2}$, 12.

Note, Any arithmetical series may be constructed, from any given term, and with any given difference, thus, for an ascending series, add the difference to the first term, the sum will be the second; add the difference to the second, the sum will be the third; and so on, continually adding the difference to the term last found for the next succeeding term. And for a descending series, subtract continually the common difference for the several succeeding terms.

3. If

3. If the extreams be 8 and 128, and the number of terms 31; what is the common difference, and sum of the series? Answ. The difference is 4 and sum 2108.

4. What debt can be discharged in 2 years by weekly payments in Arithmetical Progression, whereof the first term or payment is 2s and last 15l. 11s. and what is the common difference of the series of payments?

Answ. The difference is 4s. and the debt 813l. 16s.

III. *Given* l, g *(the extreams,) and* d *(the common difference,) to find* n *and* s.

1 Rule. $n = \overline{g-l+d} \div d$: That is to the difference of the extreams add the common difference; divide the the sum by the common difference and the quote will be the number of terms.

2. Rule. $s = \overline{g+l} \times \overline{g-l+d} \div d$: i.e. $s = \overline{g+l} \times \frac{1}{2}n$.

EXAMPLES.

1 Given the least extream (l) 5, the greatest (g) 158, and common difference (d) 3; required (n) the number of terms, and (s) the sum of the series.

1. Rule. $n = \overline{g-l+d} \div d = \overline{158-5+3} \div 3 = 52$, the number of terms.

2. Rule. $s = \overline{158+5} \times \frac{1}{2}52 = 4238$, the sum of the series.

2. If the extreams be 8 and 128, and the common difference 4, what is the number of terms and sum of the series? Answ. The number of terms is 31, and the sum is 2108.

3. What debt can be discharged, and in what time, supposing the first weeks payment is 2s. and the payments every week following to increase by 3s. till the last payment be 15l. 11s? Answ. The debt is 831l. 16s. and will be discharged in two years.

IV.

IV. *Given* l, g, *and* s; *to find* n, *and* d.

1. RULE. $n=2s\div g+l$: That is, divide twice the ſum, by the ſum of the extreams, the quote will be the number of terms.

2. RULE. $d=\overline{gg-ll}\div\overline{2s-g-l}$: Or having found *n*, then *d*, may be found as in Problem II.

EXAMPLES.

1. If the extreams be 5 and 158, and the ſum of the ſeries 4238, what is the number of terms and common difference?

1. RULE. $n=4238\times 2\div 158+5=52$ the number of terms.

2 RULE. $d=158-5\div 52-1=3$, the common difference.

2. If the extreams be 8 and 128, and the ſum of the ſeries 2108: what is the number of terms, and the common difference? Anſw. The number of terms in 31, and the common difference 4

3. In what time will a debt of 813l. 16s. be diſcharged by weekly payments in Arithmetical Progreſſion, the firſt payment being 2s. and the laſt payment 15l. 11s. and what will the common difference of the ſeries be?

Anſw. The common difference will be 3s. and the debt will be diſcharged in two years.

V. *Given one extream, the ſum of the ſeries, and number of terms; to find the other extream, and common difference.*

1. RULE. $g=\overline{2s\div n}-l$, or $l=\overline{2s\div n}-g$.

2. $\left\{\begin{array}{l} d=2s-2nl\div nn-n, \text{ the leſs extream being given.} \\ d=2ng-2s\div nn-n, \text{ the greater ext. being given.}\end{array}\right\}$

Or having both the extreams, d, the com. diff. may be found, by prob. 2. or 4.

EXAM-

E X A M P L E S.

1. If the least term be 5 (l), the number of terms 52 (n), and sum of the series 4238 (s); what is the greatest term (g), and com. diff. (d)?

1. RULE. $g = 4238 \times 2 \div 52 - 5 = 158$, the greatest term.

2. RULE. $d = 158 - 5 \div 51 = 3$ the common difference.

2. If the greatest extream be 128 (g), the number of terms 31, and sum of the series 2108; what is the least term, and common difference?

1. RULE. $l = 2108 \times 2 \div 31 - 128 = 8$ the least extream, and $d = 4$ the com. diff.

3. A debt of 813l. 16s. can be discharged in 2 years by weekly payments in Arithmetical Progression, the least term or payment being 2s. what will be the greatest payment, and the com. diff. of the payments? Answ. The com. diff. will be 3s. and the last payment 15l. 11s.

VI. *Given* d n, *and* s; *to find* l, *and* g.

RULE. $l = \frac{s}{n} - \frac{n-1}{2} \times d$, and $g = \frac{s}{n} + \frac{n-1}{2} \times d$. That is, divide the sum of the series by the number of terms; then to and from the quotient, add and subtract half the product of the common difference into 1 less than the number of terms, and the sum and difference will be the two extreams.

E X A M P L E S.

1. If the common difference be 3, the number of terms 52, and the sum of the series 4238; what are the extreams?

First $l = \frac{4238}{52} - \frac{52-1}{2} \times 3 = 8$ the less extream.

Secondly $g = \frac{4238}{52} + \frac{52-1}{2} \times 3 = 158$ the greatest extr.

2. If the common difference be 4, the number of terms 31, and the sum of the series 2108; what are the extreams? Answ. 8, and 128.

3. If

3. If a debt of 813l. 16s. can be discharged in two years, by weekly payments in Arithmetical Progression, whose common difference is 3s. what are the first and last payments? Answ 2s. and 15l. 11s.

VII. *Given one of the extreams, the common difference* (d) *and the sum of the series* (s) *to find the other extream and number of terms* (n.)

1. Rule. $$\left\{\begin{array}{l} g = \sqrt{\overline{l \backsim \tfrac{1}{2}d}\,^{2} + 2ds} - \tfrac{1}{2}d \\ l = \tfrac{1}{2}d \pm \sqrt{\overline{g + \tfrac{1}{2}d}\,^{2} - 2ds} \end{array}\right\}$$

2. Rule. $$\left\{\begin{array}{l} n = \dfrac{\sqrt{2sd + ll + \tfrac{1}{4}dd - ld} \pm \tfrac{1}{2}d \backsim l}{d} \\ n = \dfrac{\tfrac{1}{2}d + g \mp \sqrt{\overline{g + \tfrac{1}{2}d}\,^{2} - 2ds}}{d} \end{array}\right.$$

Or, having now both extreams and the common difference, the number of terms may be found by the first rule of Problem III.

Note, That $\backsim$ denotes the difference or excess of any two numbers or quantities.

E X A M P L E S.

1. If the least term be 5 (l,) the common difference 3 (d), and the sum of the series 4238 (s); what is the greatest term, and the number of terms.

1. Rule. $g = \sqrt{\overline{5 - \tfrac{1}{2}3} \times \overline{5 - \tfrac{1}{2}3} + 4238 \times 2 \times 3} - \tfrac{1}{2}3 = 158$ the greatest term.

2. Rule. $n = \dfrac{\sqrt{4238 \times 2 \times 3 + 5 \times 5 + \tfrac{1}{4} \times 3 \times 3 - 5 \times 3}}{3} - \dfrac{5 - \tfrac{1}{2}3}{3} = 52$ the number of terms.

Or, $n = \overline{158 - 5 + 3} \div 3 = 52$. See Rule 1. Prob. 3.

2. If

2. If the greateſt term be 128 (g), the com. diff. 4 (d), and the ſum of the ſeries 2108 (s); what is the leaſt term, and the number of terms ?

1ſt. $l=\frac{1}{2}4+\sqrt{\overline{128+\frac{1}{2}4\times128+\frac{1}{2}4-2108\times2\times4}}=8$ the leaſt term

Secondly $n=\overline{128-8+4}\div4=31$ the number of terms.

3. In what time will a debt of 813l 16s. be diſcharged by weekly payments in Arithmetical Progreſſion, the firſt term or payment being 2s. and the common difference 3s. and what will the laſt payment be ? Anſw. The laſt payment will be 15l. 11s. and the debt will be diſcharged in 2 years.

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§ XXV. *Geometrical Progreſſion.*

GEometrical Progreſſion, (or proportion continued), is when a ſeries of numbers do proceed by equal ratio's, that is, by one common ×er or ÷ſor, either increaſing or decreaſing ; that is, either aſcending or deſcending.

As { 1, 2, 4, 8, 16, 32, &c. is an aſcending Geom. ſeries.
   { 32, 16, 8, 4 2, 1, is a deſcending Geom. ſeries.

THEOREMS.

In any number of continued geometrical proportionals,

1. If the number of terms be even ;
The product of the extreams, and that of every two terms, equally diſtant from them are equal.

2. If the number of terms be odd ;
Then thoſe products are each equal to the ſquare of the middle term.

As 1, 3, 9, 27, 81, 243, 729, 2187, even.

Thus $1\times2187=3\times729=9\times243=27\times81$.
Or, as 1, 3, 9, 27, 81 243, 729, odd ;
Thus, $27\times27=9\times81=3\times243=729$.

In

In Geometrical Progreſſion, theſe five things are to be obſerved, viz.

$$\text{The}\left\}\begin{array}{l}\text{Firſt or leaſt term.}\\ \text{Laſt or greateſt term,}\\ \text{Ratio (or com.} \times \text{er or} \div \text{ſor)}\\ \text{Number of terms}\\ \text{Sum of the ſeries}\end{array}\right\}\text{which put}\left\{=\left\{\begin{array}{l}\text{a.}\\ \text{z.}\\ \text{r.}\\ \text{n.}\\ \text{s.}\end{array}\right\}\right.$$

Any three of theſe terms being given, the other two are eaſily found.

P R O B L E M S.

I. *Given one of the extreams (*a *or* z*), the ratio (*r*), and the number of terms (*n*) to find the other extream* (z *or* a), *and the ſum of the ſeries* (s).

1. Rule $\left\{\begin{array}{l} z = a \times r^{n-1}, \text{ when a is given.}\\ a = z \div r^{n-1}, \text{ when z is given.}\end{array}\right\}$ That is, multiply or divide the given extream by ſuch power of the ratio whoſe index is one leſs than the number of terms, and the product or quotient will be the required term, according as it is the greater or leſs extream.

2. Rule. $\left\{\begin{array}{l} s = \overline{rz - a} \div \overline{r - 1}\\ s = \overline{z - a} \div \overline{r - 1} + z\end{array}\right\}$ That is, $\times$ the greateſt term by the ratio, from the product ſubtract the leaſt term, then divide the difference by the ratio leſs 1, and the quotient will be the ſum of the ſeries. Or, divide the difference of the extreams by the ratio leſs 1 ; to the quotient add the greater extream, and it will give the ſum of the ſeries.

E X A M P L E S.

1. Given the leaſt term 3 (a), the ratio 2 (r), and the number of terms 12 (n) ; required the greateſt term (z) and the ſum of the ſeries (s).

Here $r=2$ which being involved to the 11th power is 2048: viz. to a power lefs by 1 than the number of terms: then by the

1. Rule. $z=3\times 2048=6144$ the greateft term. And by the

2. Rule. $s=\overline{6144\times 2-3}\div 2-1=12285$ the fum of the feries.

2. If the greateft term be 1310720 (z), the ratio 4 (r), and the number of terms 10 (n); what is the leaft term, and the fum of the feries?

Here, $r=4$, whofe 9th power is 262144, then by the

1. Rule. $a=1310720\div 262144=5$ the leaft term.

2. Rule. $s=1310720-5\div 4-1+1310720=1747625$ the fum of the feries.

3 What debt will be difcharged in a year or 12 months, by paying 3s. the firft month, 9s. the fecond, 27s. the third, and fo on, each fucceeding payment being trebble the laft; and what will the laft payment be? Anfw. the debt is 39858l. and the laft payment 26572l. 1s.

II. *Given* a, r, *and* z, *to find* s, *and* n.

1 Rule. $s=zr-a\div r-1$: the fame as 2d Rule, in prob I.

2. Rule. $n=\dfrac{\log z-\log. a.}{\text{Log. } r}+1$: That is, divide the difference of the Logarithms of the extreams by the logarith of the ratio; add 1 to the quotient, and the fum will be the number of terms. Or divide the greateft term by the leaft; find what power of the ratio is equal to the quotient; then add 1 to the index of that power, and the fum will be the number of terms.

E X A M P L E S.

I. *Given the extreams* 3 (a), *and* 6144 (z), *and the ratio* 2 (r); *required the fum of the feries* (s), *and the number of terms* (n).

1 Rule.

1. Rule $s=\overline{6144\times2-3}\div2-1=12285$ the sum as before.

1. Rule. { Log. of 6144 is 3. 7884512
Log. of 3 is 0. 4771213
Log. of 2 is 0. 301300)3. 3113299(11
+1 }

The number of terms $n.=12$

Or thus, $6144\div3=2072$, which being continually divided by the ratio (2), the number of those divisions will be (11) the number of terms wanting one.

2 If the extreams be 5 and 1310720, and the ratio 4; what is the sum of the series, and the number of terms?

Answ. The sum is 1747625, and the number of terms 10.

3. What debt will be discharged by monthly payments in Geometrical Progression, whereof the first is 3s. and the last 265721. 1s. the ratio being 3; and in what time will it be discharged?

Answ. The debt is 39858l. and will be discharged in a year.

III. *Given* r, s, *and one of the extreams; to find the other extream, and the number of terms* (n).

1 Rule. { $z=\overline{rs+a-s}\div r$; when the extream a is given.
$a=\overline{zr+s-rs}$; when the extream z is given. }

2 Rule. { $n=\log.z-\log\overline{s+rz-sr}\div\log.r$: by having (z) the greater extream.
$n=\log.\overline{sr+a-s}-\log.a\div\log.r$: by having (a) the less extream. }

Or, having found both the extreams, the number of terms may be found as in problem 2.

E X A M P L E S.

1. If the least term be 3 (a), the sum of the series 12285 (s), and the ratio 2 (r;) what is the greatest term (z), and the number of terms (n)?

 1. Rule.

1. Rule. $z = 12285 \times 2, + 3 - 12285 \div 2 = 6144$ the greatest term.

And the number of terms (found by prob. 2.) 12.

2. Given the greatest term 1310720, the sum of the series 1747625, and the ratio 4; what is the least term, and the number of terms?

1. Rule $a = 1310720 \times 4 + 1747625 - 1747625 \times 4 = 5$ the least term. And the number of terms 10.

3. In what time will a debt of 132860 l. be discharged by monthly payments in Geometrical Progression, whereof the first term is $\frac{1}{2}$l. and the ratio 3; and what will the last payment be? Answ. The last payment will be $88573\frac{1}{2}$l and the debt will be discharged in a year.

IV. *Given* a, n, *and* z; *to find* r, *and* s.

1. Rule. $r = \overline{z \div a}\rceil^{\frac{1}{n-1}}$: That is, $\div$ the greater extream by the less, and extract such root of the quotient whose index is equal to the number of terms less 1.

2. Rule. s $\overline{\overline{z-a}, \div \overline{z \div a}\rceil^{\frac{1}{n-1}} - 1}, + z$: Or having found the ratio, the sum of the series may be found, as in problem 1.

E X A M P L E S.

1. Given the extreams 3 (a), and 6144 (z), and the number of terms 12 (n); to find the ratio (r), and the sum of the series (s.)

1. Rule. $r = \overline{6144 \div 3}\rceil^{\frac{1}{11}} = 2$ the ratio: That is, $6144 \div 3 = 2048$, whose 11th root is 2.

2. Rule. $s = \overline{6144 - 3 \div 2}, + 6144 = 12285$ the sum of the series.

2. If

2. If the extreams be 5, and 32805, and the number of terms 9; what is the ratio, and the ſum of the ſeries?

Anſw. The ratio is 3; and the ſum 49205.

3. What debt can be diſcharged in a year by monthly payments in a Geometrical Progreſſion, whereof the firſt payment is 10s. and the laſt 88573 5l. and what will the ratio of the ſeries be?

Anſw. The ratio will be 3, and the debt 132860l.

V. *Given* a, z, *and* s; *to find* r, *and* n.

1. Rule. $r = \overline{s - a} \div \overline{s - z}$.

2. Rule, $n = \dfrac{\log. z - \log. a}{\log. \overline{s-a} - \log. \overline{s-z}} + 1$. Or, having found r, n may be found as in Problem 2.

E X A M P L E S.

1. Given the extreams 3 (a), and 6144 (z), and the ſum of the ſeries 12285 (s); to find the ratio, and number of terms.

1. Rule. $r = 12285 - 3 \div 12285 - 6144 = 2$ the ratio. And 2dly $n = 12$ found by Problem 2.

2. If the extreams be 5, and 32805, and the ſum of the ſeries 49205; what is the ratio, and number of terms? Anſw. The ratio is 3, and the number of terms 9.

3 In what time will a debt of 132860l. be diſcharged by monthly payments in Geometrical Progreſſion, the firſt payment being 10s. and the laſt 885731. 10s and what is the ratio of the ſeries? Anſw. The ratio is 3, and the debt will be diſcharged in a year.

VI. *Given* r, n, *and* s, *to find* a, and z.

1. Rule. $a = \overline{rs - s} \div \overline{r^n - 1}$.

2. Rule $z = \overline{r - 1}, \div \overline{r^n - 1}, \times sr^{n-1}$. Or, having found the leaſt term, the greateſt may be found by Problem 1.

E X-

EXAMPLES.

1. If the ratio be 2 [r], the number of terms 12 [n], and the ſum of the ſeries 12285 [s]; what are the extreams?

Here r^n = 12th power of 2. viz. 4096; alſo r^{n-1} = 2 involved to the 11th power, viz. 2048; $\frac{r-1}{r^n-1} = \frac{2-1}{4095} = \frac{1}{4095}$: Then per

1. Rule. $a = \frac{1}{4095} \times 12285 = 3$ the leaſt term. And by the

2. Rule. $z = \frac{1}{4095} \times 12285 \times 2048 = 6144$ the greateſt term.

2. Given the ratio 3, the number of terms 9, and the ſum of the ſeries 49205; what are the extreams?

Anſw. 5 and 32805.

Of decreaſing Geometrical Progreſſion.

IN *finite decreaſing progreſſion* the ſame rules will ſerve the like Problems, if the ſeries be inverted, ſo that the leaſt term be the firſt, and the greateſt the laſt. But in an infinite decreaſing progreſſion or ſeries, that is, when the number of terms are infinite, then ſhall (z) the laſt term, be equal to (o) nothing. For becauſe n, and conſequently r^{n-1} is infinite, $z = a \div r^{n-1} = 0$ for it is leſs than any aſſignable number whatever. The ſum of ſuch a ſeries $s = a \times r \div r - 1$; which is a finite ſum, though the number of terms be infinite.

PROBLEMS.

I. *Given the firſt term* [a], *and ratio* [r], *of an infinite decreaſing progreſſion, or ſeries, to find the ſum of the ſeries* [s].

Rule

1. Rule. $s = a \times r \div \overline{r - 1}$. That is, $\div$ the product of the first term and ratio, by the ratio less 1.

EXAMPLES.

1. What is the sum of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ &c. infinitely continued ?

Here $s = a \times r \div \overline{r - 1} = 1 \times 2 \div \overline{2 - 1} = 2$ the sum of the series required.

2. What is the sum of the infinite series ,666 &c. whose ratio is 10, and first term ,6 ?

Answ. $s = ,6 \times 10 \div 9 = \frac{2}{3}$ the sum required.

3. Suppose a body to move eternally in this manner, viz. the first hour 20 miles, the 2d 19 miles, the 3d $18\frac{1}{20}$, and so on in the same Geometrical Progression whose ratio is $\frac{20}{19}$, how far will it move in an eternity ? Answ. 400 miles.

That is, a moveable body continuing its motion in that ratio eternally, would only run 400 *miles, or more than any thing that is less than* 400 *miles.*

II. *Given the ratio* [r], *and the sum* [s] *of an infinitely descending series; to find the first term* [a].

Rule. $a = \overline{rs - s} \div r$. That is, from the product of the ratio and sum, subtract the sum, and divide the remainder by the ratio.

EXAMPLES.

1. Given the ratio 5 [r] and the sum $\frac{1}{4} = ,25$ [s] of an infinite series; to find the first term (a).

Rule. $a = \overline{5 \times ,25 - .25} \div 5 = ,2$ *or one fifth* the first term.

2. What is the first term of the infinite series whose ratio is 10, and sum $\frac{2}{3}$? Answ. $\frac{3}{5}$.

3. Suppose, that a body, if continued moving eternally, would move over 400 miles; and in such manner,

that

that the ſpaces paſſed over at the end of each hour, are in a deſcending progreſſion, whoſe ratio is $\frac{20}{19}$; how much does it move the firſt hour? Anſw. 20 miles.

III. *Given the ſum* [s], *and firſt term* [a], *of an infinite deſcending ſeries, to find the ratio* [r].

Rule $r = s \div s - a$. That is, divide the ſum by the difference between the ſum and firſt term, for the ratio.

EXAMPLES.

1. Given the ſum $\frac{1}{4}$=,25 [s], and firſt term *one fifth*=.2 [a], of an infinite deſcending ſeries; to find the ratio [r].

Rule. $r = s \div s - a = .25 \div .25 - .2 = 5$ the ratio ſought.

2. What is the ratio of an infinite ſeries, whoſe firſt term is $\frac{3}{5}$, and ſum $\frac{2}{3}$? Anſw. 10

3. If a body, in an eternity, move over 400 miles, in ſuch ſort, that at the end of any equal intervals of time, the ſpaces moved over be in a geometrical progreſſion; what will the ratio of the ſeries be? Anſw. $\frac{20}{19}$.

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§XXVI. *Of* PERMUTATION.

PERMUTATION is the changing or varying the order of things.

Rule. Multiply all the given terms continually one into another, and the laſt product will be the Anſwer.

Exam. 1. How often might a family of 7 perſons dine together, and be placed every day in a different poſition?

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$ days the Anſw. required.

Exam. 2. How many changes may be rung on 12 bells, and how long would they be ringing but once over, allowing 3 ſeconds to every round?

Anſw. 479001600 changes. The time 1437004800 ſeconds. = 45 years, 27 weeks, 6 days, 18 hours.

§ XXVII.

§ XXVII. *Interest and Annuities.*

TO write all the *rules* in words at length, relating to the business of Interest and Annuities, would swell this Treatise beyond its intended limits: therefore I shall express the rules (or Theorems) algebraically.

I. *Of a single sum of money paid either before, or after it is due.*

1. SIMPLE INTEREST.

For the definition, see page 98.

Let p, n, m, be put to represent the same things here as in page 98, and r = the ratio of the rate, or interest of 1l. for a certain time, as a year, &c. Then by proportion.

As $\begin{cases} 1\text{ l.} : r :: p : pr, \text{ the interest of the principal (p) for a year.} \\ 1\text{ yr.} : pr :: n : prn, \text{ the interest of p for the time n.} \end{cases}$

Then $p + prn$, = the amount or arrear at the end of the time n.

Hence we have these four Theorems.

1. $m = p + prn$, when p, r, n, are given.

By this Theorem Table I. was constructed.

2. $p = m \div \overline{rn + 1}$, when m, r, n, are given.
3. $n = m - p \div pr$, when m, p, r are given.
4. $r = m - p \div pn$, when m, p, n, are given.

Note, The ratio (r) or interest of 1l. for a year is found thus: as 100 l. : is to its rate of interest : : so is 1 l. to its rate of interest for a year.

Thus as 100 l. : 3 l :: 1l. : ,03 = r, at 3 per cent.
100 l. : 3.5 :: 1l. : .035 = r, at $3\frac{1}{2}$ per cent.
100 l. : 4 :: 1l. : .04 = r, at 4 per cent. &c.

2. COM-

COMPOUND INTEREST.

Let p, n, r, m represent the same as before, and put $x = r + 1$, the amount of 1l. for any given time, as a year &c. at the rate r. Then x may be found thus, viz.

$$\text{As}\left\{\begin{array}{l} 100 : 104.5 :: 1 : 1{,}045 = x, \text{ at } 4\tfrac{1}{2} \text{ per cent.} \\ 100 : 105 \ :: 1 : 1{,}05 \ = x, \text{ at } 5 \text{ per cent.} \end{array}\right\}$$

and so on for any other rate of interest.

$$\begin{array}{l}\text{In geometrical} \\ \text{prop. continued.}\end{array}\left\{\begin{array}{l} 1\text{ l.} : x :: p \ : px \ = 1\text{st.} \\ 1\text{ l.} : x :: px \ : pxx \ = 2\text{d.} \\ 1\text{ l.} : x :: pxx : pxxx = 3\text{d.} \end{array}\right\}\begin{array}{l}\text{year's} \\ \text{amount} \\ = m.\end{array}$$

And as 1l. : x^n :: p : px^n = nth, years amount (= m). That is any principal (p) or sum of money ×ed by the amount of 1l. involved to a power whose index is denominated by the number of years, gives its amount. Whence we have these four theorems.

1. $m = p \times x^n$, or $m = \log. p + \overline{\log.}\ x \times n$, when p, n, x are given.

2. $p = m \div x^n$, or $p = \log. m - \overline{\log. x \times n}$, when m, n, x are given.

3. $n = x^n \div\div x$, or $\log. n = \overline{\log. m - \log. p} \div x$, when p, n, x are given.

Note. $\div\div$ signifies a continual division by x, (see Prob. 1st, in Geom. Prog.) and $x^n = m \div p$; the amount of 1l. at the given rate in the required time.

4. $x = \overline{m \div p}\big|^{\frac{1}{n}}$, or $x = \overline{\log\ m - \log.\ p \div n}$, when p, m, n, are given.

Note. Table II. is constructed by successively involving to the 30th power these numbers, 1.03, 1.035, 1.04, 1.045, and 1.05, being the amount of 1l. for a year at these several rates of interest viz. 3, 3-half, 4, 4-half, and 5 per cent.

The

The ſeveral powers (or tabular numbers) are the amounts of 1l. for ſuch a number of years as is expreſſed by the index of the power to which the firſt years amount of 1l. is raiſed. Any ſum of money multiplyed by the amount of 1l. for any number of years, (found in the table) will give the amount of that ſum of money for the ſaid number of years The ſame as expreſſed by Theorem 1ſt. from which the other three are deduced.

QUESTIONS *to excercise the firſt Theorem, both of ſimple and compound Intereſt.*

1. If 575 l. (p), be put out to intereſt, what will it amount to in 7 years (n), at 4 per cent. *ſimple* or *compound Intereſt?*

Here $r=.04$ and $x=r+1=1.04$; then $m=p+prn$ $=575+575\times.04\times7=736$l. the amount required at ſimple intereſt. Again, $m=p\times x^n=575\times 1.04)^7=756,6607$l. Or 756l. 13s. $2\frac{1}{2}$d the amount required at compound intereſt.

By the Tables. Look into Table I. under 4 per C and againſt 7 years you'll find 1.28 the amount of 1l. for 7 years at 4 per cent.

Then $m=575\times1.28=736$l. as before. And in Table II. you'll find the amount of 1l. for 7 years at 4 per cent. com. intereſt to be 1.31593; then 575×1.31593 $=756.65975$ nearly as before.

2. If 125l. be put out to intereſt, what will it amount to in 15 years, at 5 per cent. ſimple or compound intereſt? Anſw. At ſimple intereſt 218l. 15s. at compound 259l. 17s. 3d. 3.6q.

3. If 50l. 10s. 6d. be put out at intereſt, what will it amount to in 30 years, at 5 per cent. ſimple or compound intereſt? Anſw. At ſimple intereſt 126l. 6s. 3d. and at compound 218l. 7s. 3d. 3.37q.

QUESTIONS *to exercise the second Theorem, both of simple and compound interest.* By the 2d Theorem (both simple and compound) discount is calculated.

1. If 736 l. (m) was to be paid 7 years hence (n) what is the present value (p), discounting at 4 per cent. simple interest?

By Tab. I. the amount of 1 l. for the given time and rate is $1.28 = \overline{n\times r+1}$; then $p = m \div \overline{n\times r+1} = 736 \div 1{,}28 = 575$ l. the present value sought.

2. If 756,65975 l (m), was to be paid 7 years hence (n). what is its present worth, discounting at 4 per cent. compound interest?

By Table II. the amount of 1 l. is 1.31593 (=1.04 raised to the 7th power) for the given time and rate; then $p = m \div x^n = 756.65975 \div 1.31593 = 575$ l. the present worth sought.

3. What principal put out for 15 years will amount to 218 l. 15 s. at 5 per cent. simple interest, and 259 l. 17 s. 3 d. 3. 6 q. at compound? Answ. 125 l.

4. What principal put out for 30 years at 5 per cent. will amount to 126 l. 6 s. 3 d. simple interest and to 218 l. 7 s. 3 d. 3. 37 qr. at compound? Answ. 50 l. 10 s. 6 d.

QUESTIONS *to exercise the third Theorem both simple and compound interest.*

1. In what time will 575 l. (p), amount to 736 l. (m) at 4 per cent. simple interest, and to 756.65975 l. (m), at 4 per cent. compound interest?

Here $n = \overline{m-p} \div pr = \overline{736-575} \div 575 \times .04 = 7$ years at simple interest.

Again, is $x^n = m \div p = 756.65975 \div 575 = 1.31593$ the amount of 1 l. which being continually divided by 1,04 (x) till nothing remains, the number of those divisions will be 7. Which $x^n \div x = 7$ years at compound. Or seek 1,31593 in Table II. col. 4. per C. and you'll find it against 7 years, the time sought.

2, In

2. In what time will 125l. amount to 218l. 15s at 5 per cent. simple intereſt, and to 259l. 17s. 3d. 3.6 qr. at 5 per cent. compound intereſt? Anſw. In 15 years.

3. In what time will 50l. 10s. 6d. amount to 126l. 6s 3d. at ſimple intereſt, and to 218l 7s. 3d. 3.37 q at compound intereſt, the rate being 5 per cent. each?
Anſw. 30 years.

QUESTIONS *to exerciſe the 4th Theorem, both of ſimple and compound intereſt.*

1. At what rate will 575l. (p), amount to 736l. (n), at ſimple intereſt, and to 756.659751. (m), at compound intereſt, in 7 years?

Firſt $r=\overline{m-p}\div p\times n=\overline{736-575}\div 575\times 7=.04$ the ſimple intereſt of 1l. and as 1l. : .04 :: 100l. : 4 the rate per cent.

2dly. x^n or $x^7=m\div p=756.65975\div 575=1.31593$ the amount of 1l. for 7 years at compound intereſt; then

$x=\overline{m\div p}\rceil^{\frac{1}{n}}=\overline{1.31593}\rceil^{\frac{1}{7}}=1,04$ the amount of 1l. for a year at 4 per cent. Or thus, ſeek the number 1.31593 (m÷p) over againſt 7 years in Tab. II. and you'll find it under 4 per cent. the rate ſought.

2. At what rate will 125l. amount to 218l. 15s. at ſimple intereſt, and to 259l 17s. 3d. 3.6 q. at compound intereſt, in 15 years? Anſw. At 5 per cent.

3. At what rate will 50l. 10s. 6d. amount to 126l. 6s. 3d. at ſimple intereſt, and to 218l. 7s. 3d. 3.37qr. at compound intereſt in 30 years? Anſw. At 5 per cent.

II. *Of Annuities, Rents, or Penſions, &c. in Arrear.*

ANNUITIES, RENTS, or PENSIONS, &c. are ſeveral equal payments, due at ſeveral equal times, as yearly, half yearly, or quarterly: and when they remain unpaid for any number of payments, they are then ſaid to be in arrears.

1. *Allowing simple Interest.*

Put a = *annuity*, *rent*, or *pension*; n = years, or number of payments it is forborn; r = interest of 1l. for a year, &c. m = whole arrear 'Tis evident, 1 : r :: a : ar, that is, the interest of a. at the rate r, per 1l. per annum is ar.

But in arithmetical progression. $\left\{\begin{array}{l} a \\ a + 1\,ar \\ a + 2\,ar \\ a + \overline{n-1} \times ar \end{array}\right\}$ is the $\left\{\begin{array}{l} 1st \\ 2d \\ 3d \\ nth \end{array}\right\}$ years amount (m)

Therefore by the 2d Rule. Prob. 1. in Arithmetical Progres. the sum of those amounts is $2a + n - 1 \times ar \times \frac{1}{2}n$, or the amount (m) of the annuity (a) at the nth years end. Whence we have these four Theorems.

1. $m = \overline{2a + \overline{n-1} \times ar} \times \frac{1}{2}n$, when a, n, r, are given.

Note, By this Theorem, Table III. was constructed.

2. $a = 2m \div \overline{2 + nr - r} \times n$, when m, n, r, are given.

3. $n = \sqrt{\overline{2m \div ra, + zz}} - z$; (putting $\frac{1}{r} - \frac{1}{2} = z$), when m, r, a, are given.

4. $r = 2m - 2na \div an^2 - an$, when m, n, a, are given.

2. *Allowing Compound Interest.*

Let a, n, r, m, represent the same as before, and let $x = 1 + r$, the amount of 1l. for a year. Now since the amount of 1l. in n years is x^n, its interest for that time will be $x^n - 1$; but for a single year is $x - 1 = r$. Therefore say, as the interest of 1l. for a year : is to its interest for the given time :: so is the annuity : to the amount. Thus, as $x - 1 : x^n - 1 :: a : m$. Therefore, by $\times$ing means and extreams, $m \times \overline{x-1} = \overline{x^n - 1} \times a$. Whence we have these four Theorems.

1. m

1. $m = \overline{ax^n - a} \div r$, when a, n, r, are given. (For $r+1 = x$.)

Note, By this Theorem Table IV. was constructed.

2. $a = m \times r \div \overline{x^n - 1}$, when m, r, n, are given.

3. $x^n = \overline{m \times r \div a, +1}$; or $n = \log.\overline{\overline{rm+a}} - \log. a \div \log. x$, when m, r, a, are given.

4. $\overline{m \div a,} \times x - x^n = \overline{m - a} \div a$, when m, a, n, are given

From the *equation* above, x may be found, and then r. But this must be done by an Algebraic process. Or, by help of Table. IV.

QUESTIONS *to exercise Theorem 1st both of simple and compound Interest.*

1. If 60l. yearly rent (a), be forborn 10 years (n), what will be in arrear (m) at that time, at $4\frac{1}{2}$ per cent. simple and compound interest?

1. $m = \overline{\overline{60 \times 2 + 9} \times 60 \times .045 \times 5} = 721,5$l. the amount at simple interest.

Or, thus $\overline{n^2 - n,} \div 2, \times a \times r = \overline{10 \times 10 - 10} \div 2 \times 1 \times ,045 = 2,025$l. the simple interest of 1l. annuity for 10 years, to which add 10l. the 10 years rent, and the sum is 12,025l. the amount of 1l. annuity for 10 years at $4\frac{1}{2}$ per cent. as in Table III. which was in this manner constructed. Then $12,025 \times 60 = 721,5$l. $= 721$l. 10s. the amount.

2dly. $r = ,045$ and $x = 1,045$, whose 10th power is 1,55297

Then $m = \overline{ax^n - a} \div r = \overline{1,55297 \times 60 - 60} \div ,045 = 737,2933$l. the arrear sought, at compound interest.

Or thus, $\overline{1,55297 \times 1 - 1} \div ,045 = 12,28821$l. the amount of 1l. annuity for 10 years at $4\frac{1}{2}$ per cent. compound interest, the same as against 10 years under $4\frac{1}{2}$ per c. in Table IV. which in this manner was constructed. Then $12,28821 \times 60 = 737,2926$l. $= m$, the arrear as before, *nearly*.

2. If 125l. yearly rent be forborn 16 years, what will be in arrear at that time, at $3\frac{1}{2}$ per cent. simple and compound interest?

By Table III. the amount of 1l. annuity for 16 years is 20,2l.; then $125\times 20,2=2525$l. the arrear sought, at simple interest.

By Table IV. the amount of 1l. annuity is 20,97103; therefore $20,97103\times 125=2621,37875$l. the amount at compound interest.

3. If 320l. yearly pension be forborn 24 years, what will be in arrear at that time, at 5 per cent simple and compound interest? Answ. 12096l. at simple interest, and 14240,64l. at compound.

4. What will an annuity of 50l. a year amount to in 30 years at 3 per cent. simple and compound interest?
Answ. 2152,5l at simple, and 2378,7705l. at common interest.

Note, If the rent or pension, is payable half yearly or quarterly, the method of proceeding (in all the 4 cases) will still be the same, provided n be always taken to express the number of payments, and r the interest of 1l for the time in which the first payment becomes due. The Tables being calculated for yearly payments, are of no use for half yearly and quarterly: and as the business of these payments at compound interest, is best performed by the Logarithms, I shall only give a few examples, in the several cases, at simple interest.

5. If 30l. (a) half yearly rent be forborn 10 years, what will be in arrear at that time, at $4\frac{1}{2}$ per cent. simple interest?

Here $n=20$ the number of payments, and $r=,0225$ [$\frac{1}{2}$ of ,045],

Then, $m = 30\times 2+\overline{19\times 30\times ,0225\times 10} = 728,25$l. $=728$l 5s. the arrear sought.

6. If 15l (a) quarterly rent be forborn 10 years, what will be in arrear at that time, at 4 per cent. simple interest?

Here

Here $n=40$, and $r=,01125$ ($\frac{1}{4}$ of ,045), then

$m=\overline{15\times2+39\times15\times,01125}\times20=73$l. 12s. 6d. the arrear ſought.

7. If 300l. half yearly penſion be forborn 5 years, what will it amount to at 4 per cent. ſimple intereſt?
Anſw. 3270l.

8. If 150l. quarterly penſion be forborn 5 years, what will it amount to at 4 per cent. ſimple intereſt?
Anſw. 3285l.

QUESTIONS *to exerciſe Theorem 2d, of ſimple and compound Intereſt.*

1. What yearly rent being forborn 10 years (n), will amount to 721,5l. at ſimple intereſt, and to 737,2933l. at compound, at $4\frac{1}{2}$ per cent?

1ſt. $a=721,5\times2\div\overline{2+10\times,045-,045}\times10=60$l. the yearly rent, by ſimple intereſt. Or by Tab. III. The amount of 1l. annuity is 12,025; then $a=721,5\div12,025=60$l. the rent as before.

2dly, $a=737,2933\times,045\div\overline{1,045}\,\rceil^{10}-1=60$l. the rent, by compound intereſt. Or by Table IV. The amt. of 1l. annuity is 12,28821; then 737,2926 $\div$ 12,28821 $=60$ the rent, as before.

2. What yearly rent being forborn 16 years, at $3\frac{1}{2}$ per cent. will amount to 2525l. at ſimple intereſt, and to 2621,378751. at compound? Anſw. 125l. yearly rent.

3. What yearly penſion being forborn 24 years, at 5 per cent. will amount to 12096l. at ſimple intereſt, and to 14240,64l. at compound? Anſw. 320l. yearly.

4. What half yearly rent forborn 10 years, at $4\frac{1}{2}$ per cent. will amount to 728,25l at ſimple intereſt?

$a=728.25\times2\div\overline{2+20\times.0225-.0225}\times40=30$l. the half yearly's rent.

5. What

5. What quarterly penfion forborn 10 years will amount to 731 675l. at $4\frac{1}{2}$ per cent. fimple intereft ?

$a = 731{,}675 \times 2 \div \overline{2 + 40 \times {,}01125 - .01125} \times 40 = 15$l. the quarterly penfion fought.

6. What half yearly penfion forborn 5 years, will amount to 3270l. at 4 per cent. fimple intereft ?
Anfwer 300l. half yearly.

7 What quarterly annuity forborn 5 years, will amount to 3285l. at 4 per cent. fimple intereft ? Anfw. 150l. quarterly.

QUESTIONS *to exercife Theorem 3d. of both fimple and compound intereft.*

1. In what time (n) will 60l. (a) yearly rent, at $4\frac{1}{2}$d. per cent. amount to 721.5l (m) at fimple intereft, and to 737.2933l. (m) at compound ?

Firft $z = \frac{1}{r} - \frac{1}{2} = 21{,}72$ &c. $zz = 471.845284$; then

$n = \sqrt{\overline{721.5 \times 2 \div {,}045 \times 60}, + 471.845284} - 21{,}72 =$ 10 years the time fought : by fimple intereft. Or thus, fay,

	rent	amount		rent		amount
As	60l. :	721,5	::	1 :		12,025l. the amount of 1l.

annuity ; which being found in col. $4\frac{1}{2}$ per cent Tab. III. ftands over againft 10 years the time fought.

2dly $x^n = \overline{m \times r \div a}, + 1 = \overline{737.2933 \times .045 \div 60}, + 1 =$ 1.55297, which is fuch a power of 1.045 (x) whofe index is the number of years. Therefore 1.55297 (x^n) being continually divided by 1,045 (x) till nothing remains, the number of thofe divifions will be n=10, the years fought ; by compound intereft. Or, fince 1,55297 is the amount of 1l. for the required time, at compound intereft, feek it in Tab. II col $4\frac{1}{2}$ per C. and it ftands over againft 10 years the time fought.

2 In what time will 125l. yearly rent, at $3\frac{1}{2}$ per cent. amount to 2525l at fimple intereft and to 262?,37875l. at compound intereft ? Anfw. In 16 years.

4. In

3. In what time will 50l. yearly pension at 3 per cent. amount to 2152,5l. at simple interest, and to 2378,7705l. at compound ? Answ. in 30 years.

4. In what time will 300l. (a) half yearly pension, amount to 3270l (m), at 4 per cent. simple interest ?

First r=,02, and $z=\frac{1}{r}-\frac{1}{2}=49{,}5$, then per Theo. $n=\sqrt{\overline{3270\times2\div300\times{,}02,+49{,}5\times49{,}5}}-49{,}5=10$ half years, =5 years the time sought.

5. In what time will 150l. quarterly rent amount to 3285l. at 4 per cent. simple interest ?

Here r=.01 and $z=\frac{1}{r}-\frac{1}{2}=99{,}5$ then per Theorem.

$n=\sqrt{\overline{3285\times2\div50\times{,}01,+99{,}5\times99{,}5}}-99{,}5=$ 20 quarters=5 years the time sought.

6. In what time will 30l. half yearly pension, amount to 728,25l. at $4\frac{1}{2}$ per cent. simple interest ? Answ. in 10 years.

7. In what time will 15l. quarterly rent amount to 731,675l. at $4\frac{1}{2}$ per cent. simple interest. Answ. in 10 years.

QUESTIONS *to exercise Theorem 4th, both simple and compound interest.*

1. At what rate will 60l. (a) rent, in 10 (n) years amt. to 721.5l (m) at simple interest, and to 737.2933l. (m) at compound ?

First $r=\overline{721.5\times2-60\times10\times2}\div\overline{10\times10-10}\,,\times60=$,045, then $100\times{,}045=4\frac{1}{2}$ per cent. By simple interest.

Or say, as 60 rent : 721,5 amt. :: 1 rent : 12,025 the amount of 1l. annual rent. Seek this number over-against 10 years in Tab. III. and it is found under $4\frac{1}{2}$ per C.

2dly. Say as 60 rent : 737,2933 amt :: 1 rent : 12,28821 the amount of 1l annual rent at compound intereſt. Seek this number over-againſt 10 years in Table IV. and it ſtands under $4\frac{1}{2}$ per C. the rate ſought.

2. At what rate will 125l. rent amount to 2525l. at ſimple intereſt, and to 2621,37875l. at compound, in 16 years? Anſw. At $3\frac{1}{2}$ per cent.

III. *The preſent value of Annuities, &c. Or*

RULES for finding the diſcount, &c. in buying and ſelling of Annuities, Penſions and Leaſes in Reverſion, &c.

1. *Computed at ſimple Intereſt.*

Let $v =$ preſent value, $a =$ annuity, $n =$ time, $r =$ intereſt of 1l. Hence have theſe four Theorems; which ſolves the queſtions appertaining to this head.

1. $v = na + \frac{1}{2}\overline{n^2 - n} \times ar \div rn + 1$, when a, r, n, are given:

2. $a = \overline{2 + 2nr} \times v \div \overline{2 + nr - r} \times n$, when v, r, n, are given.

3. $n = \sqrt{\overline{2v \div ar}, + z^2} - z$; (putting $z = \frac{1}{r} - \frac{v}{a} - \frac{1}{2}$) when v, r, a, are given.

4. $r = 2na - 2v \div \overline{2v + a - na} \times n$, when v, n, a, are given

2. *Computed at compound Intereſt.*

Put $x = r + 1$ the amount of the annuity for a year. Hence the four following Theorems will ſolve the queſtions appertaining to this head.

1. $v = \dfrac{a - \overline{a \div x^n}}{r}$, when a, x, n, are given.

2. $a = vr \div \overline{1 - 1 \div x^n}$, when, v, x, n, are given.

3. $x^n = a \div \overline{a - vr}$, or $n = \log.a - \overline{\log.a - vr} \div \log.x$, when v, x, a, are given.

4. $x^n + \frac{a}{v} x^n - x' + 1 = \frac{a}{v}$, when v, a, n, are given, from which equation x and r may be found. But this cannot be done but by an Algebraic procefs; or by help of the Tables.

To find, by the Tables, the prefent worth of 1*l. annuity to continue a certain number of years, at a given rate either fimple or compound Intereft.*

Look into the Tables I. and III. for fimple intereft; or II. and IV. for compound intereft. And under the given rate, and againft the number of years (if within the limit of the Tables) in both Tables, you'll find two numbers, which take out and divide the latter by the former, for the prefent worth.

Ex. 1. What is the prefent worth of 1l. annuity to continue 14 years, at 5 per cent fimple and compound intereft?

Numb. Tab. I. is 1.7 Numb. Tab. III. is 18,55; then $18.55 \div 1.7 = 10.91176$. the prefent worth at fimple intereft.

Numb. Tab. II. is 1.97993, Numb. Tab. IV. is 19,59863, then $19{,}59863 \div 1{,}97993 = 9{,}89865$ the prefent worth at comp. int.

Now having found the prefent worth of 1l. annuity for any number of years at any given rate per cent. the prefent worth of any other annuity (whether fimple or compound) may be found, thus;

As 1l. annuity : is to its prefent worth :: fo is any other annuity : to its prefent worth.

QUESTIONS *to exercife Theorem 1ft. fimple and compound.*

1. A field is let upon leafe for 18 (n) years, at 4l. (a) per annum, and the leffee is defirous to make prefent payment, provided the leffer will allow him 5 per cent. how much muft be paid down, at fimple or compound intereft?

1st. $v=\overline{4\times18+\frac{1}{2}\,18\times18-18\times4\times,05}\div\overline{18\times,05+1}$ $=54$l. at simple interest.

Or thus, numb. in Tab. I. is 1 9. numb. in Tab. III. is 25,65, then $25\,65\div1,9=13.5$l the present worth of 1l annuity for 18 years at 5 per cent. Now as 1l. : 13, 5 : : 4l. : 54l, the present worth at simple interest as before.

2dly. $v=\overline{4-4\div2,40662}\div,05=46,7583$l.$=46$l. 15s. 2d the present worth at compound interest. Or thus, number in Tab. II is, 2,40662, numb. in Tab. IV. is 28,13238d then $28,13238\div2,40662=11,689581$ the present worth of 1l annuity for the given time and rate at compound interest. Now as 1l. : 11,68958 : : 4l. : 46,75832l. the present worth as above.

2. What is the present worth of 100l. to continue 7 years at 4 per cent simple and compound interest?

Answ. At simple interest 612l. 10s. and at compound interest 600l. 4s.

3 How much present money must be paid for a field let for 18 years at 4l. per annum, half yearly payments, allowing 5 per cent. simple interest?

Here a=2l. n=36 payments, and =,025 then

$v=\overline{2\times36+\frac{1}{2}\,36\times36-36\times2,\times025}\div\overline{36\times,025+1}=$ $54\frac{2}{19}$l. the present worth required.

4 What is the present worth of 600l. per annum, payable quarterly, for 5 years at 4 per cent. simple interest?

Answ. 2737l. 10s.

QUESTIONS *to exercise Theorem 2d, simple and compound.*

1. What annuity to continue 18 years (n), will 54l. (v) ready money purchase at 5 per cent. simple interest, and 46.75832l. (v) at compound interest?

1st.

1st. $a=\overline{2+18\times2\times,05}\times54\div\overline{2+18\times,05-},05\times18=4$l. the year's rent required. Or by the Tables I. and III. find the present worth of 1l. annuity, which is 13,5l. then as 13,5l. : 1 :: 54l : 4l. as above.

2dly. $r=,05$, $x=1,05$ and $x^{18}=2,40662$ (see Tab. II. under 5 per C. and against 7 years), and $1-\overline{1\div2,40662}$ $=,5845$ then $a=46,75832\times,05\div,5845=4$l. the yearly rent required, by compound interest.

Or by the Tab. II. and IV. find the present worth of 1l. annuity, which is 11,689581.

pr. w. ann. pr. w.
Then as 11,68958 : 1 :: 46,75832 : 4 annuity.

2. What annuity (or yearly rent) to continue 7 years at 4 per cent. will 612,5l. ready money purchase, allowing simple interest, and 600,2l. allowing compound interest? Answ. 100l. a year.

3. There is a field let upon lease for 18 years to come, payable half yearly, what is the half year's rent, the present worth at 5 per cent. simple interest being $54\frac{9}{19}$ l.? Answ. 2l. the half year's rent.

4. What quarterly pension to continue 5 years, will 2737l. 10s. ready money purchase, at 4 per cent. simple interest? Answ. 150l. quarterly pension.

QUESTIONS *to exercise Theorem 3d. simple and compound.*

1. How long may one have a lease of 100l. (a) a year, for 612,5l. (v) ready money at 4 per cent. simple interest, or for 600,2l. (v) at compound interest?

Here $r=,04$, and $z=\frac{1}{r}-\frac{v}{a}-\frac{1}{2}=18,375$, then per

Theo. $n=\sqrt{\overline{612,5\times2\div100\times,04}+18,375\times18,375}$ $-18,375=7$ years, the time required.

rent pre. w. rent
Or thus, as 100 : 612,5 :: 1 : 6,125l. the present worth of 1l. annuity for an unknown time. Then take

A a some

ſome year by gueſs, and find the amount by Tab. III. and the preſent worth of that amount by Tab. I If this agree not with 6,125l. try again, and by a few eaſy trials you'll come to the truth. In ſhort thus, ſet down the correſponding numbers in Tab. III. and I fractionwiſe, in order to approach continually to 6,125l. which at laſt you'll obtain.

Suppoſe 3 years $\frac{3,12}{1,12}$ = 2,6 &c. too little.

5 years $\frac{5,4}{1,2}$ = 4,5 too little.

7 years $\frac{7,84}{1,28}$ = 6,125 juſt. So 7 years is the time required.

And in like manner you may proceed with Tab. IV, and II. for compound intereſt. Or as follows, by the Theorem.

$x^n = 100 \div \overline{100 - 600,2 \times ,04} = 1,31593$ the amount of 1l. at compound intereſt. Seek this number in Tab. II. col. 4 per C. and you'll find it againſt 7 years, the time ſought.

2. If an annuity of 4l. a year, at 5 per cent. be ſold for 54l ready money, at ſimple intereſt, or for 46,7583l at compound intereſt, what is the time of its continuance?

Anſw. 18 years.

3. A leaſe of 4l. per an. payable half yearly is ſold for $54\frac{2}{19}$l. at 5 per cent. ſimple intereſt, what is the time of continuance, or number of payments? Anſw. 18 years.

4. An annuity of 600l. per annum payable quarterly, is ſold for 2737l 10s. ready money, allowing 4 per cent. ſimple intereſt, what was the time of continuance?

Anſw. 5 years.

QUESTIONS *to exerciſe Theorem 4th. ſimple and compound.*

1. At what rate per cent. will an annuity of 100l. (a) a year produce 612,5l (v) preſent worth, at ſimple intereſt, and 600,2l. at compound, in 7 years (n)?

r=

$r = \overline{100 \times 7 - 612,5 \times 2} \div \overline{612,5 \times 2 + 100 - 100 \times 7]} \times 7$ $= ,04$, therefore the rate is 4 per cent. simple interest.

Again, say, as 100 ann. : 600,2 pr. :: 1 ann. : 6,002, the present worth of 1l. annuity, at an unknown rate of compound interest. Take some rate of interest by guess and find the amount for 7 years by Tab. IV. and the present worth of that amount by Tab. II. repeat this work with other rates, till the result be 6,002. Or in short thus, set down the correspondent numbers in Tab. IV. and II. fractionwise, and you'll approach to the rate sought by a few trials. Thus,

Suppose 3 per cent. $\dfrac{7,6}{1,2} = 6,3$, too great.

$3\frac{1}{2}$ per cent. $\dfrac{7,77}{1,27} = 6,1$, too great,

4 per cent. $\dfrac{7,89829}{1,31593} = 6,002$, just.

Therfeore 4 per cent. is the rate required, by compound interest.

2. At what rate per cent. will 4l. yearly pension produce 54l present worth at simple interest, and 46l. 15s. 2d. at compound interest, for 18 years? Answ. At 5 per cent.

3. An annuity of 4l. per annum payable half yearly, having 18 years to come, is sold for $54\frac{2}{79}$l. what is the rate per cent simple interest? Answ. 5 per cent.

Note, The questions relating to annuities, leases, &c. taken in reversion, are solved (in all the 4 cases) by the application of the Theorems relating to the interest of money, and those concerning the present worth of annuities, both simple and compound. As for example (in the first case).

What is the present value of the reversion of a lease of 100l. *per annum to continue* 7 *years, but not to commence till the end of* 5 *years, allowing* 4 *per cent. simple or compound interest?* By Theorem 1st. page 269.

$v = \overline{100 \times 7 + 7 \times 7 - 7 \div 2 \times 100 \times ,04 \div 1} + 7 \times ,04 =$ 612,5l. the present worth of the annuity for 7 years, viz. the time of its continuance.

And by Theo. 2. (page 258) $p = 612,5 \div \overline{5 \times ,04 + 1} =$ 510,416l. &c. the present value required; by simple interest.

Again, $r = ,04$; $x = 1,04$; $x^n = x^7 = 1,31593$; and $x^5 = 1,21665$; as found in Table II. Then by Theorem 1. page 269.

$v = \overline{100 - 100 \div 1,31593} \div ,04 = 600,2$l. the present value of the lease for 7 years, the time of its continuance. Now find what sum put out for 5 years at 4 per cent. will amount to 600,2l. By Theo 2. (page 259.)
$p = 600,2 \div 1,21665 = 493,346481$ the present value required; by compound interest.

I think it needless to give examples in the other three cases, as it would but only be treading the old path over again, as may be seen by the example above.

IV. *The valuation of Freehold Estates.*

RULE.

WHEN Freehold Estates are to be valued; divide 1 by (r) the rate of 1l. the quotient shews how many years purchase it is worth, at compound interest.

Or if the annuity or rent be required; multiply the purchase money by (r) the rate of 1l. for the annuity.

EXAMPLES.

1. What is an estate of 60l. a year worth at 5 per cent?

Here $1 \div ,05 = 20$ years purchase.

Or $60 \times 20 = 1200$l. the purchase money.

2. What

2. What annuity can I buy for 1200l. at 5 per cent ? Here $1200, \times ,05 = 60$l. the annuity.

3. If 60l. (a) a year can be bought for 1200l. (v) ; what's the rate of interest (r) ? Here $r = a \div v = 60 \div 1200 = ,05$ and $,05 \times 100 = 5$, the rate per cent.

To find the value of a Freehold Estate, in Reversion, at compound Interest.

Theorem 1. $v = \frac{a}{r} \div x^n$, when a, r, n, are given.

2. $a = v \times r \times x^n$, when v, r, and n, are given.

3. $x^n = a \div vr$, and $n = x^n \div x$. when r, a, v, are given.

4. $x^{n+1} - x^n = a \div v$. when n, a, v, are given. From which equation x may be found and then r : but this cannot be done, but by an algebraic process.

Question 1. If a freehold estate of 60,5l. (a) per annum, to commence 10 years hence, is to be sold, what is it worth allowing the purchaser 5 per cent. for present payment ? Here x^n or $x^{10} = 1,6289$, and $n = 10$, then per Theo. 1.

$$v = \frac{60,5}{,05} \div 1,62889 = 742,83715\text{l. the value required.}$$

Question 2. If a freehold estate to commence 10 years hence (n) is sold for 742,83715l. (v), allowing the purchaser 5 per cent. What is the yearly rent ?

Theo. 2. $a = 742,83715 \times ,05 \times 1,62889 = 60,5$l. the yearly rent.

3. A freehold estate of 60,5l (a) per annum, being in reversion, is sold for 742,83715l. (v), at 5 per cent ; how long must the purchaser be before he enters upon it ?

$x^n = 60,5 \div 742,83715 \times ,05 = 1,62889$, being such a power of $x = 1,05$; whose index is the number of years : Therefore $x^n \div x = 10$ years, the time required.

S C H O L I U M.

It is contrary to law to let out money at compound interest. Yet in the valuation of annuities, it is always the custom to allow compound interest ; for by simple interest they would be over-valued.

TAB-

TABLE I.

A Table of the amount of 1l. for Years, at simple interest.

Years.	3. per C.	3½ per C.	4 per C.	4½ per C.	5 per C.
1	1,03	1,035	1,04	1,045	1,05
2	1,06	1,070	1,08	1,090	1,10
3	1,09	1,105	1,12	1,135	1,15
4	1,12	1,140	1,16	1,180	1,20
5	1,15	1,175	1,20	1,225	1,25
6	1,18	1,210	1,24	1,270	1,30
7	1,21	1,245	1,28	1,315	1,35
8	1,24	1,280	1,32	1,360	1,40
9	1,27	1,315	1,36	1,405	1,45
10	1,30	1,350	1,40	1,450	1,50
11	1,33	1,385	1,44	1,495	1,55
12	1,36	1,420	1,48	1,540	1,60
13	1,39	1,455	1,52	1,585	1,65
14	1,42	1,490	1,56	1,630	1,70
15	1,45	1,525	1,60	1,675	1,75
16	1,48	1,560	1,64	1,720	1,80
17	1,51	1,595	1,68	1,765	1,85
18	1,54	1,630	1,72	1,810	1,90
19	1,57	1,665	1,76	1,855	1,95
20	1,60	1,700	1,80	1,900	2,00
21	1,63	1,735	1,84	1,945	2,05
22	1,66	1,770	1,88	1,990	2,10
23	1,69	1,805	1,92	2,035	2,15
24	1,72	1,840	1,96	2,080	2,20
25	1,75	1,875	2,00	2,125	2,25
26	1,78	1,910	2,04	2,170	2,30
27	1,81	1,945	2,08	2,215	2,35
28	1,84	1,980	2,12	2,260	2,40
29	1,87	2,015	2,16	2,305	2,45
30	1,90	2,050	2,20	2,350	2,50

TAB.

TABLE II:

A Table of the amount of 1l. for years, at compound interest.

Years.	3 per C.	$3\frac{1}{2}$ per C.	4 per C.	$4\frac{1}{2}$ per C.	5 per C.
1	1,03000	1,03500	1,04000	1,04500	1,05000
2	1,06090	1,07122	1,08160	1,09202	1,10250
3	1,09273	1,10872	1,12486	1,14116	1,15762
4	1,12551	1,14752	1,16986	1,19252	1,21550
5	1,15927	1,18769	1,21665	1,24618	1,27628
6	1,19405	1,22925	1,26532	1,30226	1,34009
7	1,22987	1,27228	1,31593	1,36086	1,40710
8	1,26677	1,31681	1,36857	1,42210	1,47745
9	1,30477	1,36290	1,42331	1,48609	1,55132
10	1,34391	1,41060	1,48024	1,55297	1,62889
11	1,38423	1,45997	1,53945	1,62285	1,71034
12	1,42576	1,51107	1,60103	1,69588	1,79585
13	1,46853	1,56395	1,66507	1,77219	1,88565
14	1,51259	1,61869	1,73167	1,85194	1,97993
15	1,55797	1,67535	1,80094	1,93528	2,07893
16	1,60470	1,73398	1,87298	2,02237	2,18287
17	1,65285	1,79467	1,94790	2,11338	2,29202
18	1,70243	1,85749	2,02582	2,20848	2,40662
19	1,75350	1,92250	2,10685	2,30786	2,52695
20	1,80611	1,98979	2,19112	2,41171	2,65330
21	1,86029	2,05943	2,27877	2,52024	2,78596
22	1,91610	2,13151	2,36992	2,63365	2,92526
23	1,97359	2,20611	2,46471	2,75216	3,07152
24	2,03279	2,28333	2,56330	2,87601	3,22510
25	2,09378	2,36324	2,66583	3,00543	3,38635
26	2,15659	2,44596	2,77247	3,14068	3,55567
27	2,22129	2,53157	2,88337	3,28201	3,73345
28	2,28793	2,62017	2,99870	3,42970	3,92013
29	2,35656	2,71188	3,11865	3,58403	4,11613
39	2,42726	2,80679	3,24340	3,74532	4,32194

TAB-

TABLE III.

A Table of the amount of 1 l. annuity for yea[illegible]s, at simp. interest.

Years	3 per C	3½ per C	4 per C	4½ per C	5 per C
1	1,00	1,000	1,00	1,000	1,00
2	2,03	2,035	2,04	2,045	2,05
3	3,09	4,105	3,12	3,135	3,15
4	4,18	3,210	4,24	4,270	4,30
5	5,30	5,350	5,40	5,450	5,50
6	6,45	6,525	6,60	6,675	6,75
7	7,6[illegible]	7,735	7,84	7,945	8,0
8	8,84	8,980	9,12	9,260	9,40
9	10,08	10,260	10,44	10,620	10,80
10	11,35	11,575	11,80	12,025	12,25
11	12,65	12,925	13,20	13,475	13,75
12	13,98	14,310	14,64	14,970	15,30
13	15,34	15,730	16,12	16,510	16,90
14	16,73	17,185	17,64	18,095	18,55
15	18,15	18,675	19,20	19,725	20,25
16	19,60	20,200	20,80	21,400	22,00
17	21,08	21,750	22,44	23,120	23,80
18	22,59	23,355	24 12	24,885	25,65
19	24,13	24 935	25,84	26,695	27,55
20	25,70	26,650	27,60	28,550	29,50
21	27,30	28,350	29,40	3[illegible],450	31,50
22	28,93	30,085	31,[illegible]4	32 395	33,55
23	30,59	31,855	33,1[illegible]	34,3[illegible]5	35,65
24	32,28	33,660	35,[illegible]	36,420	37,80
25	34,00	35,500	3[illegible],00	38,500	40,00
26	35,75	37 375	39,00	40,625	42,25
27	37,53	39,285	41,04	42,795	44,55
28	39,34	41,[illegible]30	43,12	45,010	46,90
29	41,18	43,2[illegible]0	45,24	47,270	49,30
30	43,05	45,225	47,4[illegible]	49 575	51,75

TAB.

TABLE IV.

A Table of the amount of 1l. annuity for years, at comp. intereft.

Ys	3 per C.	$3\frac{1}{2}$ per C.	4 per C.	$4\frac{1}{2}$ per C.	5 per C.
1	1,00000	1,00000	1,00000	1,00000	1,00000
2	2,03000	2,03500	2,04000	2,04500	2,05000
3	3,09090	3,10622	4,12160	3,13702	3,15250
4	4 18363	4,21494	3,24646	4,27819	4,31012
5	5,30913	5,36246	5,41632	5,47071	5,52563
6	6,46841	6,55015	6,63297	6,71689	6,8019
7	7 66242	7,77941	7,89829	8,01915	8,14201
8	8,89233	9,05169	9,21422	9,38001	9,54911
9	10,15910	10,36849	10,58279	10,80211	11,02656
10	11,46388	11,73139	12,00611	12,28821	12,57789
11	12,80779	13,14199	13,48635	13,84118	14,20679
12	14,19203	14,60196	15,02580	15,46403	15,91713
13	15,61779	16,11303	16,62684	17,15991	17,71298
14	17,08632	17,67698	18,29191	18,93211	19,59863
15	18,59891	19,29568	20,02359	20,78405	21,57856
16	20.15688	20,97103	21,82453	22 71934	23,65749
17	21,76159	22,70501	23,69751	24 74171	25,84036
18	23,41443	24,49969	25,6454	26,85508	28,13238
19	25,11687	26,35718	27,67123	29,06356	30,53900
20	26,87037	28,27968	29,77808	31,37142	33,06595
21	28,67648	30,26947	31,96920	33,78314	35,73925
22	30,53678	32,32890	34,24797	36,30338	38,50521
23	32,45288	34,46041	36,61789	38,93703	41,43047
24	34,42647	36,66653	39,08260	41,68919	44,50200
25	36,45926	38.94986	41,64591	44,56521	47,72710
26	38,55304	41.31310	44,31174	47,57064	1,11345
27	40,70963	43,75906	47,08421	50,71132	54,66912
28	42,93092	46,29063	49,96758	53,99333	58,40258
29	45,21885	48,91080	52,96628	57,42303	62,32271
30	47,57541	51,62263	56,08494	61,00707	66,43885

§ XXVIII.

§ XXVIII. *The Rule of False Position.*

THIS rule takes its name from the method made use of to discover the true answer to the question propounded, viz from guessing, or the making a supposition of false numbers as if they were the true ones, and by their means discovering the true numbers sought.

This rule consists of two parts, single and double: of which I shall treat distinctly, and

I. OF SINGLE POSITION.

Where one false position sufficiently enables us to resolve the question proposed by the following RULE.

Suppose some fit number, at pleasure, and try according to the tenor of the question whether your supposed number be true or false: if true no more's to be done; but if false, you must observe what the false result or conclusion is Then say by the rule of Three. As the false conclusion : Is to the true number given :: So is the whole or any part of the false number: To the whole or respective part of the true number sought.

Thus, let a = the absolute number given, x = any supposed number; s = the sum produced by x, when ordered according to the tenor of the question.

Then s : a :: x : ax ÷ s = the number sought.

QUESTIONS.

1. A man dying left 1000l. amongst his three sons A, B, and C; in such proportion, that A's part shou'd be one sixth of B's part, and B's part two thirds of C's part, what is the share of each?

Suppose A to have 100l. then B must have 600l. and C 900l. by the tenor of the question. Now 100 + 600 + 900 = 1600l. the false conclusion.

			l.	s.	
Then 1600 : 1000 ::	100	:	62	10	A's share.
	600	:	375	0	B's share.
	900	:	562	10	C's share.
	Proof		1000.		

2. There

2. There were 50 corporals, 40 ſerjeants, 36 enſigns, and 24 adjutants, who ſpent 380 ſhillings at a meeting; to which reckoning 5 corporals paid as much as 4 ſerjeants, 12 ſerjeants as much as 9 enſigns, and 6 enſigns as much as 8 adjutants; how much did each company pay?

Find 4 numbers to expreſs the above proportions.

Thus, 12 ſerj. : 9 enſ. :: 4 ſerj. : 3 enſ. = 5 corporals.
6 enſ. : 8 adj. :: 3 enſ. : 4 adj = 5 corp.

That is, 5 corporals paid as much as 4 ſerjeants, or 3 enſigns, or 4 adjutants. Suppoſe each company paid 1 ſhilling,

then 1 man in each company will pay $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{4}$
which ×ed by the number of men 50, 40, 36, 24

produces 10, 10, 12, 6
and 10+10+12+6=38; then it will be

$$\begin{array}{l} \text{As } 38 : 380 :: \\ \text{Or } 1 : 10 :: \end{array} \left\{ \begin{array}{lll} 10 : 100s & \text{for the} & \text{corporals.} \\ 10 : 100s. & \text{———} & \text{ſerjeants.} \\ 12 : 120 & \text{———} & \text{enſigns.} \\ 6 : 60 & \text{———} & \text{adjutants.} \end{array} \right.$$

proof 380

3. What number is that which ×ed by 8, and the product ÷ed by 5, will leave the quote 48? Anſw. 30.

4 A man being asked what number of guineas he had about him, reply'd, if I had as many, half as many, and one quarter as many more, I ſhould have 220; how many had he? Anſw. 80.

5. A poulterer laid out 1l. 12s 6d. in fowls conſiſting of turkeys, geeſe and ducks; he gave 2s 6d. a piece for the turkeys, 1s. 8d. a piece for the geeſe, and 9d. a piece for the ducks; moreover he had twice ſo many geeſe as turkeys, and thrice ſo many ducks as geeſe; how many had he of each ſort? Anſw. 3 turkeys, 6 geeſe, and 18 ducks.

II.

II. *Of Double Position.*

IN this rule we make two suppositions of false numbers, in order to find out the true number which answers the question.

RULES.

1. Suppose two fit numbers, and proceed with each of them (as with the true one) according as the conditions of the question directs; if either of the numbers thus arbitrarily taken, happens to solve the queston, there needs no more to be done, but if not; find the errors, that is, find how much the results are different from the result in the question, and observe whether they are in excess or defect.

When the errors are in excess, mark them with +, when in defect with −

Multiply alternately the first supposed number by the second error, and the second supposed number by the first error. And divide the sum of the products by the sum of the errors, when the errors are of different kinds, (that is, when one is greater and the other less than the truth;) or ÷ the difference of the products by the difference of the errors, when both errors are of one kind, (that is both too great, or both too little) and the quote will be the answer.

Thus, let P, p, be the two suppositions, E, e, their respective errors, when ordered according to the tenor of the question; $\backsim$ the difference or excess; and n = the number sought. Then, $n=\frac{Pe+pE}{E+e}$, when the errors are of different kinds; or $n=\frac{Pe \backsim pE}{E \backsim e}$, when both errors are of one kind.

2. Or, having found the errors, say, as the sum of the errors, if they are unlike, or the diff. if like signs : is to the diff. of the supposed numbers :: so is the least error : to the correction of the supposition belonging to this error; which must be added to, or subtracted from it, according to the following directions.

Observe

Obſerve whether this be the leſs or greater ſuppoſed number, as alſo whether the errors have like or unlike ſigns.

If it is the leſs number, and the errors have like ſigns, ſubtract the correction; if unlike ſigns, add it.

If it be the greater number, and the errors have like ſigns, add the correction; if unlike ſigns, ſubtract it; and the ſum, or difference, will be the true number ſought.

Note. It is often of advantage to make 1 and 0 the two ſuppoſitions.

QUESTIONS.

1. What number is that which being ×ed by 21, the product increaſed by 28, and that ſum divided by 7, will leave the quote 120?

First, ſuppoſe $40=P$, to be the number ſought; then $\overline{40\times21+28}\div7=124$, the reſult; but it ought to have been 120; therefore the error is $+4=E$, in exceſs.

Again, ſuppoſe $30=p$, to be the number ſought; then $\overline{30\times21+28}\div7=94$, the reſult; but it ought to have been 120; therefore the error is $-26=e$, in defect; the errors alſo are of different kinds.

Whence by 1ſt Rule, $n=\dfrac{Pe+pE}{E+e}=\dfrac{40\times26+30\times4}{4+26}$ $=38\frac{2}{3}$ the number ſought.

Or, 2dly thus, $26+4=30$ the ſum of the errors, $40-30=10$ the difference of the ſuppoſitions; and 40 is the greater ſuppoſed number, to which the leſs error (4) belongs; alſo the errors have unlike ſigns:

Then, as $30:10::4:1\frac{1}{3}$ the correction; therefore $40-1\frac{1}{3}=38\frac{2}{3}$ the number ſought, as before.

But to work this by the Note, ſuppoſe, firſt, $0=P$; then $\overline{0\times21+28}\div7=4$, the reſult; but ought to be 120; therefore $120-4=-116=E$, the error, in defect. Again, ſuppoſe $1=p$; then $\overline{1\times21+28}\div7=7$, the re-

 ſult;

sult; but it should have been 120; therefore the error is 113=e, in defect also, and the errors have like signs. Whence, by the first Rule,

$$n = \frac{Pe \backsim pE}{E \backsim e} = \frac{0 \times 113 \backsim 116}{113 \backsim 116} = \frac{116}{3} = 38\tfrac{2}{3}, \text{ the}$$

number sought.

And by the 2d. Rule, $116 - 113 : 1 - 0 :: 113 : 37\frac{2}{3}$ the correction; then $37\frac{2}{3} + 1 = 38\frac{2}{3}$, the number sought.

2. A labourer was hired for 90 days, on condition that he should have 12d. for every day he wrought, and forfeit 8d. for every day he idled. At last he received 3l. 5s.=780d. for his labour. How many days did he work, and how many was he idle?

		d.
1. Suppose he wrought 50 (P) days.	$50 \times 12 =$	600
Then he idled 40 days.	$40 \times 8 =$	320
received but		280
instead of 3l. 5s.		780
1st. error (E) in defect.		−500
2. Suppose he wrought 80 (p) days,	$80 \times 12 =$	960
Then he idled 10.	$10 \times 8 =$	80
then he would receive		880
instead of		780
2d. error (e) in excess		+100

1. Rule. $n = \dfrac{P \times e + p \times E}{E + e} = \dfrac{50 \times 100 + 80 \times 500}{50 + 100} = 75$ days he wrought.

2. Rule. As $500 + 100 : 80 - 50 :: 100 : 5$, the correction; then $80 - 5 = 75$ days he wrought, consequently he idled 15 days.

3. Let

3. Let a hogshead be filled with two sorts of wine; the one worth 6s. and the other 5s. per gallon; I demand how much of each sort of wine the hogshead was filled with, it being (when full) worth 16l. 14s. ?

Answ. 19 gallons at 6s. and 44 at 5s.

4. A gentleman would pay a debt of 480l. with guineas, each 21s, and jacobuses, each 23s. and would pay 5 times so many guineas as jacobuses; how many then of each sort must he pay? Answ. 375 gui. and 75 jacobuses.

5. Two persons A and B engage at play; A has 72 guineas and B 52 before they begin; and after a certain number of games won and lost between them, A rises with three times as many guineas as B: I demand how many guineas A won of B? Answ. 21.

6 A man meeting a company of beggars, gave to each 4 pence, and has 16 pence over; but if he would have given them 6 pence a piece, he would have wanted 12 pence for that purpose: I demand the number of persons?

Answ. 14.

7. Two persons A and B were talking of their money, says A to B, give me 5s. of your money, and I shall have just as much as you will have left; says B to A, rather give me 5s. of your money, and I shall have just three times as much as you will have left: how much money had each? Answ. A had 15s. and B 25s.

8. A man being at play, lost $\frac{1}{4}$ of his money, and then won 3s. after which he lost $\frac{1}{3}$ of what he then had, and won 2s. lastly, he lost $\frac{1}{7}$ of what he then had; this done, he had but 12s. left: what had he at first? Answ. 20s.

9. A trader maintained himself for 3 years at the expence of 50l. a year; and in each of those years augmented that part of his stock, which was not so expended by $\frac{1}{3}$ thereof; at the end of the third year his original stock was doubled: what had he at first? Answ. 740l.

10. A and B play at cards; A stakes B 8s. to 6s. every game. After 28 games they leave off play, and find that neither of them are winners: how many games did each win? Answ. A won 16 games, and B 12.

11. Required a number, which ſeverally multiplied by 14, and 19; the difference of the two products may be 60? Anſw. 12.

12. If the number 18 be divided into two ſuch parts, that the difference of their ſquares may be 10 times the ſaid number 18, what are the parts required?

Anſw. 14 and 4.

13. A governor of a fort ſends out *one third* of his ſoldiers, and 25 over, and has then left $\frac{1}{2}$ his ſoldiers and 100 over: how many ſoldiers had he at firſt? Anſw. 750.

C H A P. III.

§ I. *Geometrical Definitions.*

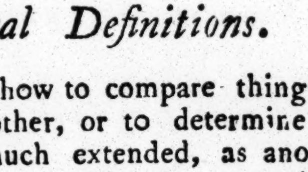
1. BY Geometry we learn how to compare things extended with one another, or to determine how one is more or leſs, or as much extended, as another. Or it is that ſcience wherein we conſider the properties of bodies as they conſiſt of three dimenſions, namely, length, breadth, and thickneſs; which may all be conceived to take their riſe from a point.

2. *A Point* is that in which we conſider neither length, breadth, nor thickneſs; and is abſolutely indiviſible.

3. *A right line* (or ſtraight line) is length without breadth, being the neareſt diſtance between two points, or places, as AB.

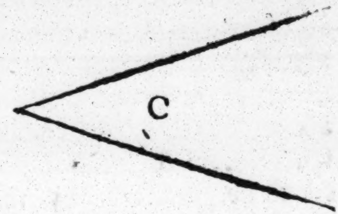
A  B

4. *A curve line*, is that which lies bending between thoſe points which limit its length, as C D. There are various ſorts of curves. A line (whether ſtraight or curved) is generated by the motion of a point.

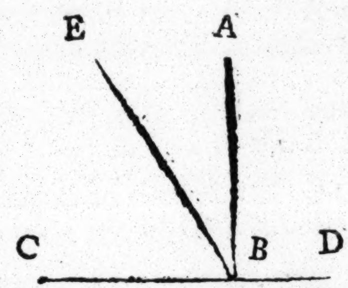
C  D

5. *A*

5. *A right lined angle* is that which is formed by the inclination of two right lines, meeting each other in a point, as C.

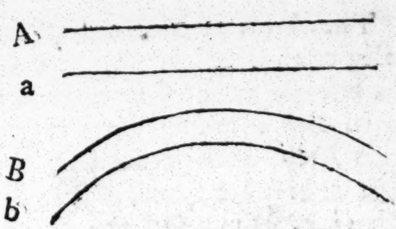


6. There are three ſorts of right lined angles.—1. When one right line A B ſtands any where upon another C D, ſo as to incline no more towards one end than the other, making the angles on both ſides A B equal, then thoſe angles are called right angles; and the two right lines, A B and C D, are then ſaid to be perpendicular to each other.—2. When the angle (EBD), is greater than a right angle (ABD), it is called an Obtuſe-angle.—3. If the angle (EBC) is leſs than a right angle (ABC), it is called an Acute-angle.



Note. When an angle is denoted by three letters (ABC), the middle letter ſtands at the angular point, and the other two at the extremeties of the lines which form the angle: Thus in the preceding definition, the letter B is the angular point of the right, obtuſe, and acute angles, there ſpecified.

7. *Parallel lines*, whether ſtraight or curved, are ſuch as are equally diſtant in all their parts, though infinitely extended; as Aa, or Bb.



OF PLANES, and SURFACES.

8. A *figure* is either a ſurface (viz. a ſuperficies) or a ſolid.

9. A *Plane surface* is any figure which lies evenly between its extreams, or bounds; and if those extreams, or bounds, are right-lines, the figure is called a Rectineal (or right-lined) plane; but if the extreams, or bounds, of a plane, are crooked, or curve-lined, the figure is then called a Curvilineal surface, or plane. A surface may be conceived to be generated by the motion of a line.

Of SOLIDS.

10. A Solid is that which has three dimensions, viz. length, breadth, and thickness:—The figure of a solid may be conceived to be generated either by the motion of a plane (or surface) in some certain direction, or by the revolution of a plane round some line as an axis.

11. The bounds or extreams of a solid, are either plain or curved surfaces.

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§ II. *MENSURATION.*

TO write a complete Treatise of Mensuration would make a Volume of itself: therefore my intent here, is to treat of such things as are generally useful in common Business, with a design to give the Learner a small insight only, into this Affair, and to excite his curiosity to look into Authors that have treated this useful and delightful Subject more at large.

I. Of SUPERFICIAL MEASURE.

The Area, or Measure, of any plane surface, geometrically considered, is the whole Space contained, under the bounds of the figure, without any regard to thickness; as in the Mensuration of Land, Painters Work, &c. This Area, or Superficial Content of the space, is computed from another space, of a determinate form and magnitude; i. e. from a square whose side is one inch, foot, yard, chain, &c. called the Measuring Unit, and the number of such squares, or units (and parts of an unit), that are contained in any plane figure, is called the content or measure of that figure.

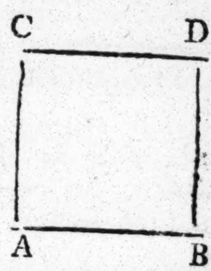
PRO-

PROBLEMS.

I. *To measure a square* ABCD.

Definition. A geometrical square is a figure consisting of four equal sides, and as many right angles.

RULE. Multiply the side of the square by itself, and the product will be the area in the same measure the dimensions were taken in.



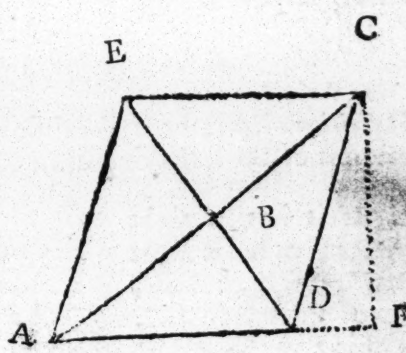
Ex. 1. Suppose the side of the square $AB=50$ inches; required its area in feet.

Operation. $50\times50=2500$ the area in inches; then $2500\div144=17,36$ the area in feet.

2. Suppose the side of a square be 21,269 inches, what is its area in feet? Answ. 3.1413 feet.

2. *To measure a Rhombus*, ADCE.

Definition. A Rhombus has four equal sides; and four angles, the opposite angles equal; two being obtuse, and two acute.



Rule. Multiply the two diagonals (AC and DE) together, and half the product will be the area. Or $\times$ the side by a perpendicular let fall from an angle thereon (if within the figure, or on the side produced if without) and the product will be the area.

Ex. Suppose the diagonal $AC=200$, and $ED=120$ yards; required the area of the Rhombus.

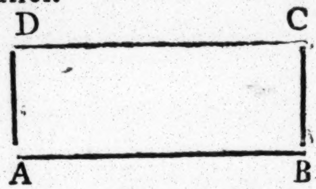
Operation. $120\times200\div2=12000$ the area Or, suppose the side $AD=114$, and perpend. $CF=105,263$; then

then 105,2632 × 114 = 12000,0048 the area as before, nearly.

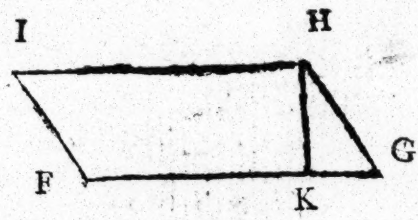
3. *To measure a Parallelogram.*

Definition. A parallelogram is a four ſided figure, having its oppoſite ſides equal and parallel.

Rule. If it be a rectangled parallelogram, as ABCD: multiply the longeſt ſide by the ſhorteſt; if otherwiſe as FGHI (called by ſome a Rhomboides); then × the longeſt ſide FG, by the perpendicular HK, and the product will be the area.



Ex. Suppoſe the longeſt ſide AB (or FG) = 180, the ſhorteſt ſide BC (or perpend. HK) = 60; required the area.
Oper. 180 × 60 = 10800 the area, required.



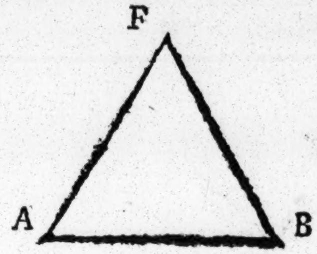
4. *To measure a Triangle.*

Definition. All three ſided figures are called triangles but admit of the following diſtinctions;

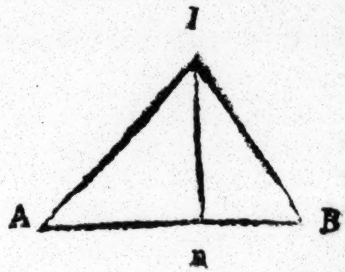
1. If it has a right angle it is called a right angled triangle as ABC, the two ſides AB, BC, containing the right angle, are called legs; the ſide AC oppoſite the right angle is called the hypothenuſe.

2. If

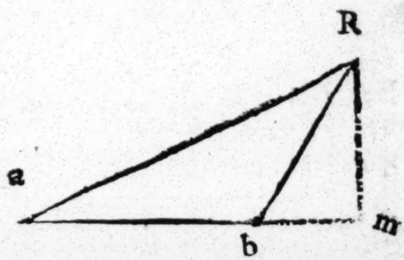
2. If the three ſides are equal it is called an equilateral triangle, as ABF.



3. If only two ſides are equal it is called an Iſoſceles triangle, as ABI.



4. If the three ſides are all unequal, it is called a ſcalene triangle, as abR. They are all meaſured by the ſame rule.



Rule. From any one of the angles let fall a perpendicular upon the ſide oppoſite (produced if needful); multiply the ſide and perpend. together, and half the product will be the area.

Ex. The baſe AB of the triangle ABC (or AIB or abR) is 90, and perpend. CB (or, In Rm) is 60; what is the area? Operation $90 \times 60 \div 2 = 2700$ the area required.

To find the area of a plane triangle by having the three ſides given. This is of great ſervice in Land Surveying.

RULE. From half the ſum of the three ſides, ſubtract each ſide ſeverally, let the ſaid half ſum, and the three differences, be $\times$ed continually; the ſquare root of the product will be the area required. Ex-

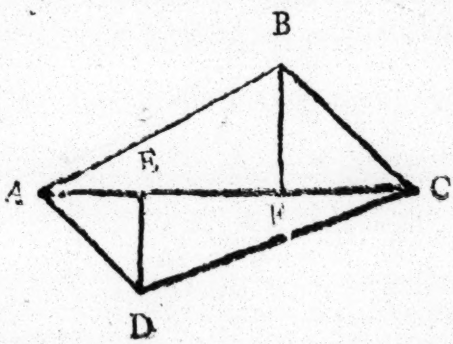
Examp. If the sides of a triangular garden be 41, 29, and 56 yards: what is the content of the garden?

Operation. $41+29+56=126$ the sum; $\frac{1}{2}$ sum $=63$.

And $63-41=22$, 1st difference, $63-29=34$, 2d diff. $63-56=7$, 3d diff. then $63\times22\times34\times7=329868$, the product, whose square root is 574,34, the area required.

5. *To measure a Trapezium.*

Definition. A Trapezium is any figure that has 4 unequal sides, as ABCD, in which the line AC is called a *diagonal*, BF and DE are perpendiculars. This Trapezium in fact is no more than two triangles ABC, and ADC, made by the diagonal AC.



Rule. Multiply the diagonal by the sum of the perpendiculars, and half the product will be the area.

Ex. Suppose the diagonal AC=120, and the perpendiculars DE and BF to be 40 and 50 respectively; required the area.

Operation $\overline{50+40}\times120\div2=4590$ the area, required.

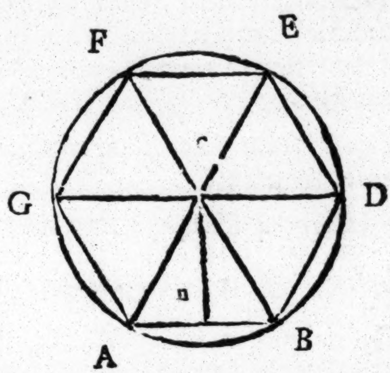
Note. The measure of any irregular figure (or polygon) may be easily found. Divide the whole figure into triangles, trapeziums, and curves; then find the area of each; add the areas together, and the sum will be the area of the whole figure (or polygon). To find the area of a curved-space. See page 55.

6. *To measure any regular Polygon.*

Definition. Any plane figure bounded by more than four right lines, is called a polygon; and is named according to the number of sides it contains: Thus if it has 5 sides it is called a pentagon; if six sides, a hexagon; if seven, a heptagon, &c. If all the sides and angles are equal, as in the figure (ABCDEFG), it is called a regular polygon; if otherwise, it is called an irregular polygon. Every regular polygon is composed of so many isosceles triangles as the polygon contains sides. Rule.

Rule. Find the area of one of the triangles (by prob. 4), and multiply that area by the number of triangles: Or × half the ſum of the ſides by a line drawn from the middle of any one of the ſides to the center of the polygon, and the product will be the area.

Ex. In the hexagon ABCDEFG, if one of its ſides AB (BD &c.) be 56, the perpendicular Cn=48,49; what is the area?



Operation. 48,49 × 56 ÷ 2 = 1357,72 the area of one triangle; then 1357,72 × 6 = 8146,32 the area of the polygon.

Note. The radius of a circle inſcribed in any regular polygon is equal to the perpendicular of it, let fall from the center to the middle of the ſide of the polygon. Now by plane Trigonometry a Table may be calculated for the more ready meaſuring of regular polygons; which Table I ſhall here ſubjoin; expreſſing the angles at the center, length of the perpendicular (or radius of the circle inſcribed in it) and area.

The names of the Polygons.	Number of ſides.	The angle at the center		The perpendicular.	The area. The ſide of the Polygon being 1.
Pentagon	5	72	00	0,6881910	1,7204774
Hexagon	6	60	00	0,8660254	2,5980762
Heptagon	7	51	26	1,0382617	3,6339124
Octagon	8	45	00	1,2071068	4,8284271
Nonagon	9	40	00	1,3737387	6,1818242
Decagon	10	36	00	1,5388418	7,6942088
Undecagon	11	32	44	1,7028437	9,3656404
Duodecagon	12	30	00	1,8660254	11,1961524

The

The use of the preceding Table.

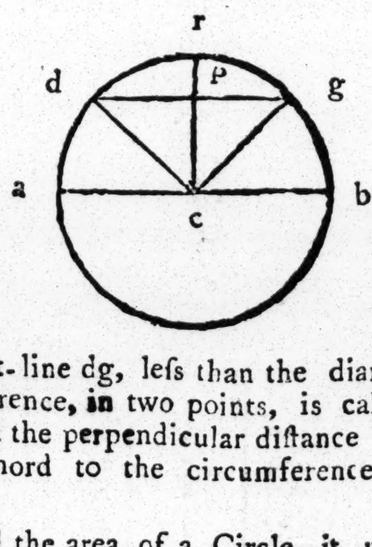
It is well known to Geometricians, that the areas of similar (or like) plane figures are in proportion to one another, as the ſquare of their correſponding ſides: Therefore multiply the ſquare of the ſide of a given regular Polygon by ſuch a number taken out of the above table, as is agreeable to the name of the Polygon, and the product will be the area thereof, in the ſame denomination as the given ſide.

Ex. If the ſide of a regular Octagon be 12 feet; what is its area?

Operation. $4.8284271 \times 12 \times 12 = 695.2935024$ the area.

7. *To Meaſure a Circle.*

Definition. A Circle is a plane figure, bounded by one continued line, called the circumference, or periphery; every part of which is equally diſtant from a point within the Circle, called its Center; from which, any right line (ac, cd, &c.) drawn to the circumference, is called the radius, or ſemi diameter of the Circle; any right line ab, drawn thro' the center, terminating each way at the circumference, is called a diameter; a right-line dg, leſs than the diameter, meeting the circumference, in two points, is called a chord, or ſubtenſe; and the perpendicular diſtance rp, from the middle of the chord to the circumference is called a verſed-ſine.



Before I proceed to find the area of a Circle, it will be neceſſary to ſhew the learner, how to find the circumference of a Circle, by having its diameter given, and the contrary.

It is now looked upon, even by Mathematicians of the firſt rank, as abſolutely impoſſible to determine the exact proportion of the diameter and circumference of a Circle.

That great Geometer Archimedes, about 2000 years ago, firſt diſcovered this proportion to be nearly as 7 to 22; that is, if the diameter of a Circle be 7, its circumference will be 22, very nearly. But 22 is too much.

Metius Snellius diſcovered the proportion to be nearly as 113 to 335; which is nearer the truth than Archimedes, but 335 is too great.

Since Archimedes's and Metius's time, various methods have been invented, whereby the ſaid proportion may be approximated to a very great degree of exactneſs.

Van Ceulen (a Dutchman) found by incredible pains, that if the diameter of a Circle be repreſented by 1, the circumference thereof will be

3,14159265358979323846264338327950288, extreamly near. This laſt number was not only confirmed, but was extended to double the number of decimal places, by that ingenious and moſt indefatigable Mathematician, the late Mr. Abraham Sharp, of Little Horton, near Bradford, in Yorkſhire. And Mr. John Machin, Profeſſor of Aſtronomy in *Greſham College*, and Secretary to the Royal Society, has carried it to 100 places. But in the ordinary practice of meaſuring, it will be unneceſſary to take any more than 3,14159 (or 3,1416): Hence it is evident, that if the diameter of any circle be multiplied by 3,1416, the product will be the circumference of that circle, very nearly.

Ex. What is the circumference of a circle, whoſe diameter is 50?

Operation. $3{,}1416 \times 50 = 157{,}08$ the circumference.

It is evident, from this example that if the circumference of a circle be divided by 3,1416, the quotient will be the diameter. For $157{,}08 \div 3{,}1416 = 50$ the diameter.

To find the area of a circle, by having the diameter and circumference given.

Rule. Multiply half the circumference by half the

 dia-

diameter, and the product will be the area. For every circle is equal to a rectangled parallelogram contained under half the circumference and half the diameter.

Ex. What is the area of a circle whose diameter is 1, and circumference 3,1416?

Operation. $\frac{1}{2}$ the circumference is 1,5708, and $\frac{1}{2}$ the diameter is .5 Then $1,5708 \times ,5 = ,7854$ the area of a circle whose diameter is 1, nearly.

Therefore if the square of the diameter of any circle be multiplied by ,7854, the product will be the area, or measure of the circle in that denomination whereby the diameter was expressed, whether inches, feet, yards, &c.

As for instance, suppose the diameter of a circle be 50 feet, the square whereof is 2500; then $,7854 \times 2500 = 1963,5$ the area sought, nearly.

The circumference of a circle being given to find the area.

Rule Multiply the square of the circumference by 0,0795776 $(= 1, \div 3,1416 \times 4)$ and the product will be the area.

Ex. If the circumference be 30, what is the area?

Operation. $0,0795776 \times 900 = 71,61984$ the area.

The area of a circle being given to find the diameter.

Rule. Multiply the square root of the area by 1,12837 $(= 2\sqrt{1 \div 3,1416})$ and the product is the diameter.

Ex. What is the diameter of a circle whose area is 2500?

Operation. $1,12837 \times \sqrt{2500} = 56,4185$ the diameter.

The diameter of a circle being given to find the side of the greatest inscribed square.

Rule. Multiply the given diameter by ,707 $(= \sqrt{\frac{1}{2}})$, and the product will be the side of the square required.

Ex. If the diameter of a circle be 50; what is the side of the inscribed square? Answ. 35,35

8. *To measure the sector of a Circle.*

Definition. A Sector of a Circle *drgc*, is a figure contained

tained by an arch (or arc) thereof, and two radius's; when these two radius's form a right angle, or the arch becomes $\frac{1}{4}$th of the circumference, the figure is called a quadrant, as *adrc*, (or *ergb*, see the last figure).

Rule. Multiply half the length of the arch, by the radius, and the product will be the area.

Ex. Let *drgc* represent a sector (or quadrant) of a circle, whose radius *dc* (or *gc*) is 45 feet, and length of the arch drg, 70,686 feet, required its area.

Operation. $45 \times \frac{1}{2} 70,686 = 1590,435$, the area.

By this proposition the area of the segment of a circle may be found: For if the area of the triangle *dgc* be subtracted from that of the sector *drgc*, there will remain the area of the segment *drgd*. As for instance, suppose the chord *dg* $=63,63$ feet, and the perpendicular *cp* $=$ $31,815$. Then $63,63 \times 31,815 \div 2 = 1012,194225$ the area of the triangle *dcg*; which taken from $1590,435$ (the area of the sector) leaves $578,240775$ the area of the segment (*drgd*). Or the area of the segment of a circle may be found by having the chord and versed sine given, by the following general rule.

Rule. Multiply the versed sine (or height) by ,626; to the square of the product add the square of half the chord: multiply twice the square root of the sum, by two thirds of the versed sine, and the product will be the area.

Ex. Required the area of the segment of a circle, whose chord is 48, and versed sine 18.

Opera. $18 \times ,626 = 11,268$, whose square is $126,967824$ which added to the square of half the chord (576), makes the sum $702,96782$; twice the square root of which, is $53,026$; which $\times$ed by two thirds of the height (12), the product is $636,312$ the area.

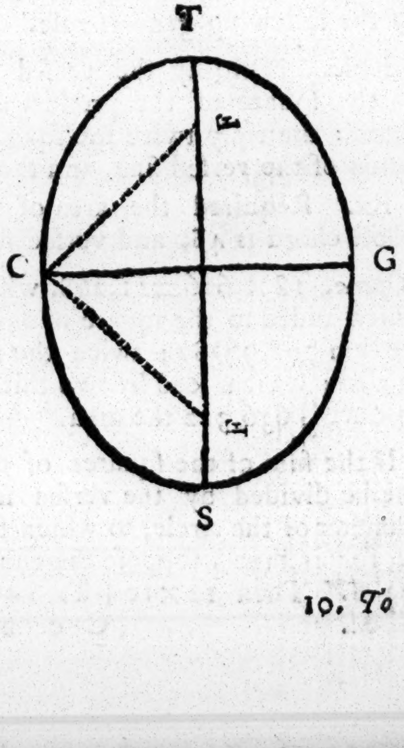
If the sum of the squares of the semi-chord and versed sine be divided by the versed sine, the quote will be the diameter of the circle, to which that segment corresponds. As for instance, suppose the chord be 24, and the versed sine 8. Then $12 \times 12 + 8 \times 8 \div 8 = 26$ the diameter required.

By having the chord and verſed ſine you may find the length of the circular arch: thus, find the diameter (as above), divide two 3ds of the verſed ſine, by the diameter leſſened by ,82 of the verſed ſine; to the quote add 1, and × that ſum by the chord, the product will be the length of the arch very nearly. For inſtance, ſuppoſe the chord of a circular arch be 40, and the verſed ſine 10. Then $\overline{20\times20+10\times10}\div10=50$ the diameter, now $10\times,82=$ 8,2, which taken from 50 leaves 41,8 the diviſor; by which divide 6,666 &c, (two thirds of 10) and the quote is ,15948; to which add 1, and the ſum is 1,15948; this ×ed by 40 gives 46,3792 the length required.

9. *To meaſure an Ellipſis, or Oval.*

Definition. An Ellipſis, commonly called an Oval, is a plane figure bounded by one continued curve line which returns into itſelf like the circle, but has two different diameters, one longer than the other. The longer diameter (TS) is called the Tranſverſe, the ſhorter (CG) the Conjugate.

Rule. Multiply the tranſverſe by the conjugate, and × that product by ,7854; the laſt product will be the area.



Exam. Suppoſe the tranſverſe (TS) = 70, and the conjugate (CG) = 50, what is the area?

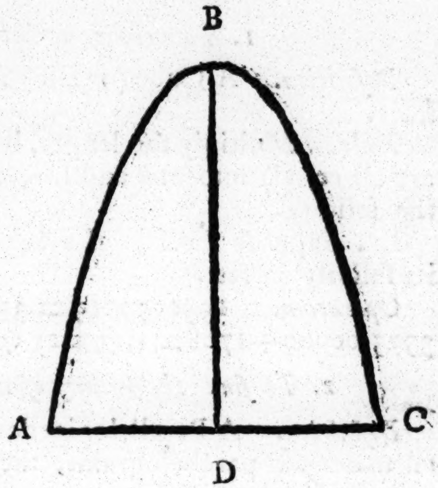
Operation. 70 × 50 × ,7854 = 2748,9, the area required.

10. *To*

10. *To measure a Parabola.*

Definition. A Parabola is a figure made by cutting a cone (see the definition of a cone farther on) by a plane parallel to one of the sides: but the figure will give the learner a better idea than any definition.

Rule. Multiply the base (or ordinate AC) by the perpendicular (or abscissa BD); and two thirds of the product will be the area. For every conical parabola, is $\frac{2}{3}$d of its circumscribing parallelogram.



Ex. Let the base of the parabola, AC = 30, and perpendicular BD = 40; required the area

Operation. $30 \times 40 = 1200$, two thirds of which is 800, the area.

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§ III. *Of* SOLID MEASURE.

THE measure of every solid figure is computed from another solid, of a determinate form and magnitude; namely, from a cube whose side is one inch, foot, yard, &c. called the measuring unit; and the number of such cubes, or units, (and parts of an unit) that any solid is found to contain, is called the measure, or content, of the solid: therefore when the measure of any solid figure in any denomination is known, it may be changed into any other denomination by the rules in reduction.

Since all solid figures are but obscurely delineated upon a plane superficies, I shall not lay down any schemes here to re-

present them; but rather advise the Master (who may have occasion to teach this) to get each solia made of wood in its true form, to delineate and explain the figures intended by each problem, in order to assist his Pupil in a true conception of what he is about.

PROBLEMS.

1. *To measure a Cube (or Die).*

Definition. A Cube (or Die) is a solid having six equal sides.

Rule. Multiply the length, breadth, and depth (which are all equal) into one another, and the product will be the solidity.

Ex. Suppose the side of a cube be 15 inches; required its solidity in feet.

Operation. $15 \times 15 \times 15 = 3375$ cubic inches: then $3375,00000 \div 1728 = 1,953125$ cubic foot.

2. *To find the solidity of a Parallelopipedon.*

Definition. A Parallelopipedon is a solid figure contained under six parallelograms, the opposite sides of which are equal and parallel.

Rule. Multiply the length by the breadth, and that product by the depth (or altitude), the last product will be the solidity.

Exam. 1. The length of rectangled parallelopipedon is 72 feet, the breadth 30, and depth 20: required the solidity.

Operation. $72 \times 30 \times 20 = 43200$ the solidity required.

Exam. 2. What is the solidity of a piece of timber whose length is $30\frac{1}{2}$ feet, breadth 18 inches, and depth 15 inches?

Operation. $30,5 \times 1,5 \times 1,25 = 57,1875$ feet the solidity.

3. *To find the solidity of a Prism.*

Definition. A Prism is a solid figure contained under several planes, its bases for the most part are regular polygons, (most commonly triangular,) and is of an equal thickness thro' out.

Rule.

Rule. Find the area of the baſe, or end, according to its form, multiply that area into the length, and the product is the ſolidity.

Exam. 1. What is the ſolidity of a triangular priſm, whoſe length is 28 feet, and one ſide of the equilateral end is $1\frac{1}{2}$ foot ?

Operation. $1,5\times3=4,5$ ſum of the ſides ; $\frac{1}{2}$ ſum 2,25 and $2,25-1,5=,75$ the 1ſt diff. and becauſe the ſides are equal, the other two will be the ſame; then $2,25\times,75\times,75\times,75=,94921875$, whoſe ſquare root is ,9742 the area of the end or baſe ; and $,9742\times28=27,2776$ feet the ſolidity required.

4. *To find the ſolidity of a Cylinder.*

Definition A Cylinder is a round body, like a round column, or a rolling ſtone of a garden, whoſe baſes are equal circles.

Rule. Multiply the area of the circular baſe (or end) by the length, and the product will be the area.

Exam. What is the ſolidity of a Cylinder, whoſe length is $3\frac{3}{4}$ feet, and the diameter of the end is $2\frac{1}{2}$ feet ?

Operation. $2,5\times2,5\times,7854=4,90865$ the area of the end, and $4,90865\times3,75=18,4074375$ feet the area.

5. *To find the ſolidity of a Pyramid.*

Definition. A Pyramid is a ſolid, which from a triangular or other figured baſe, diminiſhes gradually in thickneſs till it ends in a point, called the vertex.

Rule. Multiply the area of the baſe, or end, by the altitude, or length, and $\frac{1}{3}$ of the product will be the area. For every Pyramid is equal one third of its circumſcribing priſm.

Exam. 1. What is the ſolidity of a pyramid, whoſe height is 15 feet, and one ſide of its hexagonal baſe is $1\frac{1}{2}$ foot ?

Operation. The area of an hexagon, whoſe ſide is 1, is

is found to be 2,598076 (by the Tab. of polygons). And $1,5 \times 1,5 \times 2,598076 = 5,845671$, the area of the base. Then $5,845671 \times 15 \div 3 = 29,228355$, the solidity.

Exam. 2. What is the solidity of a pyramid, whose height is 72 feet; and the side of its square base 18 feet?

Answ. 8100 feet.

6. *To find the solidity of a Cone.*

Defin. A Cone is a solid, which, from a circular base, diminishes gradually in thickness till it ends in a point at top (or vertex), in the form of a round spire. A Cone may be conceived to be generated by the rotation of a right-angled triangle round one of its perpendicular legs.

Rule. Find the area of the cone's base (whether the diam. or circum. be given), multiply this area by the perpendicular altitude, and one third of the product will be the area. For every cone is equal to one third of its circumscribing cylinder.

Exam 1. What is the solidity of a cone, the diameter of whose base is 20 feet, and the altitude 60 feet?

Operation. $20 \times 20 \times ,7854 = 314,16$ the area of the base. And $314,16 \times 60 \div 3 = 6283,2$ feet the solidity required.

Exam. 2. If the circumference of the base of a cone be 30 feet, and the height 50 feet: what is the solidity?

Operation $30 \times 30 \times ,0795776 = 71,61984$ the area of the base. And $71,61984 \times 50 \div 3 = 119,3664$ feet the solidity.

7. *To find the convex superficies of a right cylinder, by having its length, and the diameter, or circumference given.*

Rule. Multiply the diameter by 3,1416, and the product by the length, will give the convex surface. Or, × the circumference by the length and the product will be the convex superficies.

Ex.

Exam. 1. What is the convex ſuperficies of a right cylinder whoſe diameter is 50 inches, and length 90 inches?

Operation. $3,1416 \times 50 \times 90 = 14137,2$ the convex ſuperficies.

Exam. 2. A cylinder is painted at 6d. per foot, whoſe length is 20 feet, and circumference 10 feet, what does it come to?

Operation. $20 \times 10 = 200$ the curve ſuperficies.
And $10 \times 10 \times ,07958 \times 2 = 15,916$ the area of both ends.
Then, $200 + 15,916 = 215,916$ feet, the whole ſuperficies; which at 6d. per foot, amounts to 5l 7s. $11\frac{1}{2}$d. the Anſw.

8. *The length of the ſlant ſide and the diameter, or circumference of the baſe of a right cone, being given, to find the convex ſuperficies.*

Rule multiply the $\left\{\begin{array}{l}\text{diam. by } 1,5738\ (\tfrac{1}{2} \text{ of } 3,1416) \\ \text{circum. by } 0,5\end{array}\right\}$ and the product $\times$ed by the length of the ſide, gives the convex ſuperficies.

Exam. 1. What is the convex ſuperficies of a right cone, the length of the ſlant ſide being 60, and the diameter of the baſe 22?

Operation. $1,5708 \times 22 \times 60 = 2073,456$, the convex ſuperficies.

Exam. 2. What is the convex ſuperficies of a right cone, whoſe length of the ſlant ſide is 50 feet, and the circumference of whoſe baſe is 60 feet?

Operation. $60 \times ,5 \times 50 = 1500$ feet the convex ſuperficies.

9. *To find the ſolidity of the Fruſtum of a Cone (or pyramid of any kind.)*

Definition. The Fruſtum of a Cone, or pyramid, is what remains (towards the baſe) when the top is cut off by a plain parallel to the baſe.

Rule. To the ſum of the areas of the two ends of the fruſtum, add a geometrical mean between thoſe arrears, viz.

(the

(the ſquare root of their product, ſee page 231), multiply this ſum by one third of the altitude of the Fruſtum, the product will give the ſolidity.

Note. This rule holds good, if the ends of the fruſtum be of any other figure, than that of a regular polygon.

Exam. 1. Suppoſea ſquare pyramid's fruſtum, the ſide of whoſe greater baſe is 18 feet, the ſide of whoſe leſs is 6, and whoſe altitude is 60: What is the fruſtums ſolidity?

Operation. $6\times6+18\times18=360$ the ſum of the areas of the two ends; and $36\times324=11664$, the product of theſe areas, whoſe ſquare root is 108, a geometrical mean; then $360+108=468$ the ſum, which being $\times$ed by one third of the altitude (20) the product is 9360, the ſolidity required.

Exam. 2. What is the ſolidity of an hexagonal pyramid's fruſtum, the ſide of whoſe greater end is 3 feet, that of the leſs end 2 feet, and the length 12 feet.

$$\left.\begin{matrix}3\times3\\2\times2\end{matrix}\right\}\times2{,}598076\left\{\begin{matrix}=23{,}382684 \text{ area of the greater end}\\=10{,}392304 \text{ area of the leſs end.}\end{matrix}\right.$$

$$\sqrt{23{,}382\times10{,}392}\quad=15{,}588000 \text{ geom. mean.}$$

$$49{,}362988 \text{ ſum}$$

$$4=\tfrac{1}{3}\text{d the height.}$$

ſolidity required is 197,45·952

The ſolidity of the fruſtum of a cone, may be found by the above general rule, but more expeditiouſly by the following rule.

When the diameter or circumference at each end, and the altitude are given.

Rule 1. Multiply the $\left\{\begin{matrix}\text{diam.}\\\text{circum.}\end{matrix}\right\}$ at one end, by that at the other.

2. To the product add the ſquare of thoſe $\left\{\begin{matrix}\text{diameters.}\\\text{circumfer.}\end{matrix}\right.$

3. Multiply the ſum by the given length.

4. The

4. The product ×ed by the No $\left\{\begin{array}{l}0{,}2618\ (\tfrac{1}{3}\text{ of },7854)\\ 0{,}02656(\tfrac{1}{3}\text{ of },0795776\end{array}\right.$
will give the ſolidity.

Exam. 1. What is the ſolidity of the fruſtum of a cone, the diameter of the greater baſe being 20, that of the leſs 10, and the altitude 30?

Operation. $\overline{20\times10+20\times20+10\times10}\times30\times0{,}2618=$ 5497,8 the ſolidity required.

Exam. 2. What is the ſolidity of the fruſtum of a cone, the circumference of the greater end being 40, that of the leſs end 20, and the length or height 50?

Operation. $\overline{40\times20+40\times40+20\times20}\times50\times{,}026526$ $=3713{,}64$ the ſolidity required.

Exam. 3. There is a fruſtum of a cone (by ſome called a cylindroid), one end of which is a circle whoſe diameter is 26; the other parallel end is an ellipſis whoſe tranverſe diam. is 44, and conjugate 14; the altitude of the fruſtum is 9; required its ſolidity.

Operation. Firſt reduce the ellipſis to a circle, and then proceed according to the rule; thus, $\sqrt{\overline{44\times14}}=24{,}82$ nearly, the diam. of a circle,

Then $\overline{24{,}82\times26}+\overline{26\times26}+\overline{24{,}82\times24{,}82}\times9{,}2618=$ 4564,68981458 the ſolidity required.

10. *The ſolidity of the fruſtum of a rectangular pyramid (called a priſmoid), whoſe parallel ends are any diſſimilar rectangles, and whoſe ſides are four plane ſurfaces.*

The ſolidity of this figure may be obtained by the firſt of the preceding rules, or by the following.

Rule. To the longeſt (or ſhorteſt) ſide of the rectangle at either end, add that ſide at the other end (whether it be the length or breadth) which is parallel to it; multiply this ſum by the ſum of the other two parallel dimenſions

viz.

(viz one at the top, and the other at the bottom), and to the product add the areas of the two ends; this total being multiplied by the height, one ſixth of the product will be the ſolidity.

Ex. Suppoſe there is a fruſtum of a pyramid, whoſe parallel ends are rectangles, the length at one end being 40 inches, and breadth 34; the length and breadth of the other end (which in this caſe, are reſpectively parallel to thoſe above) being 50 and 46 inches, and the length 60 inches; required its ſolidity.

Operation. $40+50=90$; $34+46=80$; and $90\times80=7200$ the product; $32\times40=1360$ the area of the leſs baſe, $46\times50=2300$ the area of the greater baſe, and $7200+1360+2300=10860$ the total;

then $10860\times60\div6=108600$ cubic inches.
1728)108600,00(84,97 feet.
282)108600,00(385,1 ale gallons.
231)108600,00(470,12 wine gallons.

11. *To find the convex ſurface of the fruſtum of a right cone, the length of the ſlant ſide, and the diameters or circumferences of the parallel ends being given.*

Rule. Multiply the ſum of the diameters by 1,5708, or the ſum of the circumferences by 0,5, and that product × by the length of the ſide, and the laſt product will be the convex ſurface.

Exam. 1. What is the convex ſurface of a right cone, the diameters of the ends being 8 and 4; and the length of the ſide 20?

Operation. $8+4=12$ the ſum of the diameters; and $12\times1,5708\times20=376,992$ the convex ſuperficies.

Exam. 2. What is the convex ſurface of the fruſtum of a right cone; the circumference of the greater end being 30 feet, that of the leſs end 10 feet, and the length of the ſlant ſide 20 feet?

Operation. $\overline{30+10}\times,5\times20=400$ feet, the convex ſurface.

12. *The*

12. *The dimension of a Pyramid or Cone being given, to find what length from the vertex will answer to a given part of the solidity.*

Rule. Say as the solidity of the whole pyramid or cone : is to the cube of the altitude :: so is any given part of the solidity : *to the cube of its altitude, reckoned from the vertex downwards ; whose cube root is the length.*

Examp. There is a conical peice of timber, the diameter of whose base is 1,5 feet, and the length 12 feet ; what distance from the vertex must a saw be applied, to cut it into two pieces of equal solidity.

Operation. $1,5 \times 1,5 \times 12 \times ,2618 = 7,0686$ feet the solidity ; half whereof is 3,5343. And $12 \times 12 \times 12 = 1728$ the cube of the length. Then as 7,0686 : 1728 :: 3,5343 : 864, whose cube root is 9,5244 ; and so far from the vertex must the saw be applied.

13. *To find the solidity of the Hoofs of the frustum of a pyramid, the linear measures of the end, and the length being known.*

Definition. If the frustum of a pyramid be cut through the extremities of both bases by a diagonal plane, each part (or Section) is called a hoof.

If the frustum be of any other form than a square pyramid, reduce it to such, by finding the side of a square of equal area ; then

Rule. To the square of the side of one base, add one half the product of the sides of the two bases ; this sum $\times$ by the height, and one third of the product will be the solidity.

Examp. Given x = one side of the greater base of a square pyramid's frustum, y = one side of the less base, and a = the altitude, required the solidity of each hoof.

Operation. $\overline{xx + \frac{1}{2}xy} \times a \div 3 =$ the solidity of the greater hoof.

And $\overline{yy + \frac{1}{2}yx} \times a \div 3 =$ the solidity of the less hoof.

Now

Now ſuppoſe $x=1,5$; $y=1,25$, and $a=9$: then $1,5 \times 1,5=2,25=xx$ the ſquare of the greater ſide. And $1,5 \times 1,25 \div 2=0,9375=\frac{1}{2}xy$ half the product of the ſides: Then $2,25+0,9375 \times 9 \div 3=9,5625$ the ſolidity of the greater hoof. Again, $1,25 \times 1,25=1,5625=yy$, then $1,5625+0,9375 \times 9 \div 3=7,5$ the ſolidity of the leſs hoof.

Note, The ſolidity of the greater or leſs hoof will be obtained according to the end of the fruſtum uſed; as evidently appears by the example above.

14. *In a conical fruſtum, the diameters, and diſtance of the two parallel ends being given, to find the ſolidity of each hoof, when the fruſtum is cut by a diagonal plane.*

1. To find the greater hoof (S).

Rule. From the cube of the greater diameter (x), take the ſquare root of the product of the cubes of the two diameters. Divide the remainder by the difference of two diameters Multiply the quote by the altitude (a). The product multiplied by 0,2618 (n) (one 12th of 3,1416), will give the ſolidity.

Theo. $S=\dfrac{x^3-\sqrt{x^3 \times y^3}}{x-y} \times na$. putting $y=$ the leſs diameter.

2. To find the leſs hoof (s).

Rule. From the ſquare root of the products of the cubes of the two diameters, take the cube of the leſs diameter. Divide the remainder by the difference of the two diameters. Multiply the quote by the height. The product multiplied by 0,2618 will give the ſolidity.

Theo. $s=\dfrac{\sqrt{x^3 \times y^3}-y^3}{x-y} \times a\,n$

Examp. There is a conical fruſtum, the diameter of the greater end is 4; that of the leſs end 2; and the altitude 9; what is the ſolidity of each hoof?

Operation

Operation. $4\times4\times4=64$ the cube of the greater diameter, and $2\times2\times2=8$ the cube of the leſs diameter. Alſo $64\times8=512$; whoſe ſquare root is 22,6272.

Then $\frac{64-22,6272}{4-2}\times9\times0,2618=48,7413$ the ſolidity of the greater hoof.

And $\frac{22,6272-8}{4-2}\times9\times,2618=17,2323$ the ſolidity of the leſs hoof.

15. *Of a Sphere (or Globe).*

1. *The diameter of a Globe being given to find its ſuperficies.*

Rule. Multiply the ſquare of the diameter by 3,1416 ($=p$) and the product is the ſuperficies.

Exam. What is the ſuperficies of a ſphere whoſe diameter is 3?

Operation. $3\times3\times3,1416=28,2744$ the ſuperficies.

2. *The circumference of a circle biſecting the ſphere being given, to find its ſuperficies.*

Rule. Multiply the ſquare of the circumference by 0,31832, ($1\div p$) and the product is the ſuperficies.

Exam. What is the ſuperficies of a ſphere whoſe circumference is 9,4248?

Operation. $9,4248\times9,4248\times0,31832=28,2753645$ the ſuperficies.

3. *The diameter and circumference of a ſphere being given, to find its ſuperficies.*

Rule. Multiply the diameter by the circumference, and the product is the ſuperficies.

Exam. What is the ſuperficies of a ſphere, whoſe diameter is 3, and the circumference 9,4248?

Operation. $9,4248\times3=28,2744$ the ſuperficies.

4. *The diameter of a ſphere being given to find its ſolidity*

Rule. Multiply the cube of the diameter by ,5236 (=p÷6) and the product is the ſolidity.

Exam. What is the ſolidity of a ſphere whoſe diameter is 3?

Operation. $3\times3\times3\times0,5236=14,1372$ the ſolidity.

5. *The circumference of a circle biſecting the ſphere being given, to find its ſolidity.*

Rule. Multiply the cube of the circumference by 0,016887 (=1÷6pp) and the product is the ſolidity.

Exam. What is the ſolidity of a ſphere whoſe circumference is 9,4248?

Operation. $\overline{9,4248}|^3\times0,016887=14,1373$ &c. the ſolidity.

6. *The ſuperficies and diameter of a ſphere being given, to find the ſolidity.*

Rule. Multiply the ſuperficies by one ſixth of the diameter, and the product is the ſolidity.

Exam. What is the ſolidity of a ſphere, whoſe diameter is 3, and the ſuperficies 28,2744?

Operation. $28,2744\times3\div6=14,1372$ the ſolidity.

7. *If the ſuperficies and circumference of a ſphere be given to find the diameter.*

Rule. Multiply the ſuperficies by the circumference, and the product ×ed by 0,05305 (=p÷6) gives the ſolidity.

Exam. What is the ſolidity of a ſphere whoſe circumference is 9,4248, and the ſuperficies 28,2744?

Operation. $28,2744\times9,4248\times0,05305=14,1372$ the ſolidity.

Note. The ſolidity of a ſphere, is equal to two thirds of its circumſcribing cylinder.

8. *The*

8. *The ſolidity of a ſphere being given, to find the diameter.*

Rule. The cube root of the ſolidity ×ed by 1,2407 ($=\sqrt[3]{6 \div p}$) will give the diameter.

Exam. What is the diameter of that ſphere whoſe ſolidity is 14,1372?

Operation. $\sqrt[3]{14,1372} \times 1,2407 = 3$ the diameter.

16. *To find the ſolidity of the ſegment of a ſphere, the diameter of its baſe* (2x) *and altitude* (a) *being given.*

Definition The ſegment of a ſphere (or globe) is a part of it cut off by a plane; and therefore the baſe of ſuch a ſegment muſt be a circle; and its convex ſurface a part of the ſurface of the ſphere.

Rule. To thrice the ſquare of the ſemidiameter of the baſe, add the ſquare of the altitude, multiply the ſum by the height; then the product ×ed by 0,5236 (n) will give the ſolidity (s).

Theorem. $s = \overline{3xx + aa} \times a \times n$.

Exam. What is the ſolidity of a ſpherical ſegment, the diameter of whoſe baſe is 26, (=2x); and its height 12 (a)?

Operation. $s = \overline{13 \times 13 \times 3 + 12 \times 12} \times 12 \times 0,5236 =$ 4090,3632 the ſolidity.

17. *The diameter of the baſe* (x) *and the altitude* (a) *of ſpherical ſegment, being given, to find its convex ſuperficies* (s)

Rule. To the ſquare of the diameter of the baſe, add the ſquare of twice the height, the ſum ×ed by 0,7854 (n) will give the ſuperficies required.

Theorem. $s = \overline{xx + 4aa} \times n$.

Exam. What is the convex ſurface of that ſpherical ſegment, the diameter of whoſe baſe is 18 (=x), and whoſe altitude is $4\frac{1}{2}$ (=a)?

Operation. $18\times 18=324$; and $\overline{4{,}5\times 2}|^{2}=81$ the ſquare of twice the height. Then $\overline{324+81}\times 0{,}7854=$ 318,087 the ſuperficies required.

18. *To find the ſolidity of a ſpheroid, the axis of rotation, and the revolving axis being given,*

Definition. A ſpheroid is a ſolid figure, formed by the rotation of an ellipſis round either of its axis. If the rotation be about the tranſverſe axis the ſolid generated is called a prolate (or oblong) ſpheroid reſembling an egg, only both its ends are the ſame. But if the rotation be about the conjugate axis, the ſolid generated is called an oblate ſpheroid. Such a form the fig. of the earth is ſaid to be.

Rule Multiply the fixed axis by the ſquare of the revolving axis, the product ×ed by 0,5236 ($=p\div 6$) will give the ſolidity.

Note. A ſpheroid (as well as the ſphere) is two thirds of its circumſcribing cylinder.

Exam. 1 What is the ſolidity of a prolate ſpheroid, whoſe tranſverſe axis is 100, and its conjugate is 60 ?

Operation. $100\times 60\times 60\times 0.5236=188496$ the ſolidity.

Ex. 2. What is the ſolidity of an oblate ſpheroid, whoſe longeſt axis is 100, and ſhorteſt axis 60 ?

Operation. $60\times 100\times 100\times 0.5236=314160$ the ſolidity.

19. *To find the ſolidity of a parabolic conoid, the diameter of its baſe, and altitude being given.*

Definition. A parabolic conoid is a ſolid formed by the rotation of a ſemi-parabola about its axis. It is equal to half its circumſcribing cylinder.

Rule. Multiply the ſquare of the diameter of its baſe by .7854 ; then the product ×ed by half the height will give the ſolidity.

Ex. What is the ſolidity of a parabolic conoid, whoſe height is 50, and the diameter of whoſe baſe is 60 ?

Operation. $60\times 60\times ,7854\times 50\div 2=70686$ the ſolidity.

20. *To*

20. *To find the solidity of a Parabolic-Spindle.*

Definition. A parabolic-spindle is a solid generated by the rotation of a parabola about its ordinate; which ordinate will be the spindle's length, and the abscis of the parabola will be the radius of its greatest circle (or its $\frac{1}{2}$ breadth)

Rule. Multiply the square of the diameter (d) by the length (b), the product ×ed by 0,41838 (=n) will give the solidity (s). Theorem. $s = dd \times l \times n$.

Ex. Suppose the length of a parabolic-spindle, to be 60 (l). and the greatest diameter 40 (d): Required the solidity.

Operation. $40 \times 40 \times 60 \times 0.41888 = 40212.48$ the solidity.

Note. The usual method of measuring irregular solids, such as craggy stone, lumps of metal, bushes of shrubs, &c. is to fill a vessel with water, and then to put in the solid, and measure the water that flows over; as suppose you immerse a solid in a regular vessel full of water, and by such immersion 5 ale gallons run over the top of the vessel, then the content of this immersed solid is 282 multiplied by 5 is equal to 1410 solid inches. Or if you put the solid into a regular vessel, and fill the vessel up with water, then take out the solid and measure the empty part of the vessel, it will give the same content.

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§IV. *Of* BOARD MEASURE.

BOARDS are measured by superficial measure, the thickness not being regarded: So that a board is mostly a parallelogram (or may be reduced to such), the measuring of which is shewn in Sect. 2. Prob. 3. The usual method of measuring boards, is to take the length along the middle between the two sides, and breadth in the middle between the two ends. Then,

Rule. Multiply the length by the breadth and the product will be feet, if both the dimensions were taken in feet, or feet and decimal parts of a foot: but if the length was taken in feet and breadth in inches (which is mostly

the

the case), divide the said product by 12. Or if both was taken in inches, divide the product by 144, and the quote will be flat feet.

Note. In measuring a quantity of boards that have the same length, but different breadths, the best way is to take (with a string) all the breadths in one dimension, and multiply that by the length of one board. And in measuring a stock of boards that have the same length and breadth, find the content of one board and multiply that by the number of boards.

EXAMPLES.

1. If a board be 20 feet (or 240 inches) long, and 18 inches ($=1,5$ feet) broad; how many square or flat feet doth it contain?

Operation $\left\{\begin{array}{l} 20\times 1,5 = \\ 20\times 18\div 12 = \\ 240\times 18\div 144 = \end{array}\right\}$ 30 feet, the content required.

2. There is a parcel of boards, the sum of whose breadths, taken together, is 87,5 feet, and the length of each is 8,75 feet; how many square feet do they all contain?

Operation. $87,5\times 8.75=765.625$ square feet.

3. Suppose a stock of tapering boards, 25 in number, the length of each to be 16.6 feet, and breadth, taken in the middle, $10\frac{1}{2}$ inches; how many square feet does the said stock of boards contain?

Operation. $16.6\times 10.5\div 12=14\ 525$ feet, the content of one board. Then $14,525\times 25=363,125$, feet the content of the whole.

Note. If a board be irregular, that is, broader in some places than in others, take breadths at the broadest and narrowest places, add all those breadths together, and divide the sum by the number of breadths, and the quotient will be a mean breadth; which will reduce the board to the form of a parallelogram.

4. There is a board whose length is $15\frac{1}{2}$ feet, but irregular in breadth; therefore five breadths are taken which

are

are as follows: 1ſt. 11,5 inches, 2d. 11 inches; 3d. 11,25 inches; 4th. 13,5 inches, and 5th. 14,25 inches; how many ſquare feet doth it contain?

Operation. $11,5+11+11.25+13\,5+14\,25\div5=12,3$, the mean breadth.

Then $15,5\times12.3=190.65$ ſquare feet, the anſwer.

5. If a board be five inches broad, how much in length will make a ſuperficial foot?

Note. Since the product of the length and breadth gives the area, its evident that if the area be divided by one of thoſe dimenſions, the quotient will be the other.

Operation $144\div5=28\,8$ inches in length.

6. If a board be 18 inches broad, how much in length will make a ſquare foot? Anſw. 8 inches.

§ V. *Of the* Meaſuring *of* TIMBER.

THERE are various ſorts and forms of timber. If it is hewed, and the ends are ſquares or rectangles, and all of a thickneſs, it is a parallelopipedon. If it is thicker at one end than the other, it is the fruſtom of a pyramid. If it be round and all of the ſame thickneſs it is a cylendar; but if it be thicker at one end than the other, it is the fruſtum of a cone For the meaſuring of each ſee Section III. But there is a cuſtomary way of meaſuring both ſquare and round timber, which, tho' it be erroneous, is uſed by all dealers therein. Workmen, and meauſurers, uſually gird the tree in the middle with a cord, and take the 4th part of that compaſs or circumference, and eſtimate it as though it was the ſide of a ſquare, whoſe area is equal to the ſection of the tree, at the place it was girded.

It is a cuſtomary allowance to the buyer, to take the girt where he pleaſes between the greater end and middle of the tree.

PROBLEM I.

To compute the ſolidity (or content) of round timber, by having the length and circumference given.

CASE I. *When the tree is ſtreight, and the ends are equal, or nearly ſo.*

Rule.

Rule. Multiply the ſquare of one fourth the circumference by the length, and the product is the content in feet, if both the dimenſions be expreſſed in feet and decimal parts thereof; which is the beſt and eaſieſt way. But if one dimenſion be feet and the other inches, divide the ſaid product by 144, and the quote will be the ſolidity; or if both dimenſions are inches, divide the product by 1728, and the quote will be the ſolidity in feet.

EXAMPLES.

1. What is the ſolidity of a tree, whoſe compaſs is 36 inches, and length 20 feet?

Operation. $36 \div 4 = 9$ inches $= ,75$ foot;

Then, $,75 \times ,75 \times 20 = 11,25 = 11\frac{1}{4}$ feet the content required. Or $9 \times 9 \times 20 \div 144 = 11\frac{1}{4}$ feet.

2. What is the ſolidity of a tree whoſe length is $33\frac{1}{2}$ feet, and $\frac{1}{4}$ of the compaſs 17 inches?

Operation. $17 \times 17 \times 33,5 \div 144 = 67,23$ feet the content required.

3. What is the ſolid content of a piece of timber whoſe length is 204,5 inches; and $\frac{1}{4}$ of the circumference 20,5 inches?

Operation. $20,5 \times 20,5 \times 204,5 \div 1728 = 49,734$ feet the content required.

4. What is the ſolidity of a piece of timber, whoſe length is $9\frac{3}{4}$ feet, and $\frac{1}{4}$ the circumference 39 inches?

Anſw. 102,984 feet.

Note. If the tree is crooked, its length muſt not be meaſured on either the convex, or concave ſide of the curve.

CASE II. *When the tree is unequally thick, or taper.*

Rule. Gird the tree in as many places as are thought neceſſary: add the ſeveral girds together and divide that ſum by the number of girds, the quote will be (as thought by the workmen) a mean compaſs; the fourth of this mean compaſs ſquared and multiplied by the length, gives the ſolidity.

5. A

5. A tapering tree 20 feet long, is girded in four places; in the first it is 72 inches, in the second 60, in the third 52, and in the fourth 48 : what is the solidity in feet?

Operation. $72+60+52+48\div4=58$ a mean compass, and $58\div4=14,5$ inches; then $14,5\times14,5\times20\div144=29,2$ feet, the content required.

CASE III. *Any piece of wood that is 24 inches in compass (or 6 inches square) is esteemed timber; therefore if any branches or boughs measure 24 in compass, their solidity must be found and added to that of the tree.*

Note. So much of the trunk, as measures less than 24 inches compass, is not esteemed timber.

6. Required the solidity of a tree 40 feet long; 27 inches quarter compass; one branch 14 feet long by 28 inches circumference, and another 10 feet long, by 24 inches compass.

Operation. First, $27\div12=22,5$ feet, $28\div4=7$in. $=,583$ foot, and $24\div4=6$ in. $=,5$ foot: then $2,25\times2,25\times40=202,5$ feet in the tree, $,583\times,583\times14=3,35846$ feet in one branch, $,5\times,5\times10=2.5$ feet in the other branch. And $202,5+3,35846+2,5=208,35846$ the solidity required.

CASE IV. *When the trees have their bark on.*

In measuring such timber for sale, it is common to make an allowance to the buyer on account of the bark; thus, in oak, one tenth or one twelfth part of the circumference is deducted; but the allowance for the bark of ash, beach, elm, &c. is less.

Rule. From the given circumference, subtract the allowance for bark; and with the remaining compass, find the solidity, as before.

7. Required the solid content of an oak tree whose length is $44\frac{1}{2}$ feet, and 60 inches quarter compass; allowing one tenth for bark?

Operation. $60\div10=6$ the allowance, then $60-6=54=4,5$ feet, $\frac{1}{4}$th of which is 1,125 foot;

Then $1,125\times1,125\times44,5=56,32$ feet, the solidity.

PRO-

PROBLEM II.

To compute the solidity of square or flat timber, such as balks, or plank, &c.

Rule. Take the breadth and depth in the middle, if both ends be nearly of a thickness; but in tapering square (or flat) timber, take as many breadths and depths, as are thought necessary; from which find a mean breadth and depth as taught in Case II. Prob. I. Then multiply length, breadth and depth together, and the product will be the true solidity.

EXAMPLES.

1. If a piece of timber be 13.2 feet long, and 1,52 foot square, how many solid feet doth it contain?

Operation. $1,52 \times 1,52 \times 13.2 = 30,49728$ feet, the answ.

2. If a piece of timber be 30 feet long, 20 inches broad, and $15\frac{1}{2}$ inches deep, how many feet doth it contain?

Operation $20 \times 15,5 \times 30 \div 144 = 64,583$ feet the answ.

3. There are 5 planks, the length of each is 20,5 feet, breadth 1,2 foot, and depth 0,4 of a foot; how many solid feet of plank do they contain?

Operation. $1.2 \times 0.4 \times 20,5 = 9,84$ feet, the solidity of 1 plank; then $9,84 \times 5 = 49,2$ feet the answ.

Note. Workmen, and measurers, reckon 40 feet of unhewn, or rough timber, and 50 feet of hewn timber to a load, supposed to weigh a ton, or 20 cwt. For say they, hewn timber is measured by the square, and is very near exact; but rough timber by the girt, (or quarter compass,) which is about one fifth less than exact: therefore divide the feet in rough timber by 40, and in hewn (or sawn) timber by 50, and the quotient gives the loads (or tons).

To find how much in length is required to make any given solidity of timber. Because the product of the length, breadth, and depth, gives the content; therefore, if the given solidity be divided by the product of any two dimensions, the quotient will b the third.

Exam.

Exam. 1. A piece of ſquare timber whoſe ſide is 6 inches; how much in length will make 2 ſolid feet?

Operation. $6\times6=36$, and $1728\times2=3456$ cubic inches in 2 feet: Then $3456\div36=96$ in. $=8$ feet the Anſw.

Exam. 2. A piece of timber whoſe breadth is 9 inches, and depth 4; how much in length will make $1\frac{1}{2}$ ſolid feet?

Operation. $9\times4=36$, and 2592, the cubic inches in $1\frac{1}{2}$ feet: Then $2592\div36=72$ inches $=6$ feet the Anſw.

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§ VI. *Of* ARTIFICERS WORK.

ARTIFICERS have different ways of meaſuring according to the cuſtom of each craft; ſome give the content in feet, ſome in yards, ſome in ſquares, ſome in rods, &c.

They alſo differ according to the cuſtom of the place, ſome give the content in one denomination, and ſome in another.

Artificers ſeldom uſe ought but ſuperficial or flat meaſure, and becauſe *walls*, *doors*, *floors*, *windows*, *roofs*, &c. are *ſquares* or *parallelograms*, their areas are found by Prob. I. or III. Sect. II. but if any other form come in practice, it cannot but come under ſome of the rules in the foregoing ſections.

1. *Of Bricklayers Work.*

Bricklayers commonly meaſure their work by a rod or $16\frac{1}{2}$ feet: the ſquare of which is $272\frac{1}{4}$ feet: but in Practice, divide the area found by 272, the quotient of which are the rods required. In ſome places they meaſure by a rod of 18 feet; whoſe ſquare is 324 feet: and in others they meaſure by the rood, of 21 feet long, and 3 broad, containing $(21\times3=)63$ ſquare feet; and here they do not regard the wall's thickneſs, but moderate the price accordingly.

Note. Bricklayers mostly compute or value their work at the rate of a brick and an half thick, called standard thickness; and if the thickness of the wall happen to be more or less than standard, it must be reduced to standard thickness as follows:

Multiply the area of the wall, by the number of half bricks in the thickness of the wall; divide the product by 3, and it gives the area.

Exam, 1. How many square rods, of standard thickness, are there in a wall $72\frac{1}{2}$ feet long, $19\frac{1}{4}$ feet high, and $5\frac{1}{2}$ bricks thick? Answ. 18,8 rods.

First $72,5 \times 19,25 = 1395,625$ the superficial content; then $1395,625 \times 11 \div 3 = 5117,291$ feet the area in standard thickness; and $5117,291 \div 272 = 18,8$ rods the Answ.

Exam. 2. A wall being 72 feet long, 34 feet high, and $3\frac{1}{2}$ bricks thick, how many rods of brick work, at 18 feet the rod, are contained in it?

Here $72 \times 34 = 2448$, and $2448 \times 7 \div 3 = 5712$ the area in feet at standard thickness. Then $5712 \div 324 = 17$ rods, 204 feet, remaining; the Answ.

Exam. 3. A wall being 62 feet, 6 inches long, 14 feet, 9 inches high, how many roods of brick work, at 63 square feet or 7 square yards the rood, are contained in it? Answ. 14 roods, and 4,4305 yards.

First $14,75 \times 62,5 = 921,875$ square feet,

Then $921,875 \div 9 = 102,4305$ square yards, and $102 \div 7 = 14$ roods: 4,4305 yards remain.

Note. The usual way to take dimensions of a building is to measure half round it on the out-side, and half round it on the inside with a chord; which gives the true compass of the building. If it be all of a height, measure its height at any place from the bottom of the foundation; but if the height be unequal, take several heights, add them together, and divide their sum by the number of heights, and the quotient will be a mean height. The gable ends above the square being triangles, are measured as such.

Exam. 4. Suppose each side wall of an house be $30\frac{1}{2}$ feet long on the out side, each end wall $14\frac{1}{2}$ feet broad on the inside; and the height from the foundation to the square $55\frac{1}{2}$ feet; and the gable (or triangular part at top)

to

to riſe 42 courſe of bricks (reckoning 4 courſe to a foot). Now the firſt 20 feet in height is $2\frac{1}{2}$ bricks thick, the next 20 feet 2 bricks thick; $15\frac{1}{2}$ feet above $1\frac{1}{2}$ brick thick, and the gable end 1 brick thick; what will it amount to at 5l. 10s. per rod, of 272 ſquare feet, ſtandard thickneſs? Alſo how many ſquare yards, not minding the thickneſs, as if it were a ſtone building?

$42 \div 4 = 10,5$ the height of the gable, at each end.
$30\frac{1}{2} + 30\frac{1}{2} + 14\frac{1}{2} + 14\frac{1}{2} = 90$ the compaſs of the building.

$$\begin{array}{l|c|l}
 & & \text{ſquare feet} \\
14,5 \times 10,5 \times 2 \div 3 & & 101,5 \text{ at } 1 \\
90 \times 15,5 & = & 1395 \text{ at } 1\frac{1}{2} \\
90 \times 20 \times 4 \div 3 & & 2400 \text{ at } 2 \\
90 \times 20 \times 5 \div 3 & & 3000 \text{ at } 2\frac{1}{2}
\end{array}\Bigg\}\text{bricks thick.}$$

The area of the houſe is 6896,5 ſtandard thickneſs.

	feet	l.	feet	l.	l. s. d.
As	272 :	5,5 ::	6896,5 :	139,4512	= 139 : 9 : $0\frac{1}{4}$

the amount. And $6896,5 \div 9 = 766,277$ ſquare yards.

In meaſuring a chimney, the uſual way is, if it ſtands alone without leaning againſt a wall, &c. to gird it about below the mantle, and take that for length, and the height of the room or chimney ſo far as it keeps the ſame girt for the breadth; the product of theſe two is the content: but if it ſtand againſt a wall you muſt meaſure it round to the wall for the girt, and take the height as before. It is cuſtomary in moſt places to allow double meaſure for chimnies. But in Sheffield, and ſeveral other places, they allow one ſhilling a yard runing meaſure for chimney pipes, meaſuring from the hearth to the chimney top.

2. *Of meaſurements by the Square, as Flooring, Partitioning, Roofing, Tyling, &c.*

In meaſuring theſe works, the dimenſions are taken by a rod of 10 feet; and therefore the reſult is in ſquares of 100 feet each. Hence divide the area found by 100, and the quote will be the number of ſquares required.

In flooring they deduct the hearth-stone, except it has a border round it; and then the hearth is measured in with the floor.

In *roofing*, *tyling* and *slating*, it is customary to reckon the area and half area of the flat or floor for the area of the roof; or thus, as 2 is to 3, so is the area of the floor to that of the roof, when the said roof is of a true *pitch*.

Note 1. All roofs are said to be of a true pitch, when the rafters are three-fourths of the breadth of the building.

If the roof is more or less than a true pitch they measure on the ridge, and from the eaves on one side to the eaves on the other with a rod or string.

2. Double measure is commonly allowed for hips, vallies, gutters, &c. And in slating and tyling it is common to allow double measure at the eaves for so much as projects over the plate.

Slating is sometimes measured by the square yard, and sometimes by the rod of 49 yards square, &c.

Exam. 1. Suppose a house of three stories, besides the ground floor, is to be floored at 5l. 10s. per square, that the house measures 30 feet 6 inches, by 24 feet 9 inches; that there are 7 fire places, whose measures are, three, each of $6\frac{1}{2}$ feet by four feet; three other, each of 5 feet 9 inches, by 4 feet 6 inches; and the seventh 6 feet 9 inches, by 5 feet; and that the well hole for the stairs, is 10 feet 6 inches, by $8\frac{1}{2}$ feet; what will the whole come to?

First $6,5 \times 4 \times 3 = 78$ the 1st three fire-places.
$5,75 \times 4,5 \times 3 = 77,625$ the 2d ditto.
$6,75 \times 5 = 33,75$ the 7th fire place.
$10,5 \times 8,5 \times 4 = 357,$ the well hole upon 4 floors.

546,375 the whole of the deduction

And $30,5 \times 24,75 \times 4 = 3019,5$ the area of the 4 floors.
546,375 the area of the deduction

100)2473,125 = area of the work.

24 squares 73,125 feet.

feet l. feet l.
As 100 : 5,5 :: 2473,125 : 136,021875
Or, 136l. $5\frac{1}{4}$d. the Answer.

Ex.

Ex. 2. If the breadth of a roof with the usual allowance at the eaves be 30,5 feet, and the length of the ridge 50 feet, how many yards, squares, and rods are contained in it ?

First 30,5 × 50 = 1525 square feet, or 169 yards 4 feet, or 3 rods, 22 yards, 4 feet, or 15 squares 25 feet.

Exam 3. If a partition between two rooms be 80,5 feet in length, and $12\frac{1}{4}$, in height ; what will the workmanship cost at 6s. 6d. per square ?

First 80,5 × 12,25 = 986,125 square feet, which ÷ed by 100 = 9,86125 squares.

Then as 1 : 6,5s. :: 9.986125 : 64,098125s.

Or 3l. 4s. 1d. the Answ.

Exam. If a house measure within the walls 50 feet in length, and 30,5 in breadth ; and the roof be of a true pitch ; what will it cost roofing at 10s. per square.

First 30,5 × 50 = 1525, the area of the flat, or floor;

Then as 2 : 3 :: 1525 : 2287,5 the area of the roof.

And 2287,5 ÷ 100 = 22,875 squares, which at 10s. per square amounts to 11l. 8s. 9d. the Answ.

There is other work about a building done by the Carpenter, that is measured by the foot runing measure, as cornices, doors and cases, window frames, lintels, guttering, skirt-boards, &c.

3 *Of Measurements by the Yard Square.*

As Joiner's, Painter's, Plaisterer's and Paviour's Work.

In each of these works, it is best to take the dimensions by a decimally divided yard.

Of Joiners Work.

Joiners take their dimensions with a string, measuring round the room or floor for the length ; and for the breadth, they measure from the top of the cornice to the floor, girding the string over all the mouldings and swelling pannels ; for they say they ought to measure where their plane touches.

Note. All ſtuff [illegible] inch and half thick and under, wrought on both ſides, is b[illegible]ners reckoned at work and half work; but ſtuff of greate[illegible]neſs wrought on both ſides, is valued at double work. [illegible]ey make deductions for vacancies that fall within their work; and meaſure window boards, ſofit boards, cheeks, &c. by themſelves.

Exam. 1. What will the wainſcoting a room come to at 5s. per ſquare yard; ſuppoſing the compaſs of the room is 20,52 yards, and the height (taking in the cornice and moulding) 4,09 yards; ſix window ſhutters, each 2,3 yards by 1,3 yards; and the door 2,4 yards by 1,08 yards; the ſhutters and door as they are wrought on both ſides being reckoned work and half?

First $2{,}3 \times 1{,}3 \times 6 = 17{,}94 =$ ſix ſhutters area once.
$1{,}08 \times 2{,}4 = 2{,}592 =$ door's area once.

$2)20{,}532 =$ ſhutters and door.

$10{,}266 =$ half the ſhut. and door.
$20{,}52 \times 4{,}09 = 83{,}9268 = \left\{ \begin{array}{l} \text{rooms area including} \\ \text{the ſhut. \& door once} \end{array} \right.$

$4)94{,}1928 =$ whole work.

The Anſw. L. $23{,}5482 =$ 23l. 10s. $11\frac{1}{2}$d.

Exam. 2. If the window ſhutters about a room, or ſeveral rooms, be 70 feet broad (that is, if all their breadths in one ſum $= 70$ feet) and the height of each ſhutter 6,5 feet; what's the content in yards at work and half? Anſw. 75,8 yards.

Exam. 3. If the compaſs of a room be 96 feet 3 inches, and height 12 feet 9 inches; how many yards of wainſcoting doth it contain? Anſw. 136 yards, 3 ſquare feet.

Of Painters Work.

Painters work is meaſured in the ſame manner as that of Joiners, by a ſtring, becauſe the bruſh goes into the hollows and over the ſwellings: it being reaſonable they

ſhould

should be paid for all places whereon their colour is laid. Their price is generally proportioned to the number of times they lay on their colour.

Note. There must always be made deductions for vacancies within the dimensions taken.

Exam. Suppose the compass of a room is 24,94 yards, and height (taking in the cornice and mouldings) 3,86 yards; the door 2,5 yds. by 1,25 yd; five window shutters, each 2,22 yds. by 1,11 yd.; the window cases 0,38 yd. deep, and 2,66 yds. high; a closet the height of the room, 1,16 yd. deep, 1,58 yd. in front; the shelves taken together, 7,5 yds. by 0,28; the fire place 2,25 yds. by 1,66 yd. (which must be deducted); the shutters, door, and shelves coloured on both sides; what will the whole come to at 8d. per yard?

First $\overline{2,66\times2}+\overline{1,11\times2}=7,54$ the length of both sides, top and bottom of one window case, in one sum: And

$\overline{1,16\times2}+\overline{1,58\times2}=5,48$ the compass of the closet:

Then $7,54\times0,38\times5=14,3260$ area of the 5 wind cases.
$5,48\times3,86 = 21,1528$ area of the closet.
$7,5\times0,28\times2 = 4,2000$ area of the shelves.
$2,22\times1,11\times5=12,3210$ the shutters once.
$2,5\times1,25 = 3,1250$ the door once.
$24,94\times3,86 = 96,2684$ area of the room including the door, shutt. &c.

The sum of the areas $=151,3932$ out of which deduct.
$2,25\times1,66 = 3,7350$ the area of the fire place.

Remainder 147,6582 the area of the work.
Then $147,6582\div30=4,92194$l. $=$ 4l. 18s. $5\frac{1}{4}$d. the Answ.

Plaisterers Work.

When a room is plaistered take its compass in the inside girding in chimnies &c. where ever the work comes: and in taking over head dimensions gird in the summer &c.

Ex. What will the plaistering a room come to at 10d. per

peryard; whose compass is 47,2 yards, and height 4,3 yards; and the top 6,08 yards by 5,22 yards?

First $47,2 \times 4,3 + 6,08 \times 5,22 = 234,6976$ yards, the area; which at 10d. per yard amounts to 9l. 15s. 7d.

Note. In measuring walls, deductions must be made for doors and windows when cased with boards, but if plaistered where the casing should be, no deductions are made: Also, in rendering between quarters, you may deduct one fifth part for the quarters, braces, &c but in whitening and colouring one fourth or fifth part must be added, where the quarters and braces project beyond the wall.

Masons Work.

Masons wo k is sometimes measured bythe foot square, sometimes by the runing foot (viz. only feet in length), and sometimes by the solid foot, taking in all the three dimensions, length, breadth, and thickness. Paving (which is measured by the yard square) is also reckoned in with masons work

Ex. 1. If a wall be 112,25 feet long, and 16,5 fee high, how many rods, at 63 square feet to a rod, doth it contain?

$112,25 \times 16,5 = 1852,125$ feet $\div 63 = 29,4$ rods the Answ.

Ex. 2. How many solid feet are contained in a wall 64,5 feet long, 20,5 feet high, and 2,25 feet thick?

$64,5 \times 20,5 \times 2,25 = 2975,0625$ solid feet the Answ.

Ex. 3. A Pavement being 32 yards long, and 5,58 yards broad, how many square yards doth it contain?

$5,58 \times 32 = 178,56$ yards, the Answ.

Glaziers Work.

Glaziers measure by the square foot, and commonly take their dimensions nearer than any of the foregoing, for they'll go to the 8th or 10th of an inch.

Ex. 1. If there be 8 panes of glass each 2,23 feet high, and 1,28 feet broad, how many square feet?

$2,23 \times 1,28 \times 8 = 22,8352$ square feet, the Answ.

Ex.

Ex. 2. There is a house with three tire of windows, five in a tire; the height of the first is 7,5 feet, of the second 6,3 feet; of the third 5,2 feet; and the breadth of each, 3,32 feet: what will the glazing come to at 15d. per foot?

First $7,5+6,3+5,2=19$ the height of the windows in one row upwards that are over each other, which × by 5, the number of windows in a tire, this gives the length of all the 15 windows: thus $19\times 5=95$ feet, the length of the whole.

Then $95\times 3,32=315,4$ feet the area. And $315,4\div 16 =19l.$ 14s. 3d. the Answ.

§ VII. *Of* LAND SURVEYING.

SURVEYING of Land, or Planometria, is the art of measuring all manner of plain figures, in order to know their superficial content; and requires the practitioner (who would be a complete master of it) to be well skilled in numbers, both whole and broken, with the extraction of Roots, Geometry, and plain Trigonometry: But as this Treatise does not furnish the learner with plain Trigonometry, I refer him to a little Treatise wrote by Mr. Tho. Simpson, price 1s 6d. Those who are desirous of seeing a great variety of Trigonometry both in theory and practice, plain and spherical, may consult Mr. B. Martin's Young Trigonometer's Complete Guide, in two Volumes; price 10s. 6d.

In Surveying Land, the Instruments in common use are the Gunter's Chain (as described in Table 7th, page 23) for measuring of distances; the semicircle, plain table and theodolite for taking the angles; and a protractor, or plain scale, and compasses, for plotting or delineating the dimension of the field on paper, in order to reduce it to triangles, and thereby to find the true superficial *content* or *area*, in *acres*, *roods*, and *poles*.

In Measuring with the Chain, be careful to get the shortest distance between any two objects; otherwise you will make more of the land than there really is. In order hereunto you must provide ten small sticks like arrows

sharp

ſharp at the end to ſtick in the ground. Then having ſet up white marks at the angles of the field, and at other places where you ſee occaſion, let your aſſiſtant that leads the chain, carry all the arrows in his left hand, ſave one in the right with the chain; and when he ſticks down one of his arrows, always be ſure that it be in a right line with the white and the eye of him that follows the chain, who is alſo to gather up the ſticks. Go on thus to the diſtance you are to meaſure, whether it be a diagonal, perpendicular, or the boundary, off ſet &c. By this method he that follows the chain and gathers up the ſticks has a ſure method of counting the number of chains, without any trouble, or fear of error. For, inſtance, you find a diſtance to be 7 chains 56 links; when this is ſet down in your field book, ruled in proper columnes, for that purpoſe, it will ſtand thus 7,56 So if the diſtance is 5 chains 8 links, you enter it thus, 5,08, always minding to ſet a cypher in the place of primes in the links, if they be under ten. The reader, thus furniſhed being well acquainted with Decimal Arithmetic, and with Menſuration of Superficies, as taught in this Teatiſe, is qualified to go into the field, in order to meaſure the ſame.

How to take the Plot of a Field, by the Chain only, and to caſt up the content thereof.

In order hereunto, you muſt provide yourſelf with a ſmall inſtrument called a Croſs, which you may make of wood; thus, take a piece of Box wood turned round, and about 2 or 3 inches diameter: with a fine ſaw cut on the upper ſide two ſlits croſs ways, that ſo they may be exactly at right angles with each other; and on the under ſide of it let there be a hole for the conveniency of placing it upon a ſtaff about your own height in time of practice. Being thus prepared, go into a field, and upon a piece of paper draw a rough draught of it, as near the ſhape as you can. While you are doing this, let your aſſiſtant place pieces of white paper at the angles A, B, C, D, E, F, G, O, H, and I. Then, your aſſiſtant takingthe chain and arrows, ſtand at B and direct him to go towards I; and as you

you go along, by help of your crofs take the perpendicular KC; that is, find fuch a point in the line BI, that the ftaff being pricked down, you can fee thro' the flits the points I, B, and C, without moving your ftaff, which will be the point K; put down the chains and links BK=1,50; leave a mark at K, and meafure on to L; here, by help of your crofs looking at B and A, you will find out the point L, where the perpendicular AL doth fall; fet down the number of chains either from B to L, or from K to L, it matters not which; leave a mark at L, and meafure on to I, and fet down the whole digonal BI=3,75. Go back to L and meafure the perpendicular LA=1,40, and CK=1,78, which put down upon your rough draught. And thus the trapezium ABCI is finifhed, which note with the figure 1.

Secondly. Mark the trapezium IDOH with the figure 2. Then go to D, directing the affiftant towards H, and you will obferve the point N to fall at 3,93 chains, and the point M at 4,77 from D; where leave marks, and meafuring on to H you will find the whole diagonal DH =5,78 chains; which infert in your rough draught as before. Then meafuring the perpendicular MI, you will find it to be 1,80 and NO=1,09.

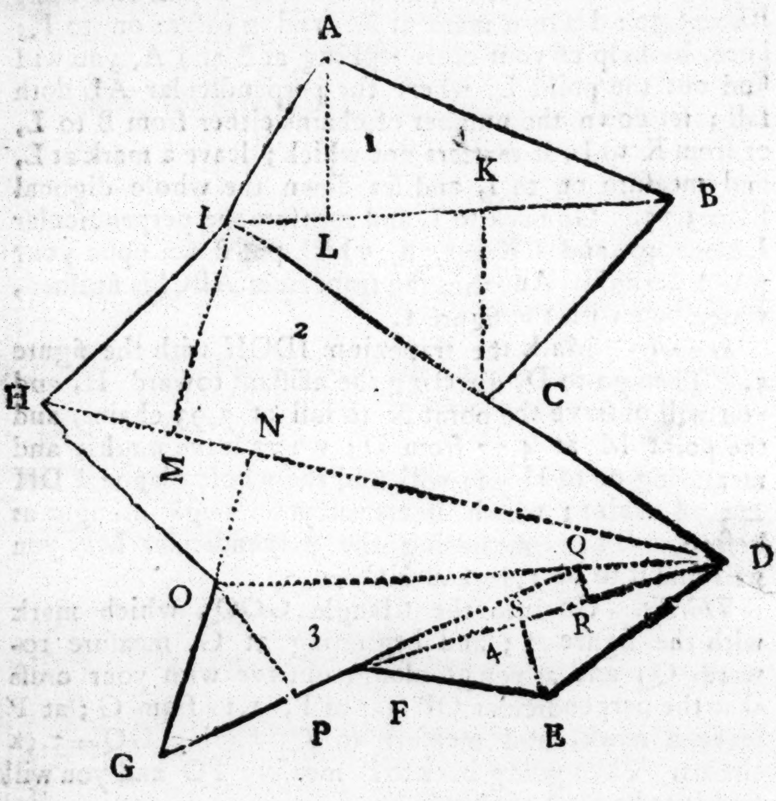
Thirdly. Go into the triangle GOQ, which mark with the figure 3; and beginning at G, meafure towards Q, and as you go along, obferve with your crofs that the perpendicular OP falls at P, 1,11 from G; at P leave a mark, and meafure to Q, finding GQ=3,54 chains. Then going back to P meafure PO and you will find it to be 1,08 chains.

Laftly. There ftill remains the Trapezium FQDE unmeafured, which note with the figure 4. Then coming to F, meafure towards D, and you will fee the perpendicular SE to fall at 1,43 chains from F, which note down in your rough draught as before; meafuring on you will find the perpendicular RQ to fall at 1,72 chains from F, which alfo note down: And meafuring to D, you will find FD =3,23 chains which alfo note down. Then coming back to the marks which you left at R and S, meafure SE and you will find it = 0,60 links, and

QR

QR =0,35 links, which set down in your field draught, and the measuring is all finished.

Note. You must remember, when in the field, to measure ID and DO, they being of use in plotting, but not in casting up of the dimensions.



For the Reader's better information, I shall here insert each particular measurement in chains and links.

	ch. links		ch links		ch links.
BK =	1, 50	DH =	5, 78	FR =	1, 72
KL =	1, 22	MI =	1, 80	FD =	3, 23
BI =	3, 75	NO =	1, 09	SE =	0, 60
AL =	1, 40	GP =	1, 11	QR =	0, 35
CK =	1, 78	GQ =	3, 54	ID =	4, 94
DN =	3, 93	PO =	1, 08	DO =	4, 15
DM =	4, 77	FS =	1, 43		

To

To plot the same. When you have finished the work in the field, you may plot and cast it up at your leisure, thus.

Take a clean piece of paper, and with a black lead pencil draw a line at pleasure. Take 1 chain 50 links from your scale and lay it from B to K; take 1,22 and lay it from K to L, and 3,75 chains from B to I. This done, take your protractor and lay the center to K; prick off the perpendicular CK, and thereon lay 1,78; lay the center of the protractor to L, and prick off the perpendicular LA, and thereon lay 1,40 chains. Draw an occult line, and thereon set 5,78 from D to H; take 3,93 chains and set them from D to M; lay the center of the protractor to M and N severally, and prick off the perpendiculars MI and NO; take 1,80 chains and set them from M to I, and 1,09 from N to O; do thus by the rest of the bases and perpendiculars; and then by drawing the boundary lines AB, BC, CD, &c. you will have the true plot, or figure of your field as is represented above.

Lastly. To cast up the dimensions, find the area of each figure, and their sum will be the content of the whole.

$$\frac{\overline{AL+CK}\times BI}{2}=\frac{\overline{140+178}\times 375}{2}=59625=1 \text{ trapez.}$$

$$\frac{\overline{IM+ON}\times HD}{2}=\frac{\overline{180+109}\times 578}{2}=83521=2 \text{ trapez.}$$

$$\frac{GQ\times PO}{2}=\frac{354\times 108}{2}=19116=3 \text{ triangle.}$$

$$\frac{\overline{SE+QR}\times FD}{2}=\frac{\overline{0,60+0,35}\times 3,23}{2}=15342=4 \text{ trapez.}$$

acre	1,77604	= the area.
	4	
roods	3,10416	
	40	
poles	4,16640	

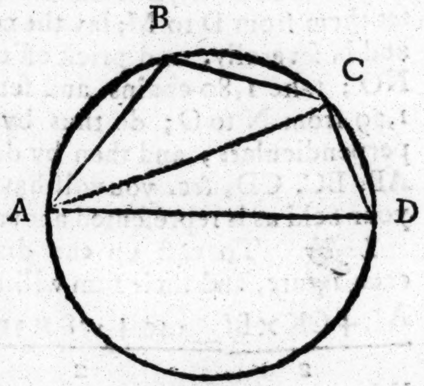
	A.	R.	P.
Answ.	1	3	4

QUESTION.

One morning in May, I went to survey,
As soon as bright Sol I espy'd;

I measured round a four corner'd ground;	Chains.
I'th Margin's the length of each side:	AB=15,60
The Angle at B, together with D,	BC=13,20
An hundred and eighty degrees,	CD=10,00
The Meadow's content, is all that I want,	DA=26 00
Assist me kind Sirs if you please.	

Solution. Because the angles B and D taken together are equal to two right angles (180 degrees), the longest side of this figure must of course be the diameter of a circle; and the other three sides drawn within the semi-circle will exactly complete the quadrangle. Now by the 31st proposition of Euclid's third Book, two right lines drawn from the extremities of the diameter meeting each other any where in the periphery of a circle, always form a right angle: Therefore by drawing the line AC to meet the line CD in the point C, the angle ACD will be a right angle.



Hence in the right angled triangle ACD, the Hypothenuse AD, and the leg CD are given, to find the leg AC; then $\frac{1}{2}$ AC × CD = the area of the triangle ACD. Thus AC = $\sqrt{26 \times 26 - 10 \times 10} = 24$; then $24 \times 10 \div 2 = 120$, the area of the triangle ACD. Now in the triangle ABC, there are given all the three sides to find the area, thus, 15,6+13,2+24=52, 8 the sum, $\frac{1}{2}$ sum=26,4, and 26,4—24=2,4 the 1st diff 26,4—13,2=13,2 the 2d diff. 26,4—15,6=10,8 the 3d diff then 26,4×13,2×2,4×10,8=9032,6016; whose square root is 95,4 square chains the area of the triangle ABC, and 120+95,4=215,4 square chains = 21 acres, 2 roods, 6,4 poles, the area required.

F I N I S.